Discrete element simulations of dense granular matter: from techniques to data analysis

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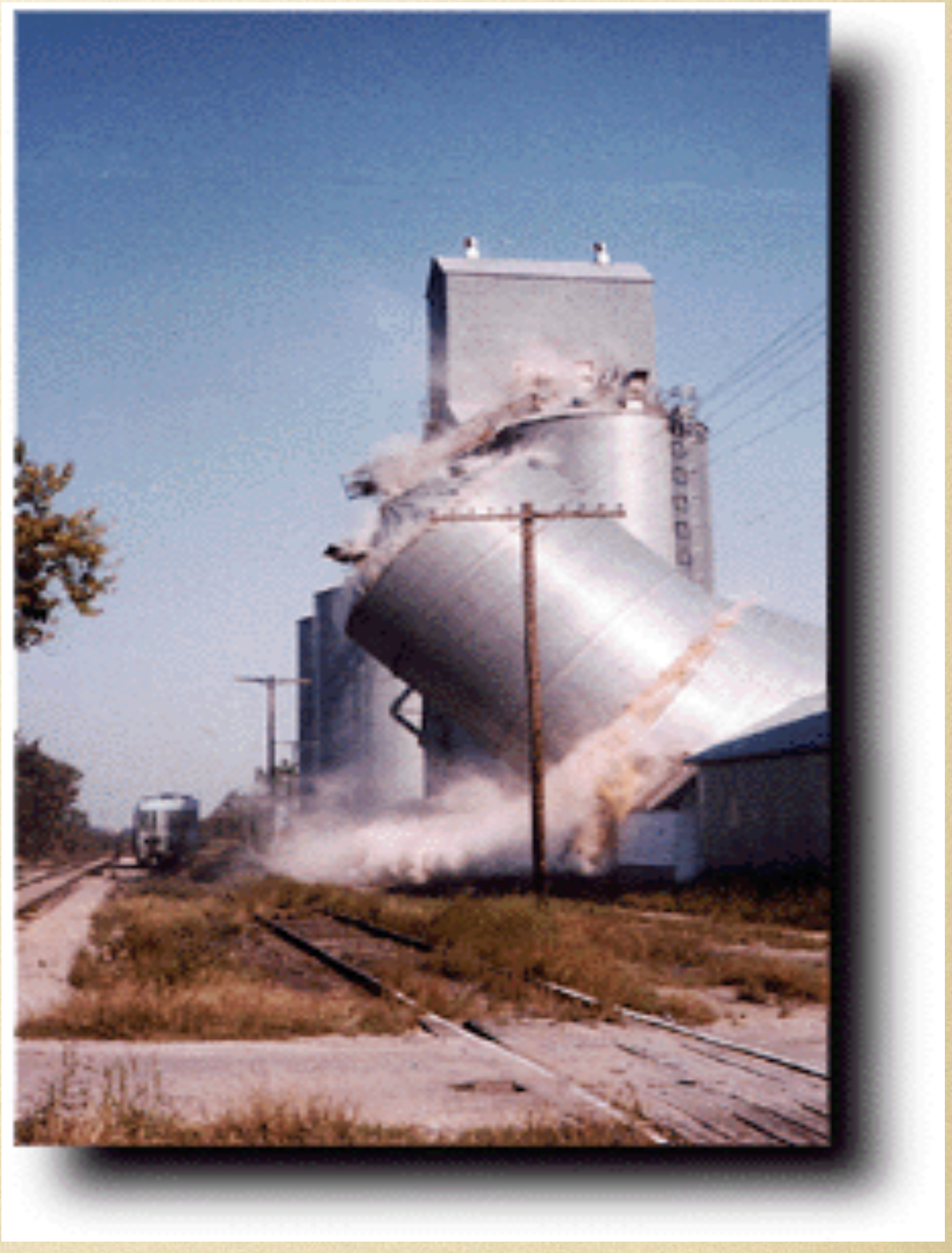
Some recent granular projects

- Relevance of energy fluctuations in dense granular matter: granular temperature and out-of-equilibrium statistical physics
- Nonlinear/linear wave propagation: 1D/2D/3D; from solitary to plane waves
- influence of microstructure on impact mechanics
- topological properties of the force chains

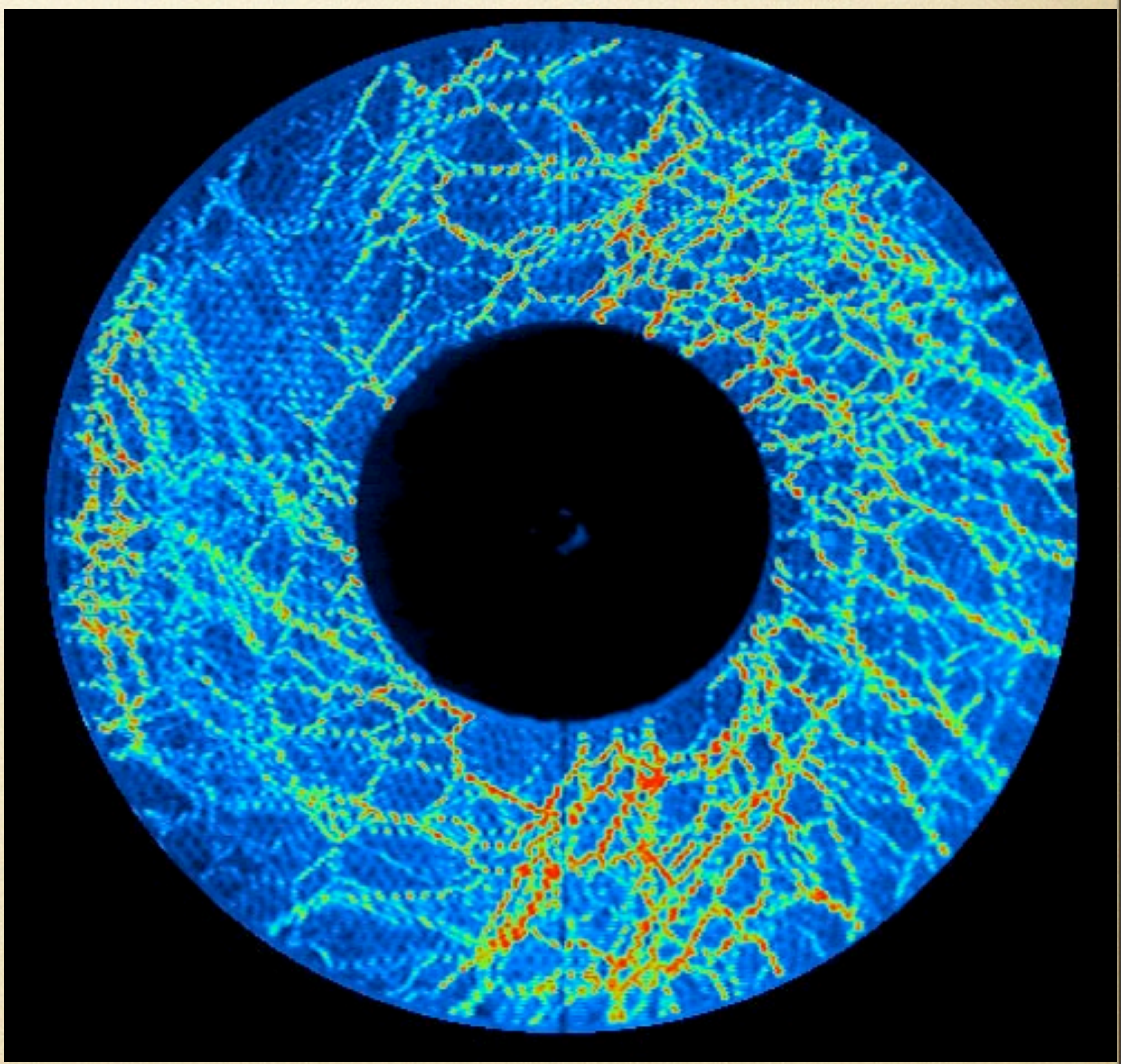
Why are granular systems so different?

- We do not understand how forces propagate through granular system
  - *Therefore we do not know how to build good silos!*
- To understand better how forces propagate consider simple systems and simple geometries



Experiment: 2D Angular Couette flow

- Experiments with photoelastic discs
- Colors show the forces which particles experience
- Note extreme nonuniformity of force field
- Behavior very different from `usual' materials



Overview

- Discrete element method (DEM)
  - Discuss different approaches towards modeling of the interaction of granular particles
- Data analysis
  - Extracting useful information from simulations
- Recent applications involving force and energy propagation, impact, and spatial structure of the relevant fields
- Important: interdisciplinary nature of the research in the field involving experiments, simulations, analytical work carried out by physicists, engineers, mathematicians
  - requires developing common `language' and joint approach to consider complex problems

DEM Techniques

- Level of complexity of simulation techniques
  - `Event driven' simulations
    - particles are considered as infinitely hard spheres/disks interacting infinitely fast
      - advantages: efficient, since no time stepping is involved
      - disadvantages: `technical' problems with inelastic collapse and difficulties when volume fraction (the part of the space occupied by granular particles) becomes large (review: Herrmann &Luding, Continuum Mech. Thermodyn. (1998) 10: 189–231)
  - `Soft' particle simulations
    - particles are considered as (elastic) objects interacting during finite time with the particles in their surrounding
      - advantages: applicable to dense systems and enduring contacts
      - disadvantages: computationally expensive, requiring small time step, since each interaction/collision has to be resolved
      - require careful simulation techniques in order to be able to simulate large systems for long times (General reviews: Allen & Tildesley, Computer Simulation of Liquids; Haile: Molecular Dynamics Simulation)

DEM Models

- Level of complexity of interaction models
  - spherical, elastic, frictionless particles interacting infinitely fast only when in contact
    - relatively easy to implement, can be connected to continuum fluid-mechanics like theories
    - certain part of physics is lost...
  - spherical particles with inelasticity and friction interacting with repulsive or attractive interactions when in contact
    - relatively easy to implement
    - typically use relatively simple force interaction laws
  - More complex approaches:
    - resolving details of individual contacts (linear/nonlinear elasticity theory) (see Johnson, Contact Mechanics)
    - aspherical particles
    - long range interactions
    - ...

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DEM Software

- `In-house' software developed by individual researchers
  - typically using simplified interaction models and relatively small system sizes
  - simplicity allows for fine-tuning and flexibility (no black-box effect)
- Publicly available software packages
  - LAMMPS, NAMD, ...
  - professionally written, complex interaction methods
  - efficient, allowing for simulations of large systems
  - may be extremely complex; certain lack of flexibility
- Commercial Software

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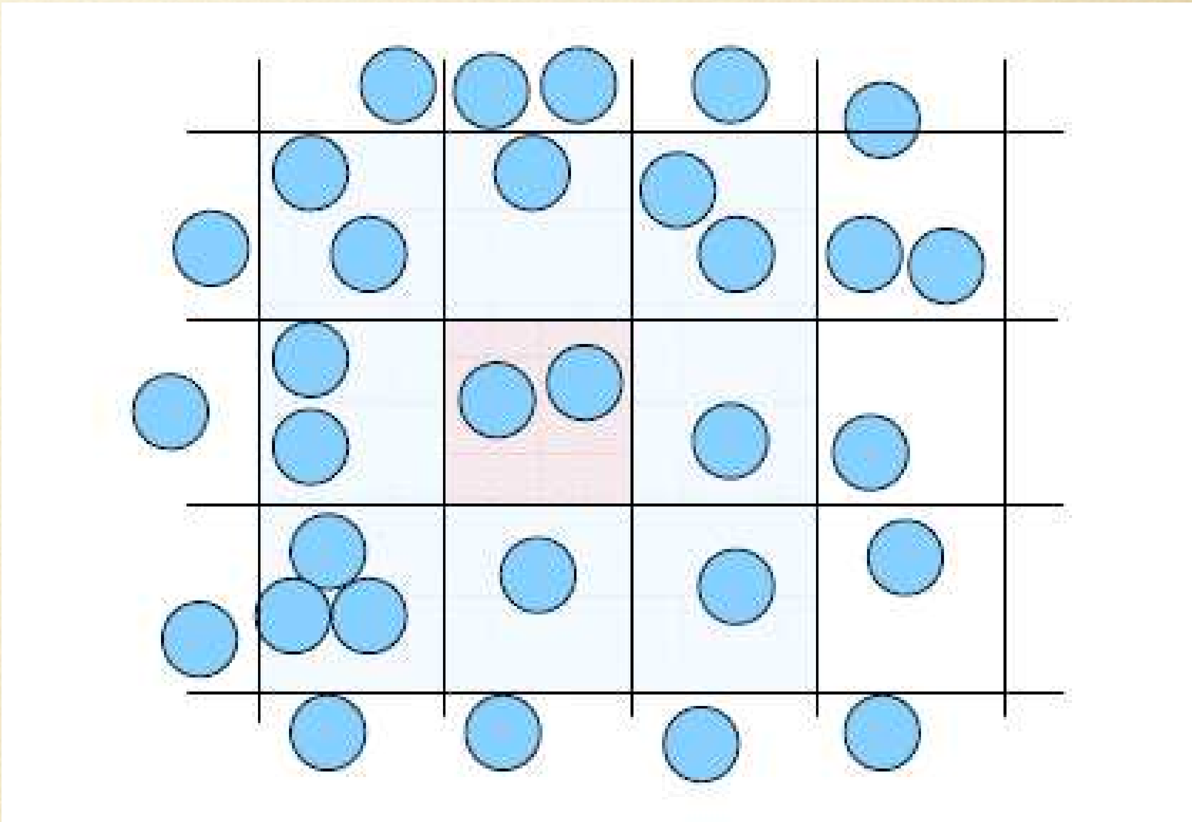
Computational Techniques

- Serial versus parallel
- parallelization becomes more and more important due to necessity to simulate large systems
  - multi-cpu and multi-core programming
  - Open MPI, Open MP, distributed and shared memory machines...
  - trade-off between speed and (human) effort involved

Implementation

- Split the system into cells
- Make the list of particles in each cell
- check for interactions between particles in a given cell and each neighbor cell
- if there is an interaction, calculate the interaction forces
- update the positions, velocities, acceleration of particles
- recalculate the lists as appropriate



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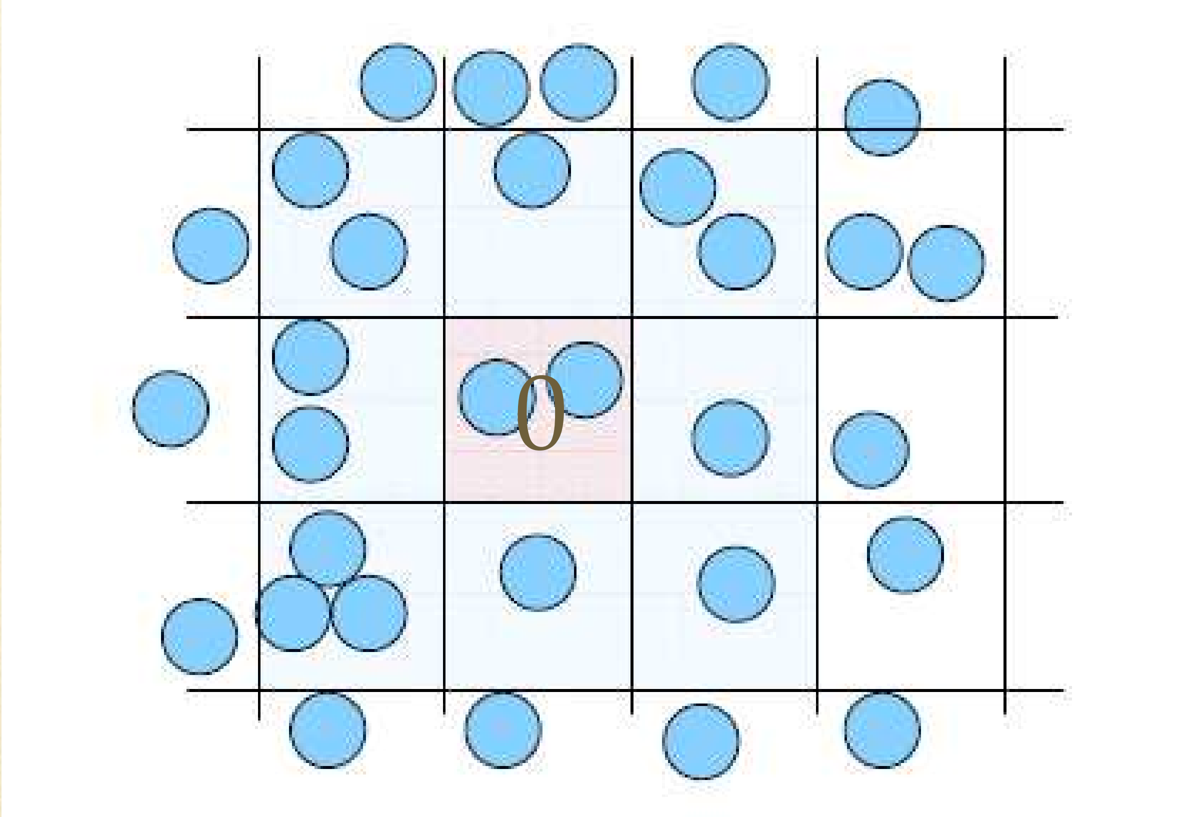

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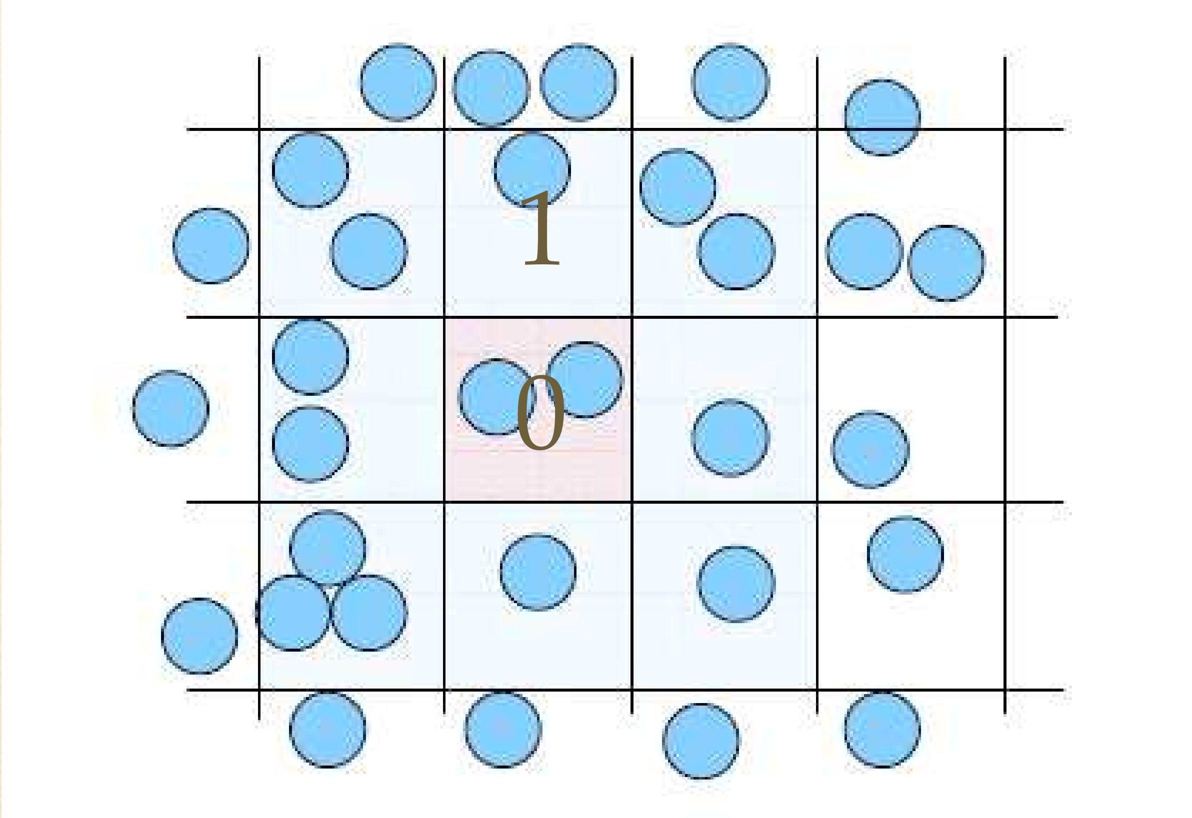

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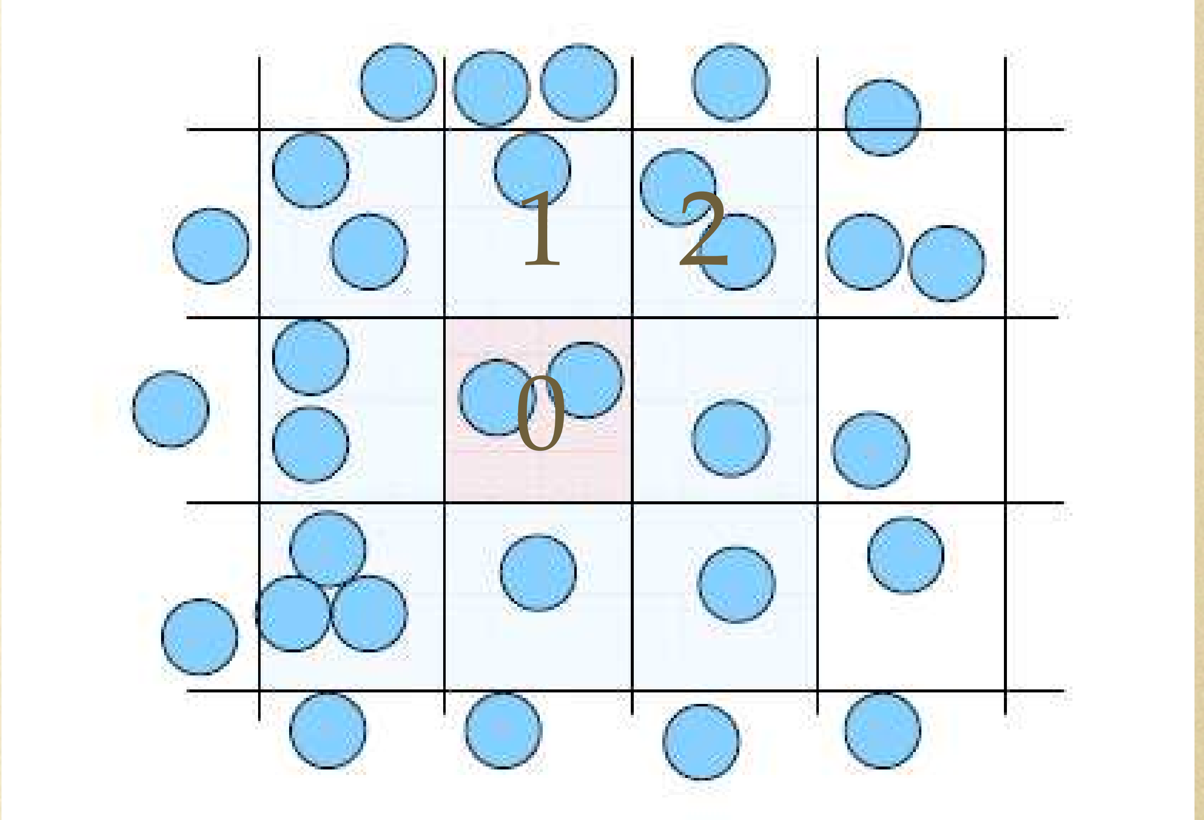

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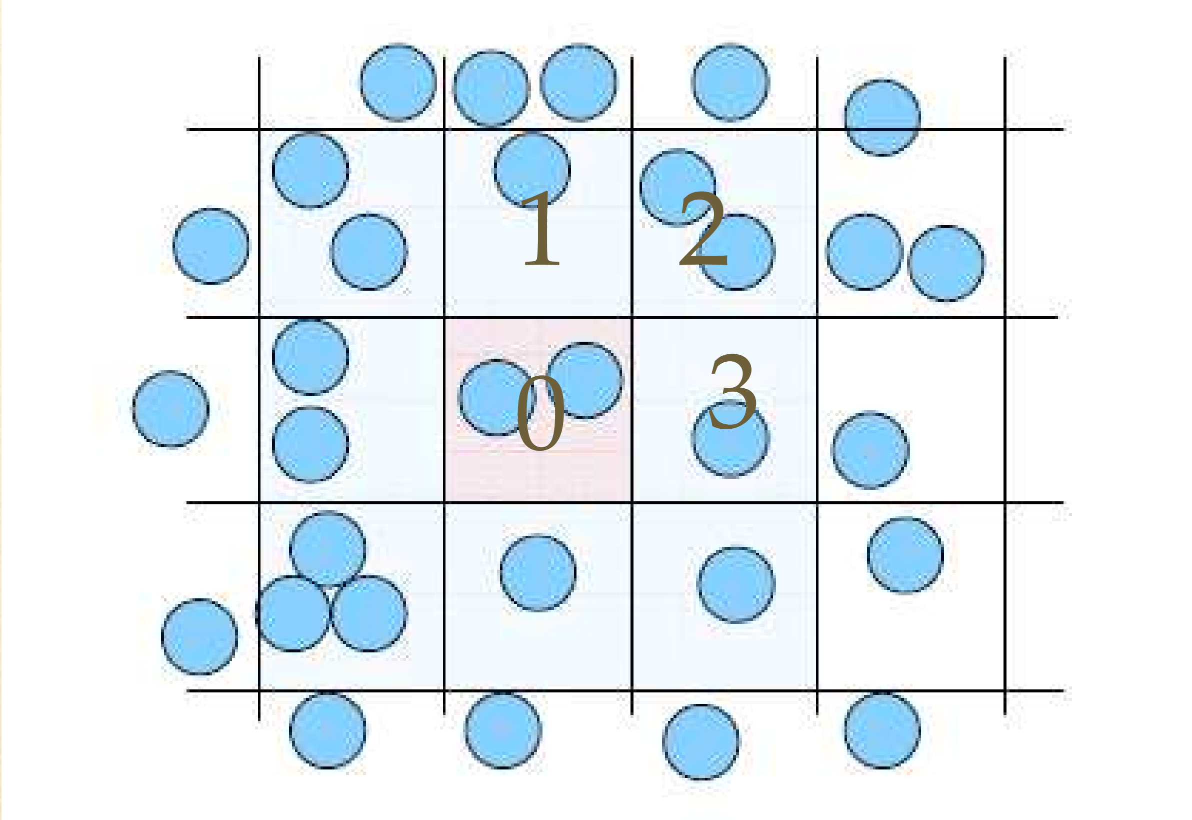

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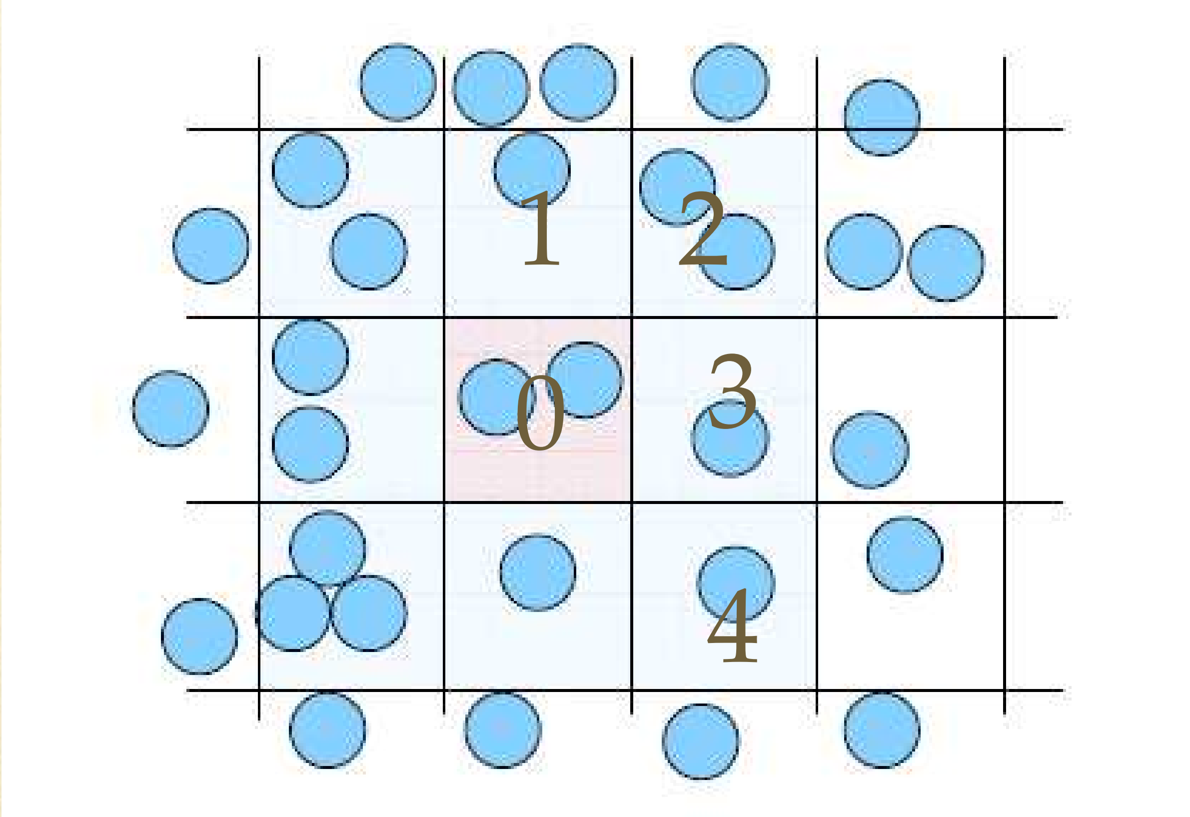

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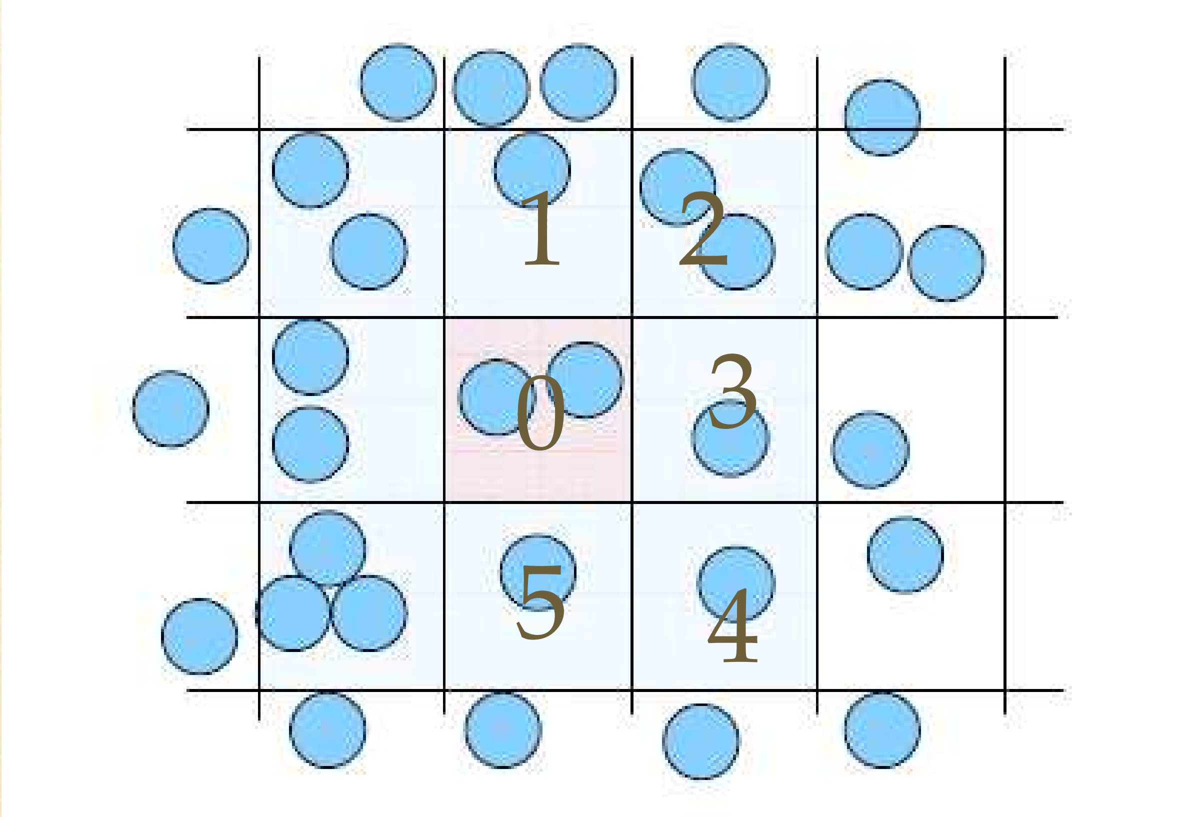

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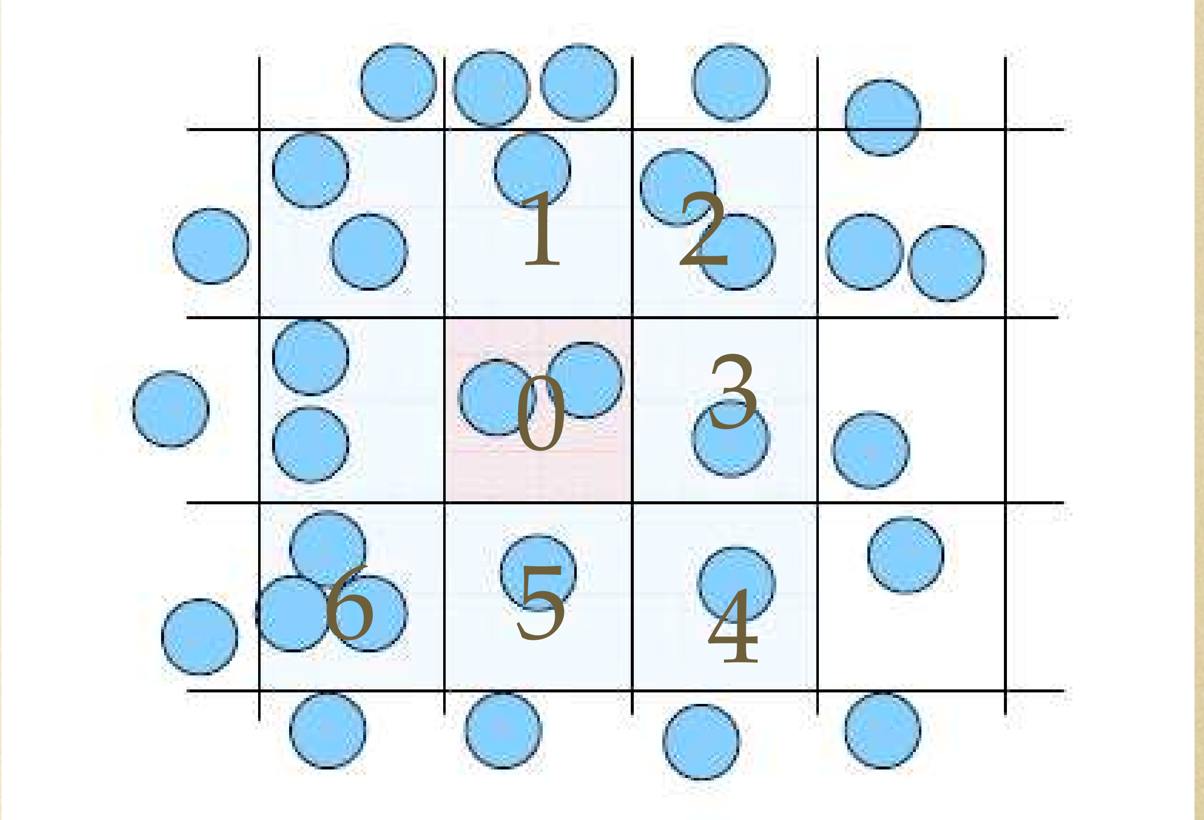

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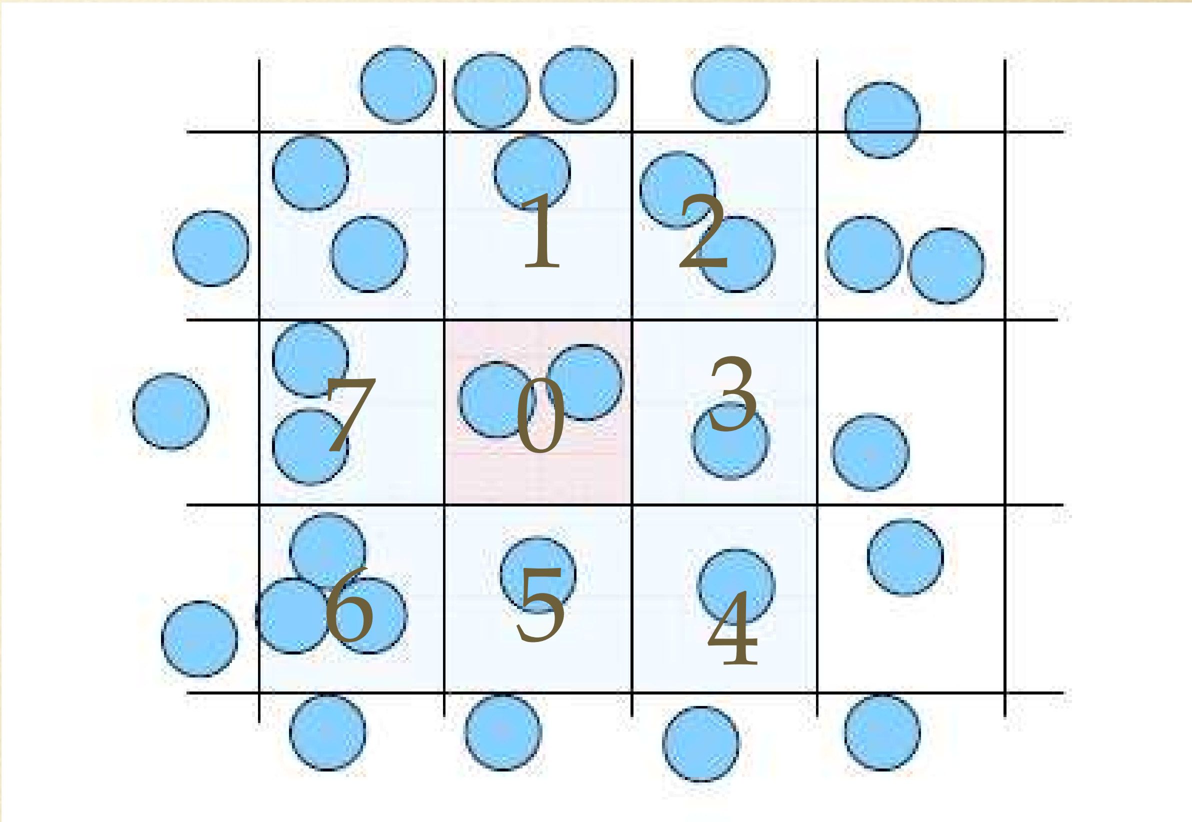

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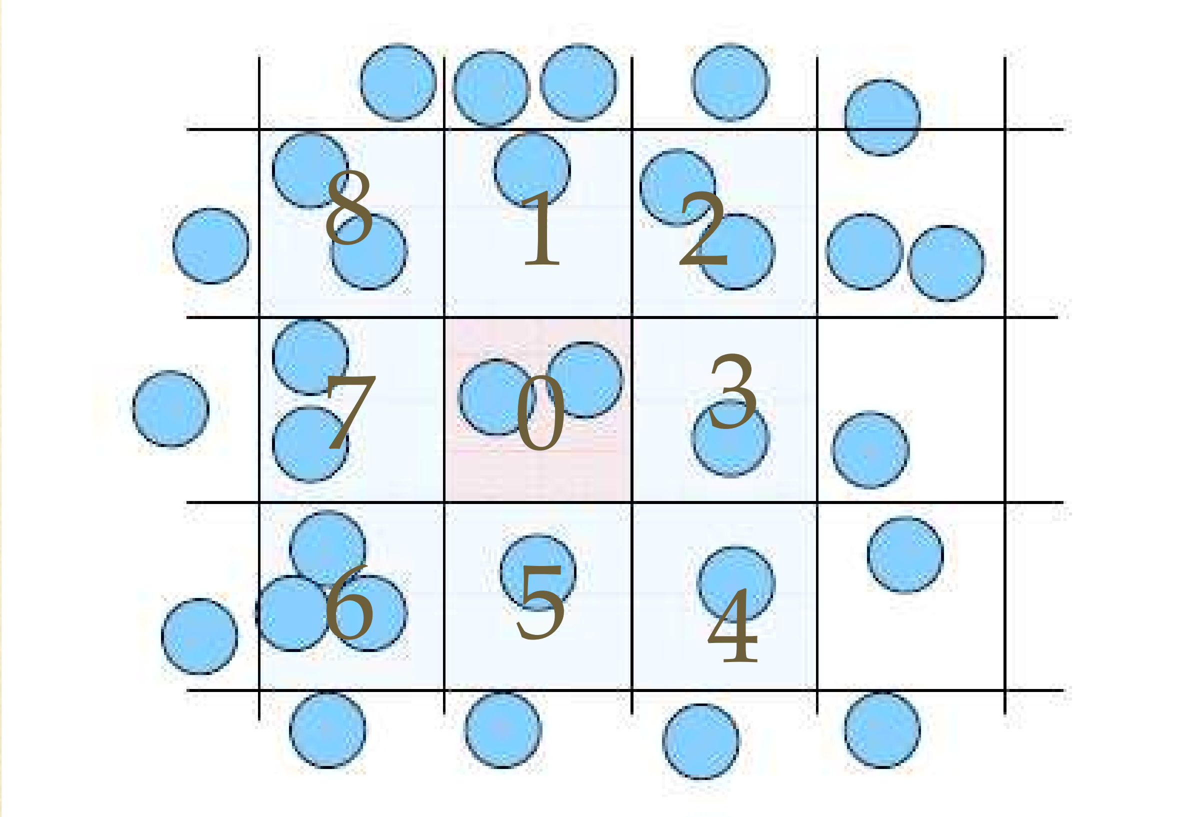

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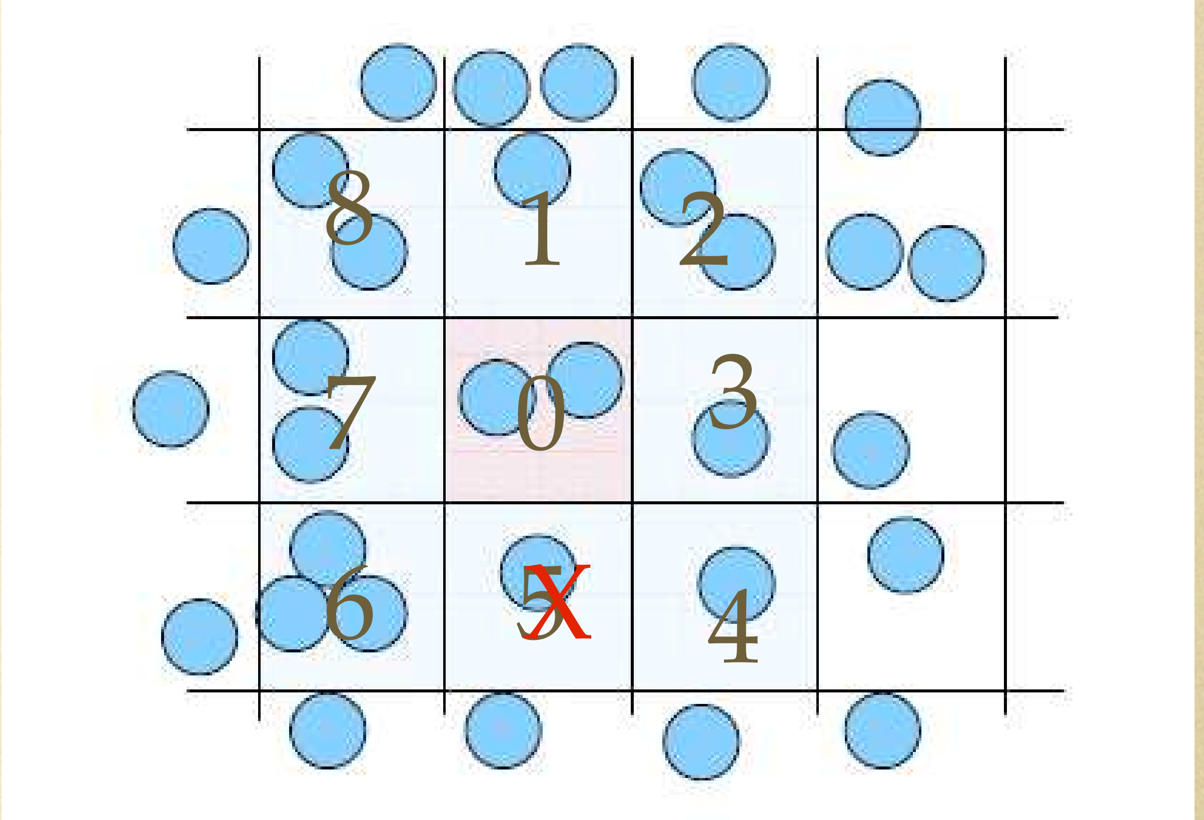

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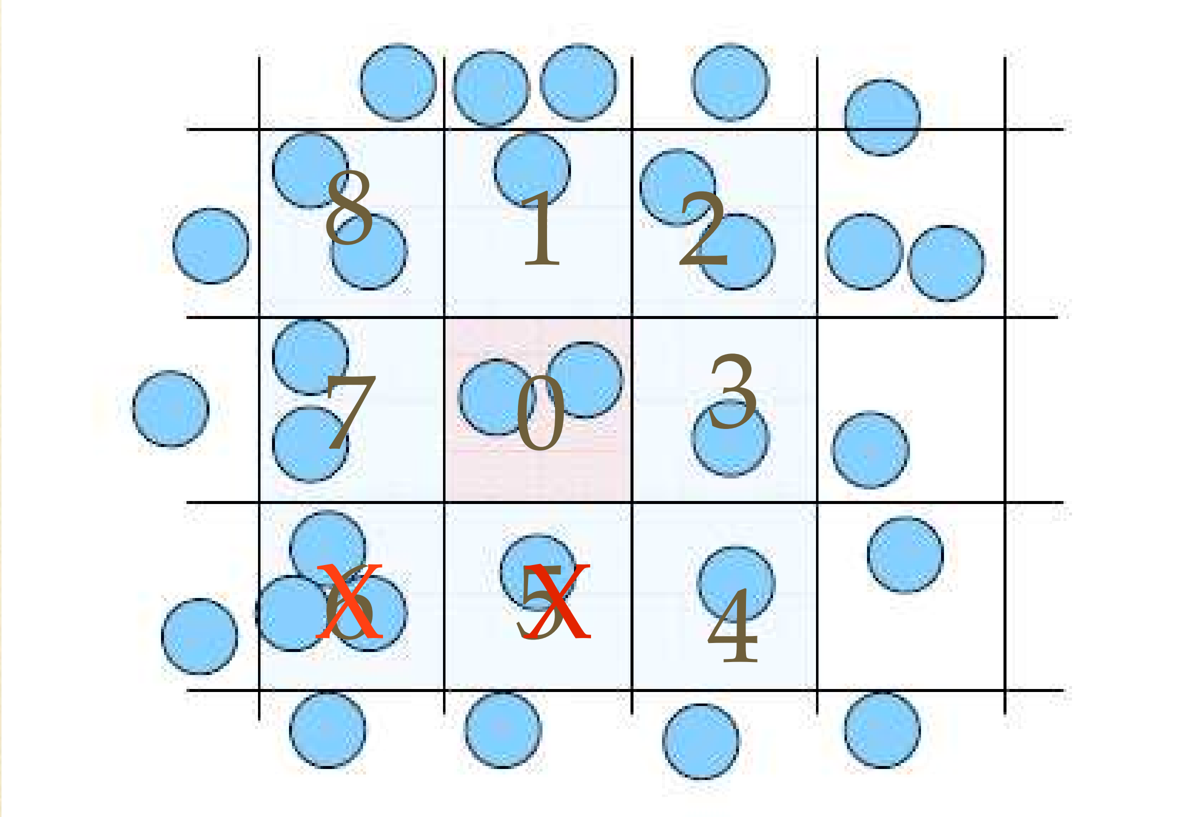

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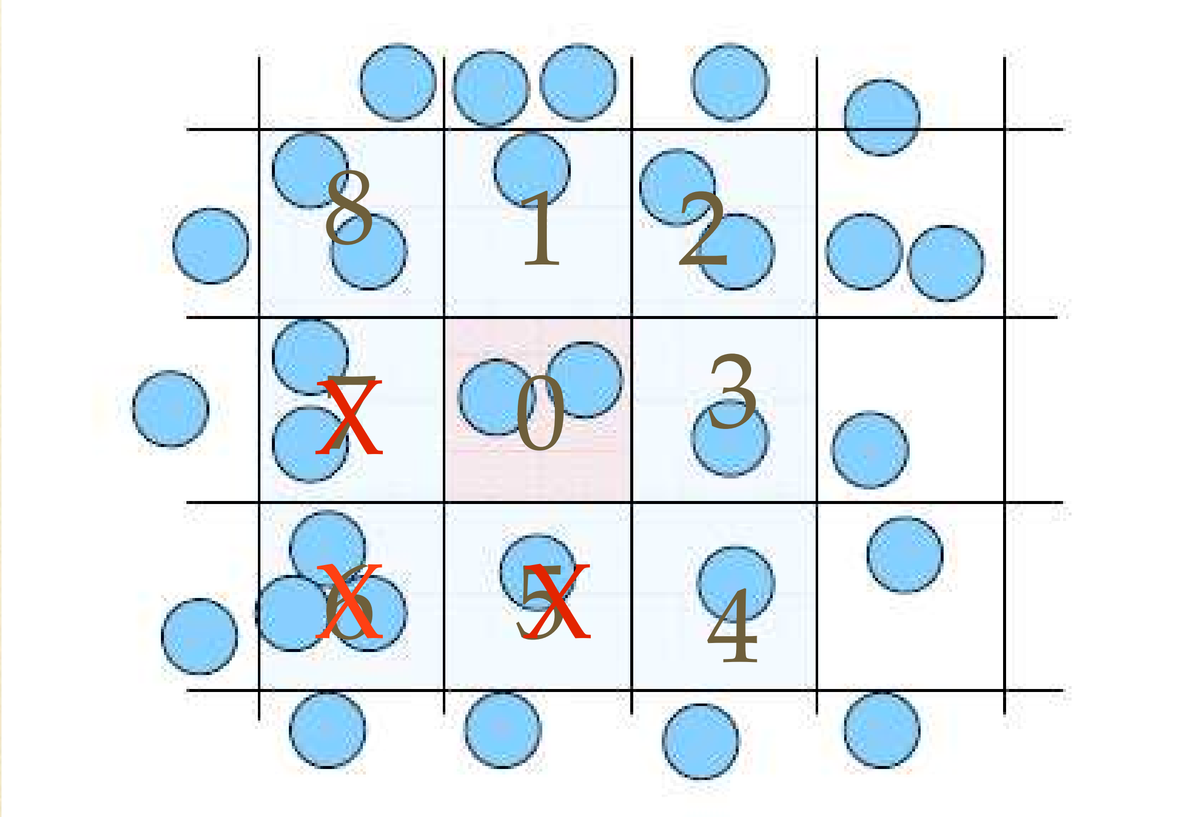

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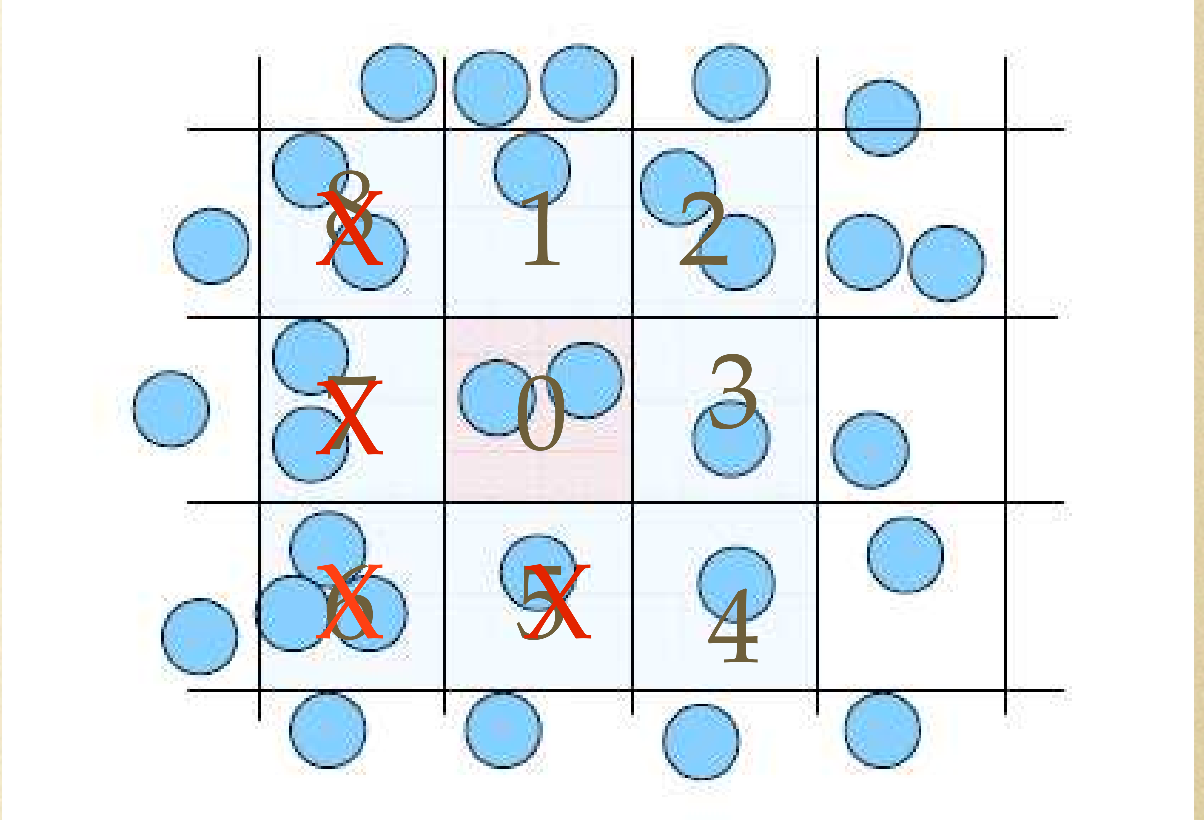
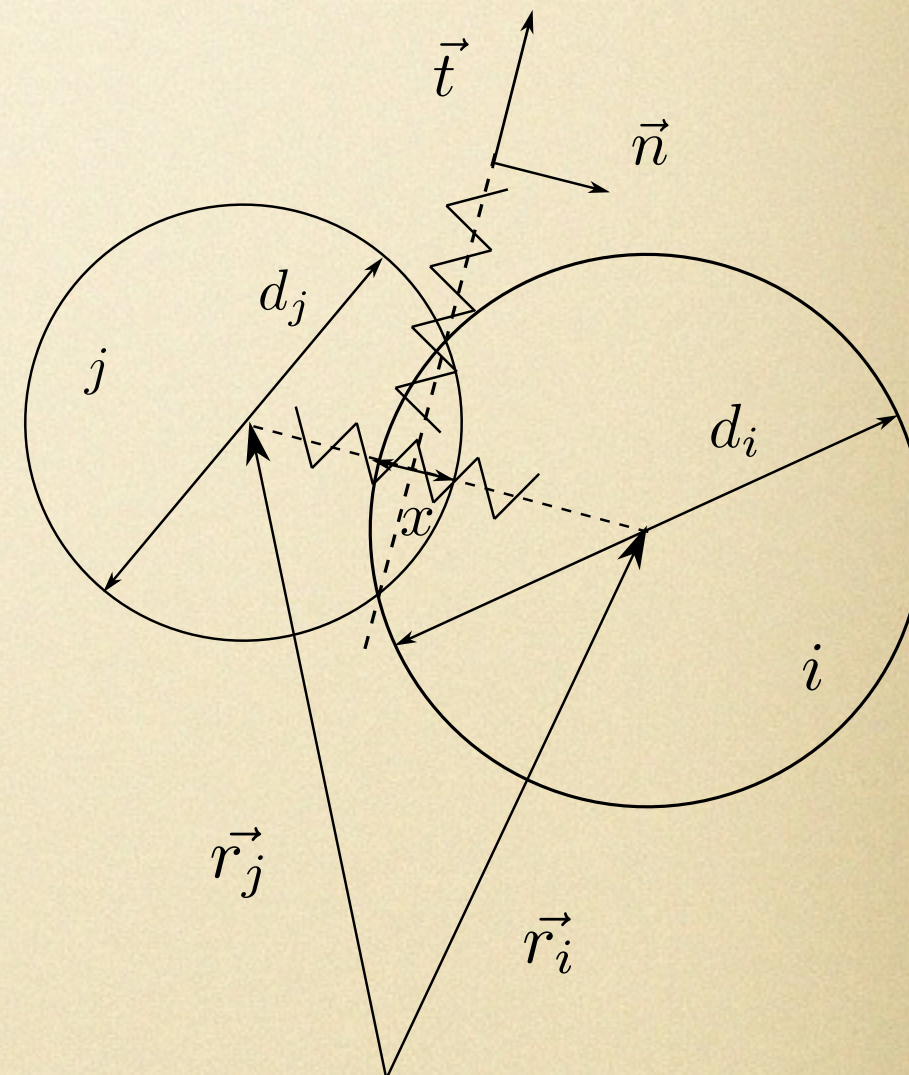


Equations

$$m_i \frac{d^2\mathbf{r}_i}{dt^2} = m_i\mathbf{g} + \mathbf{F}_{i,j}^{n}$$

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = -\frac{1}{2} d_i \mathbf{n}_i \times \mathbf{F}_{i,j}^{t}$$

$$\mathbf{n}_i = \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$



Forces

- Normal: linear force model

$$\mathbf{F}_{i,j}^{n} = k_n x \mathbf{n}_i - \gamma_n \bar{m} \mathbf{v}_{i,j}^{n}$$

$$\mathbf{v}_{i,j} = (\mathbf{v}_i - \mathbf{v}_j) - (\omega_i d_i/2 + \omega_j d_j/2)\mathbf{e_z} \times \mathbf{n}_i$$

$$\bar{m} = \frac{m_i m_j}{m_i + m_j}$$

γ_n - damping: proportional to the coefficient of restitution

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Connection of material properties and DEM parameters: see Schaffer, Dippel, Wolf, J. Phys. I France, 6, 5 (1996); Latzel, PhD Thesis, U. Stuttgart (2003); Kondic, Phys. Rev. E, 60, 751 (1999)

Forces

- Tangential:
- Cundall-Strack type of model - Geotechnique, 29, 47 (1979)
- Model I: kinetic friction (active only in the presence of relative velocity)
- Model II: includes static friction

$$F^n_{i,j} = \left[k_n x - \gamma_n \bar{m} \mathbf{v}^n_{i,j}\right]$$

$$F^t_{i,j} = -min(\gamma_s \bar{m} |v^t_{vel}|, \mu_s |F^n_{i,j}|)$$

model I

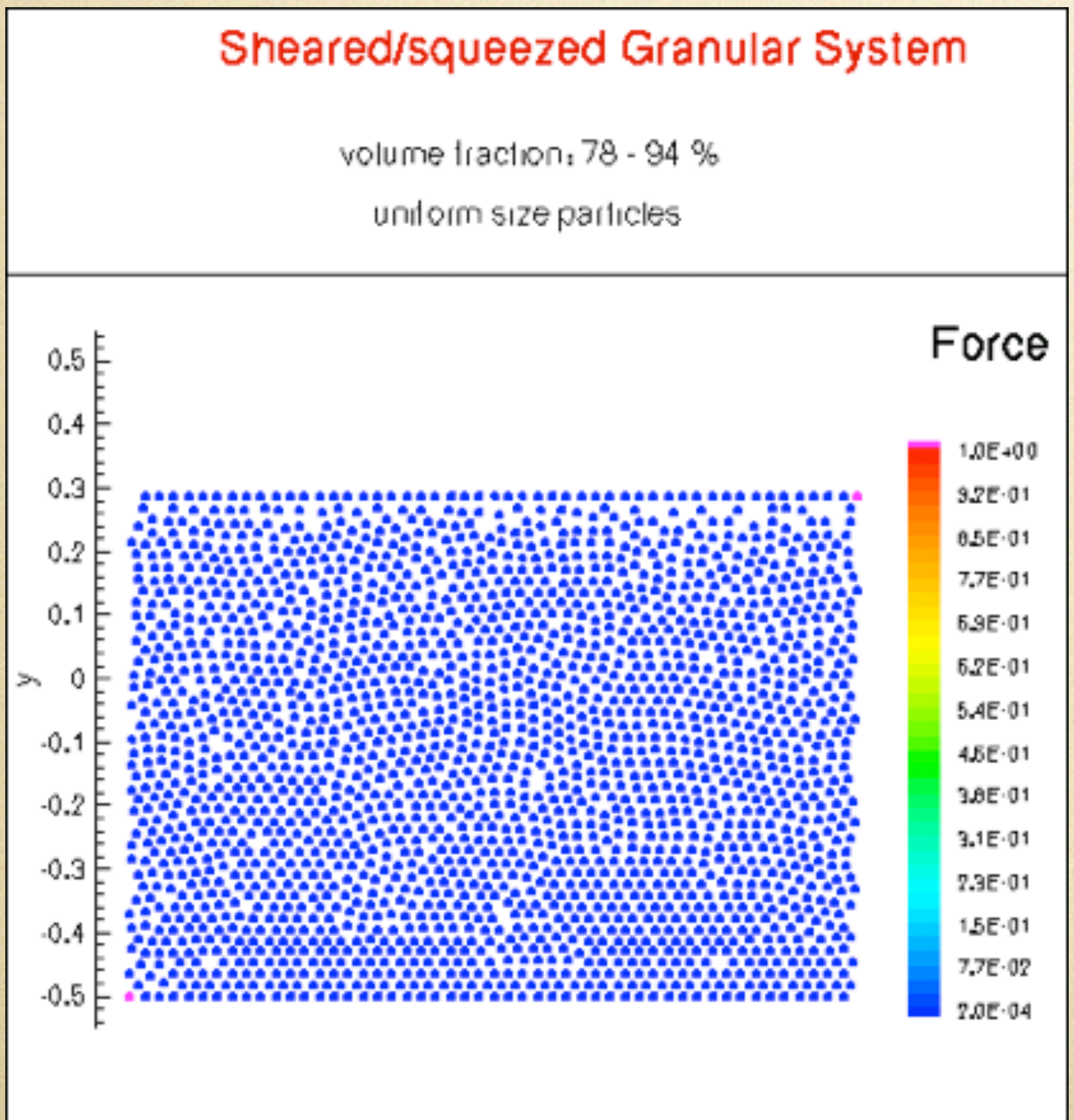
model II

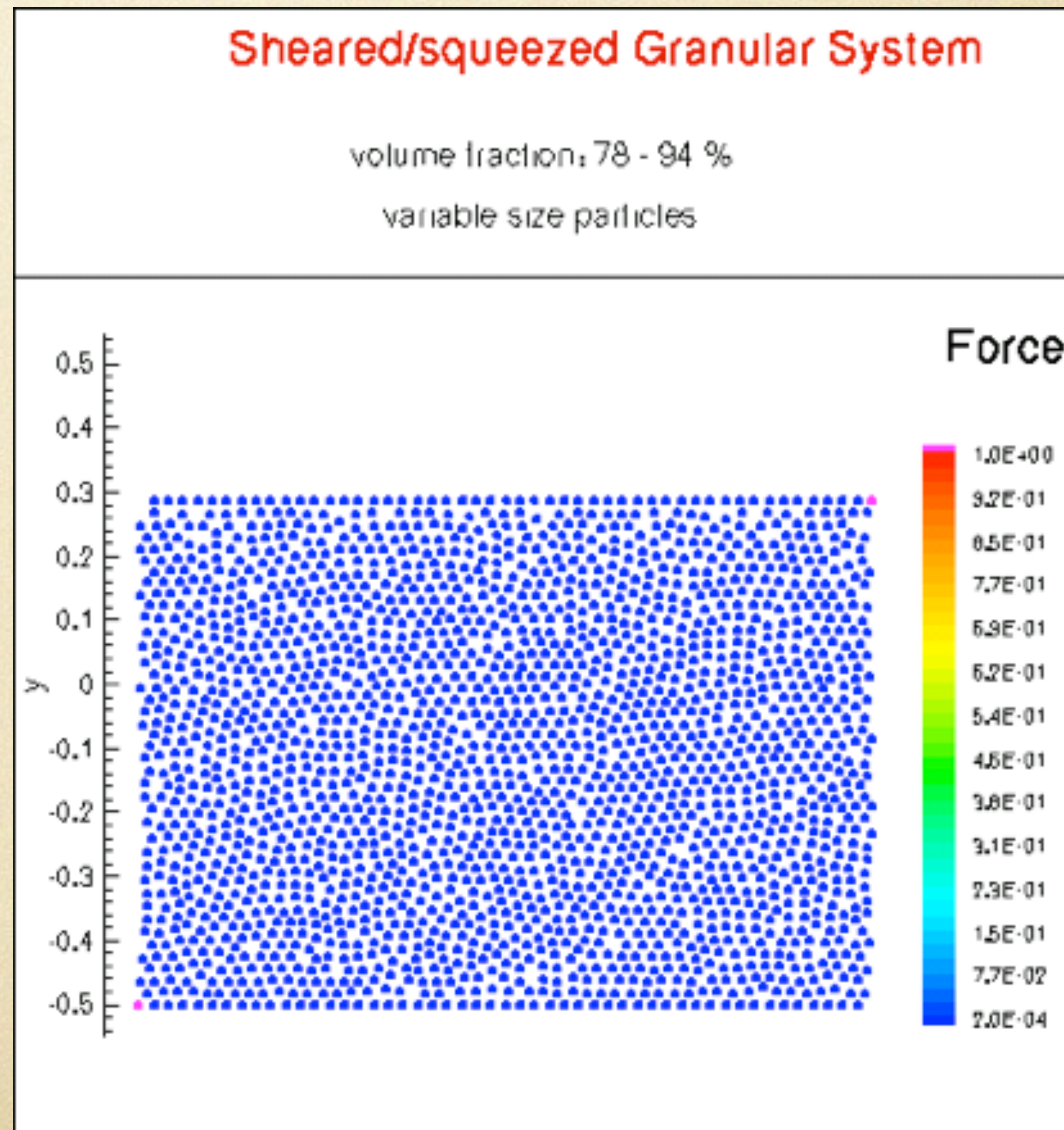
$$\mathrm{F}^t_{i,j} = -min(\mu_s |F^n|, k_t \xi) \frac{\xi}{|\xi|}$$

$$\xi = \left(\int_{t_0}^{t} \mathbf{v}^t_{i,j}\ (s)\ ds\right) \cdot \vec{t}$$

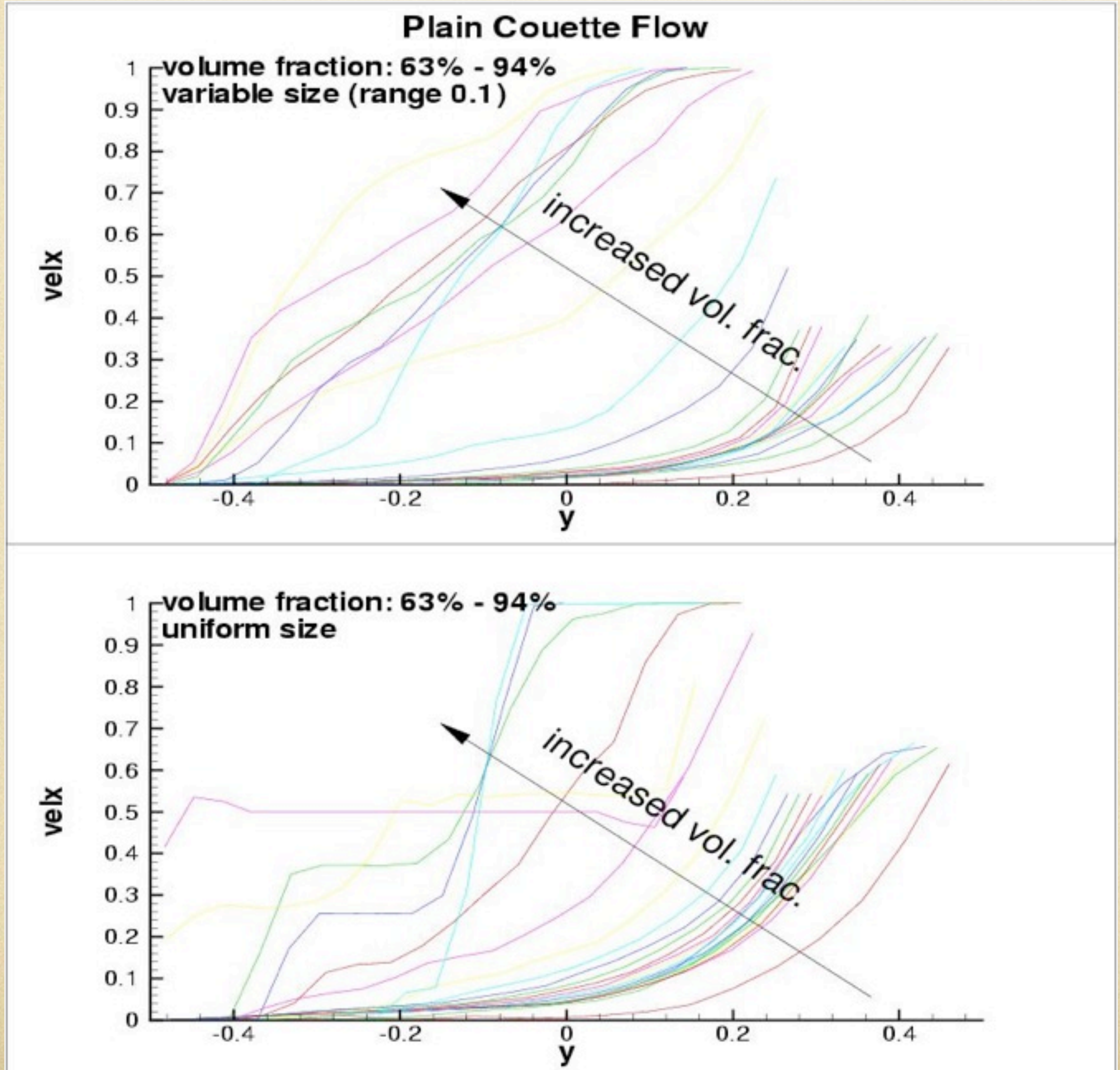
Examples

- Sheared granular systems of varied volume fraction - monodisperse and polydisperse





Velocity profiles

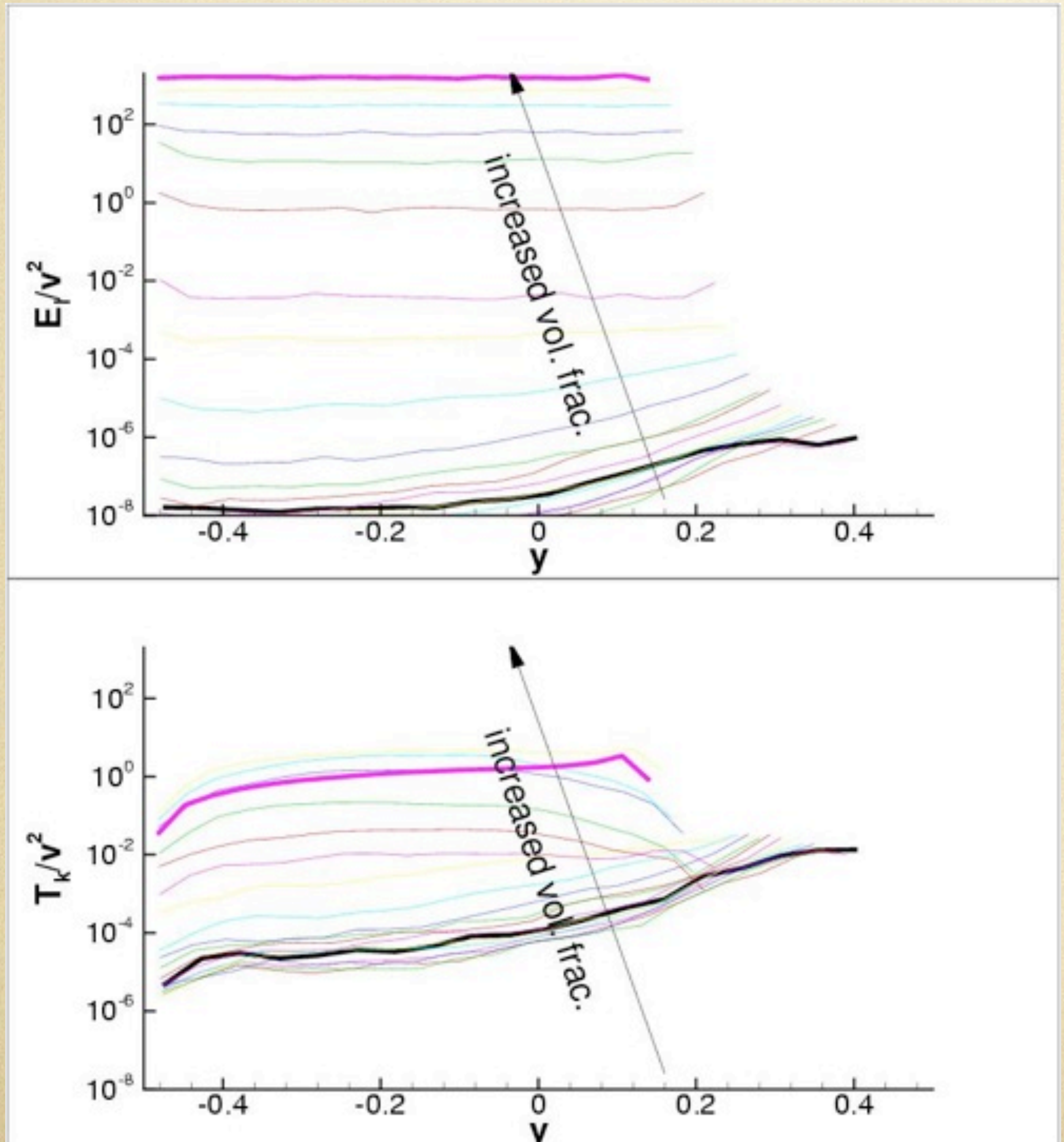


How do granular particles flow?

- Low volume fractions: exponential, shear banded velocity profiles
  - Predicted and explained by kinetic theory
- Large volume fractions
  - Monodisperse systems: fracture, formation of crystalline zones; temporarily and spatially nonuniform shear
  - Polydisperse systems: reasonably linear velocity profiles, similar as in Newtonian fluids (Xu, O'Hern, Kondic, PRL '05, PRE '05)

Energy Balance

- Note change of dominant energy from kinetic to elastic as system is compressed



E_l elastic energy

T_k kinetic granular temperature

y distance from the shearing wall

shearing wall: right
bottom wall: left

About temperatures and other averaged quantities

- Kinetic granular temperature for dense granular systems is irrelevant from the energetic point of view. Can we come up with an alternative concept?

- Application of out-of-equilbrium statistical mechanics concepts to granular matter: Berthier and Barrat J. Chem. Phys. '02; Ono etal PRL '02, O'Hern etal, PRL '04, '05

- Here: consider inclusion of fluctuations of elastic energy (compression) to define `generalized' temperature (Kondic, Behringer, EPL '04)

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$$T_g = T_k + T_e = \frac{m}{2}\left[< v^2 > - < v >^2\right] + \frac{k}{2}\left[< x^2 > - < x >^2\right]$$

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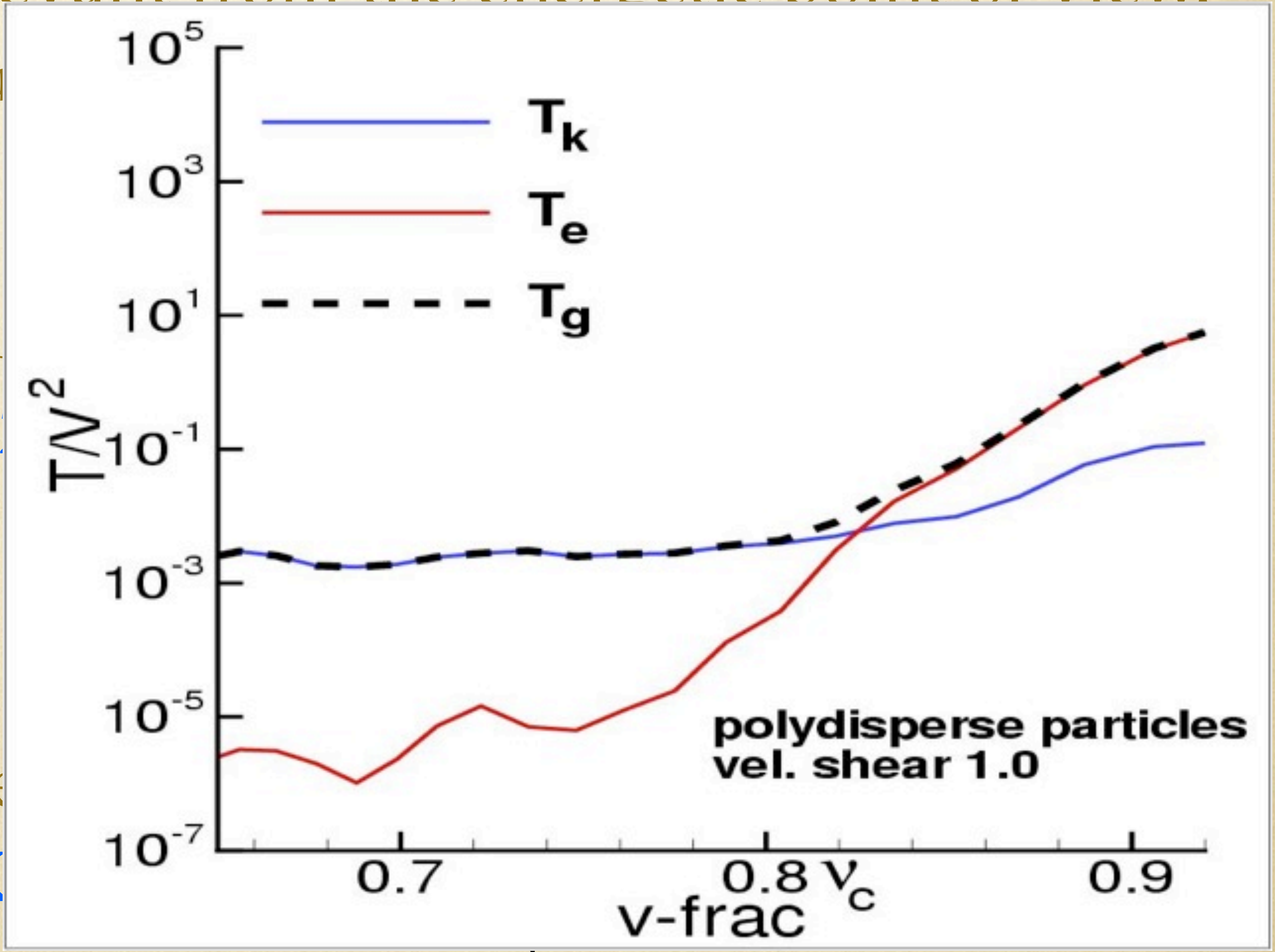

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About averages

calculating averages in l'th cell

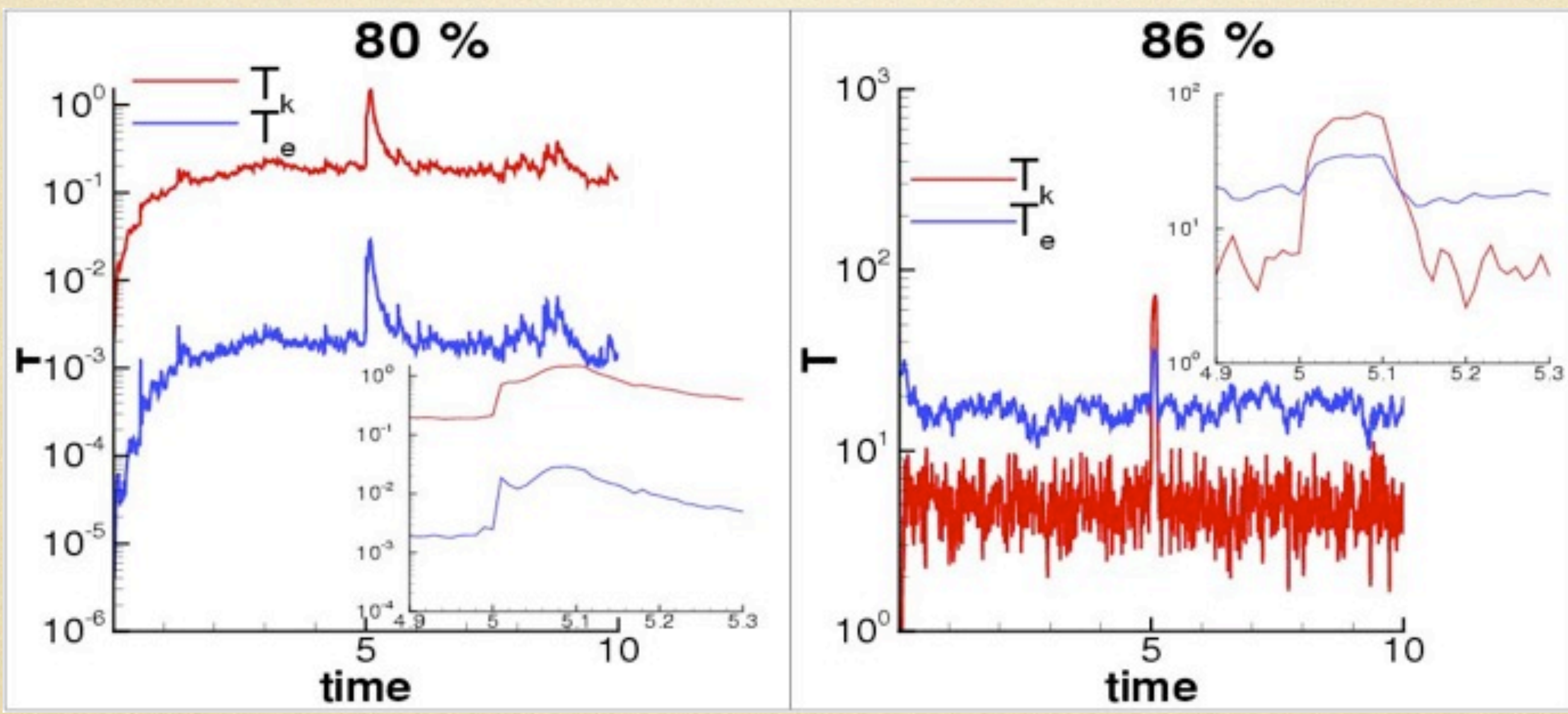
$$E_{e,l} = \frac{1}{N_t n_l} \frac{k}{2} \sum_{k=1}^{N_t} \sum_{j=1}^{n_l} \sum_{c=1}^{n_{c,j}} [x_{j,c}]^2$$

$$\langle E_{e,l} \rangle = \frac{k}{2} n_c \langle x_l \rangle^2 = \frac{k}{2} n_c \left[\frac{1}{N_t \bar{n}_l n_c} \sum_{k=1}^{N_t} \sum_{j=1}^{n_l} \sum_{c=1}^{n_{c,j}} x_{j,c} \right]^2$$

$$T_{e,l} = \frac{k}{2} n_c \langle \delta x^2 \rangle = \frac{k}{2} n_c \langle (x_{j,c} - \langle x_l \rangle)^2 \rangle = E_{e,l} - \langle E_{e,l} \rangle$$

What is temperature good for?

- Concept of temperature is a crucial step in developing a continuum model
- Temperature allows to understand heat transfer which may be analyzed on macro scale



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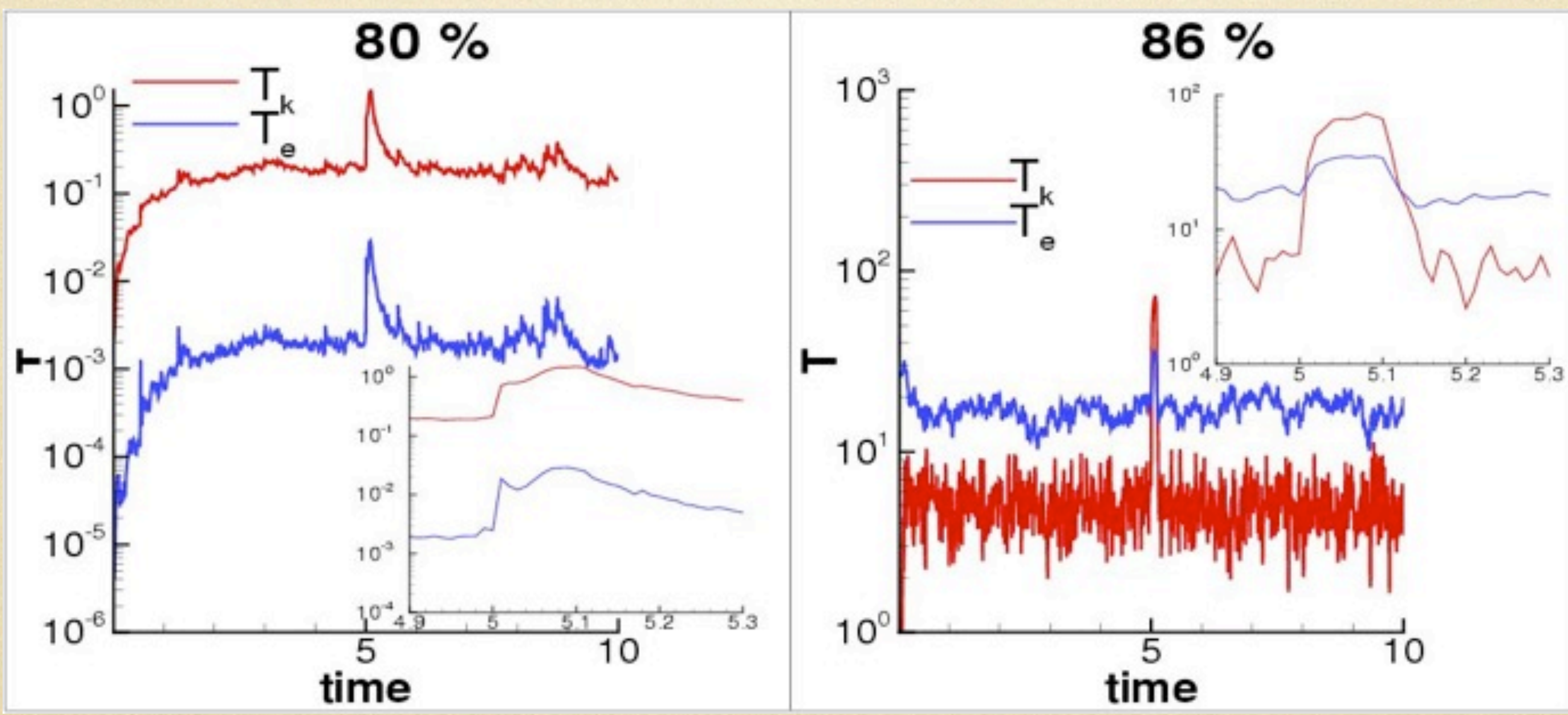

Consider role of elastic energy in signal propagation through granular matter

Previous works on energy propagation through dense granular matter

- Extensive literature considering
- response to a localized perturbation
- Reydellet & Clement PRL ‘01; Geng etal PRL ‘01; Goldenberg and Goldhirsch, PRL 02, ...
- response to spatially independent (piston-like) perturbations
- Jia etal PRL ’99, Hostler & Brennen PRE ’05, Somfai etal PRE ’05, Mouraille & Luding ’06-’09
- response of complex granular systems
- Sen etal IJMPB ’05; Baker Geophys ’99; Biot JASA ’55,...

Motivation

- Here: consider response to a space-time dependent perturbation?
- Why? Response to space-time dependent perturbations provides significant new insight
  - Nature of propagation should help us understand basic mathematical features of an appropriate continuum model
    - diffusive: Coppersmith etal PRE '96;...
    - wave-like: Bouchaud etal JPI '94; Blumenfeld PRL '04;...
    - elastic: Goldenberg and Golhirsch Nature '05; Geng etal PhysD '03;...

Basic System

- 2D system of soft discs modeled:
  - linear force model
  - typically dynamic friction
  - elasticity
  - polydispersity typically 10%
  - (constant) volume fraction typically 0.9
  - 40K particles
  - perturbation
    - standing-wave type, $\lambda \gg d$ $f \ll 1/\tau_c$ $A = 0.6d$

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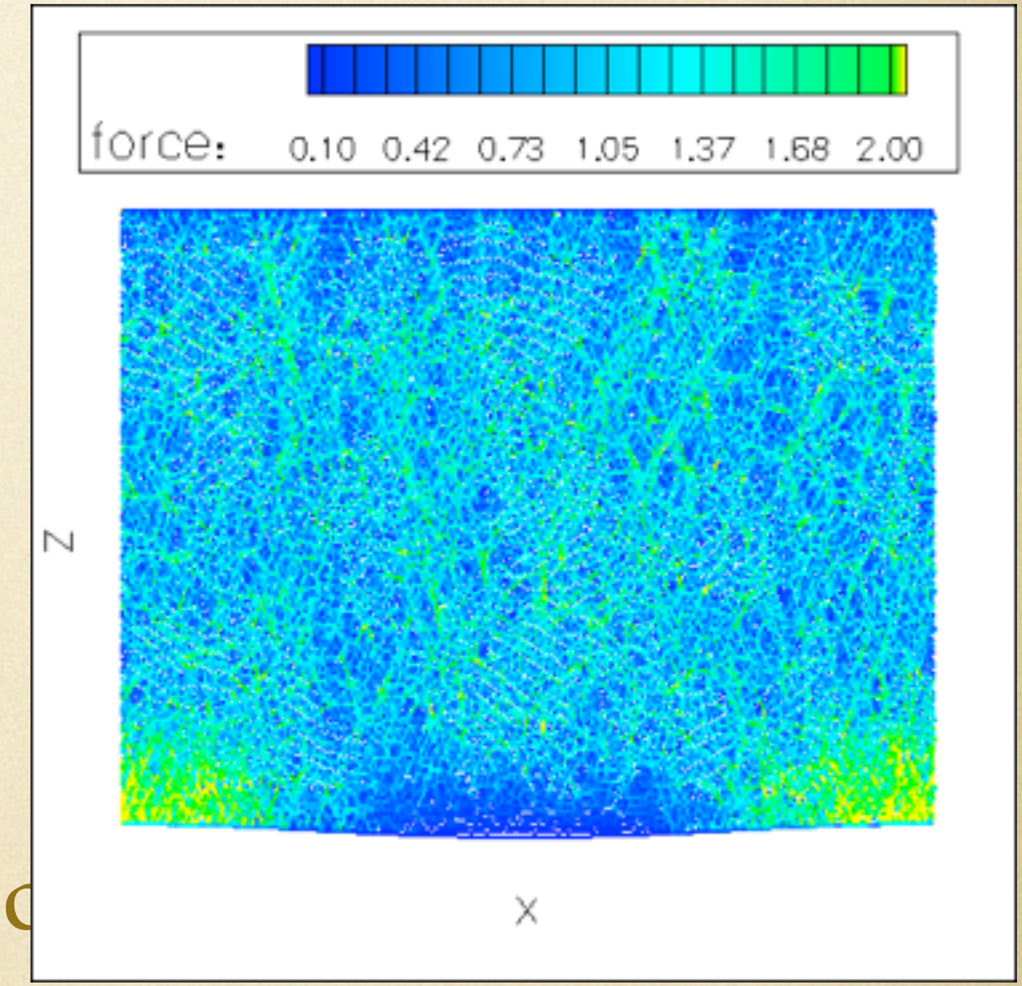


Forces and Energies

- Extensive work regarding importance of force chains on signal propagation

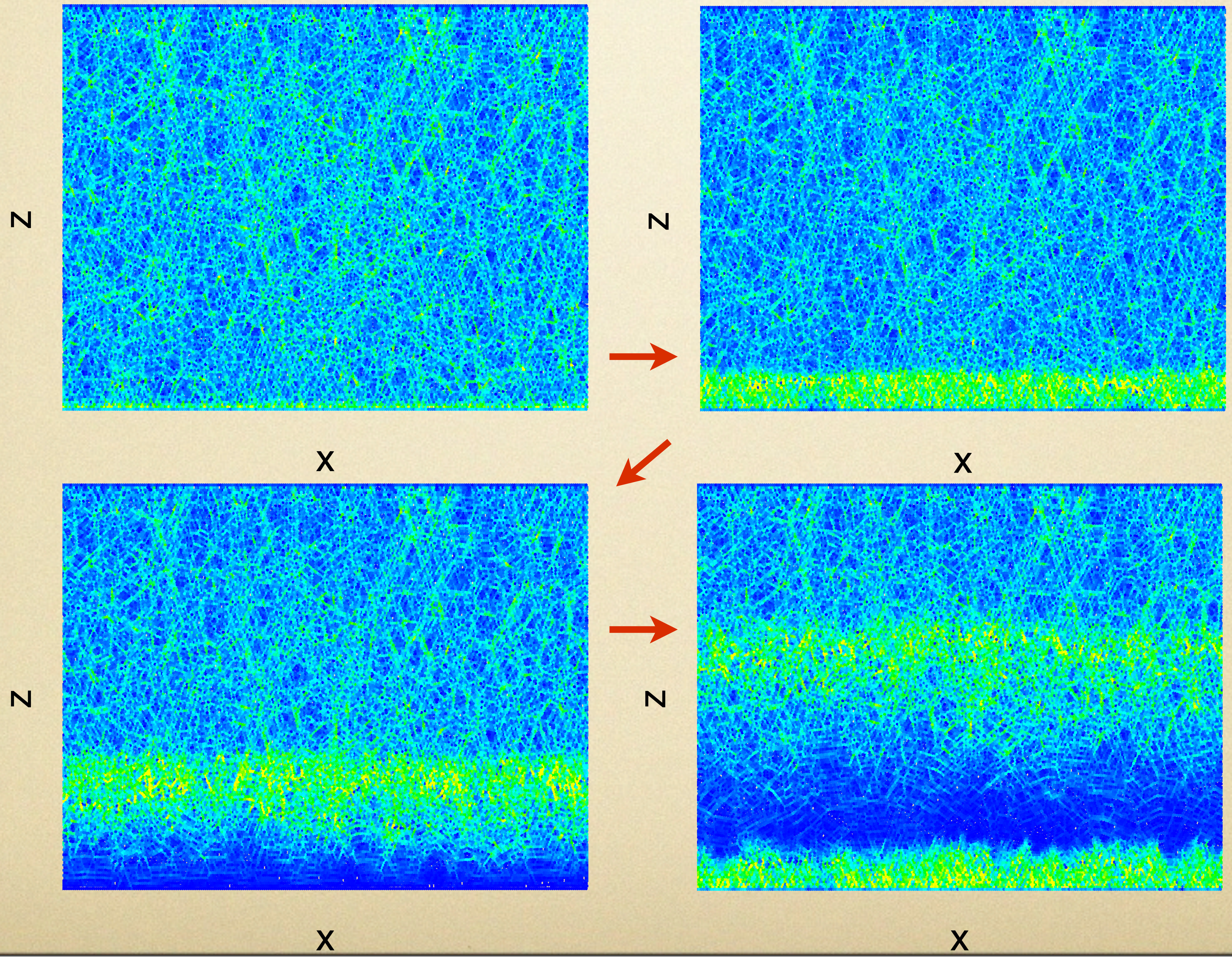
  Liu & Nagel PRL '92, Makse etal PRL '99; Somfai etal PRE '05; Hostler & Brennen PRE '05;...

- Here: average over small cells and compute space-time averages of elastic (compression) energy

$(E_{elas} \gg E_{kin})$

- Carry out Fourier transforms - typically FT in x direction, averaged over time $1/f \gg \Delta t \gg \tau_c$
- Consider first infinite wavelength perturbation

Infinite wavelength perturbation



Infinite wavelength - main results

- wave-like propagation as expected for low frequency excitations
- relation between frequency and the wavenumber in the direction of propagation consistent with linear dispersion
- speed of propagation consistent with the results found in literature

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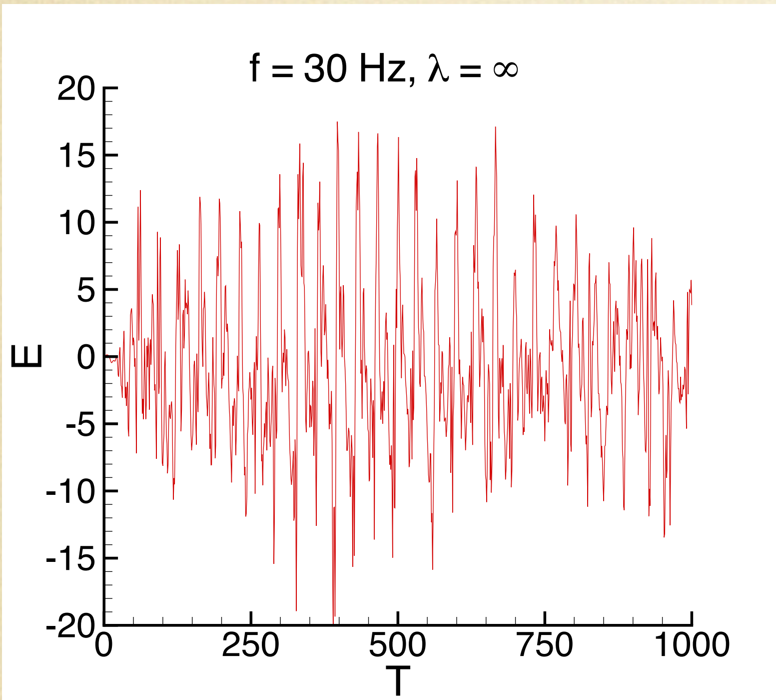

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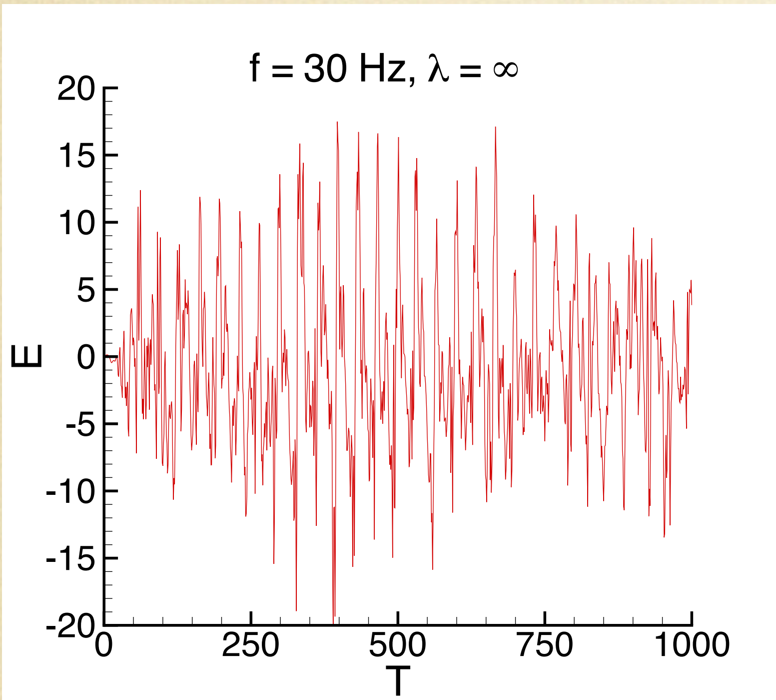

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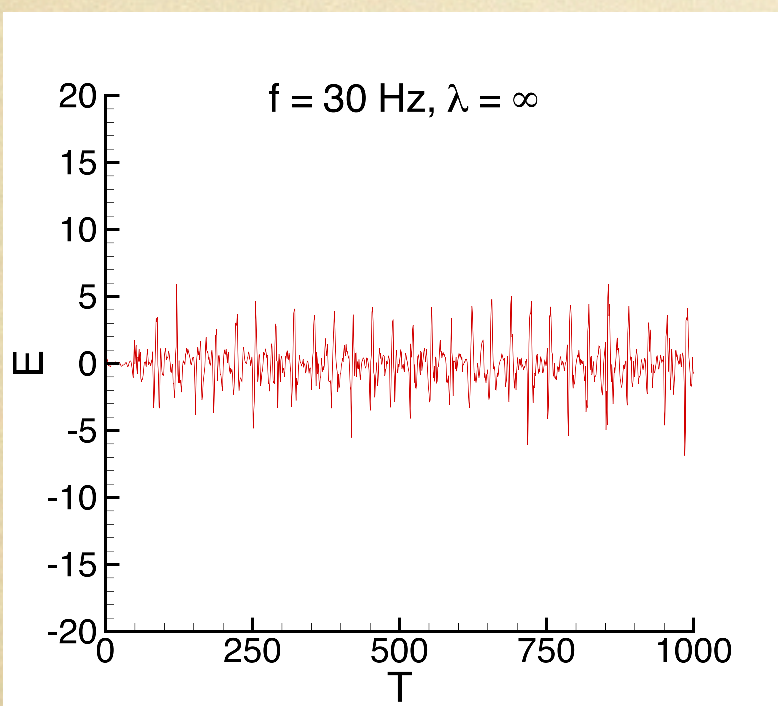

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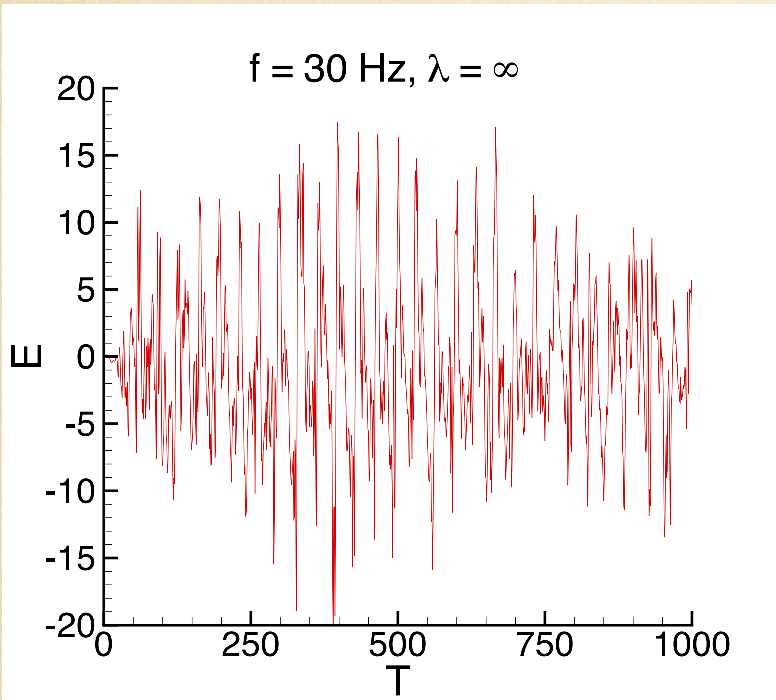

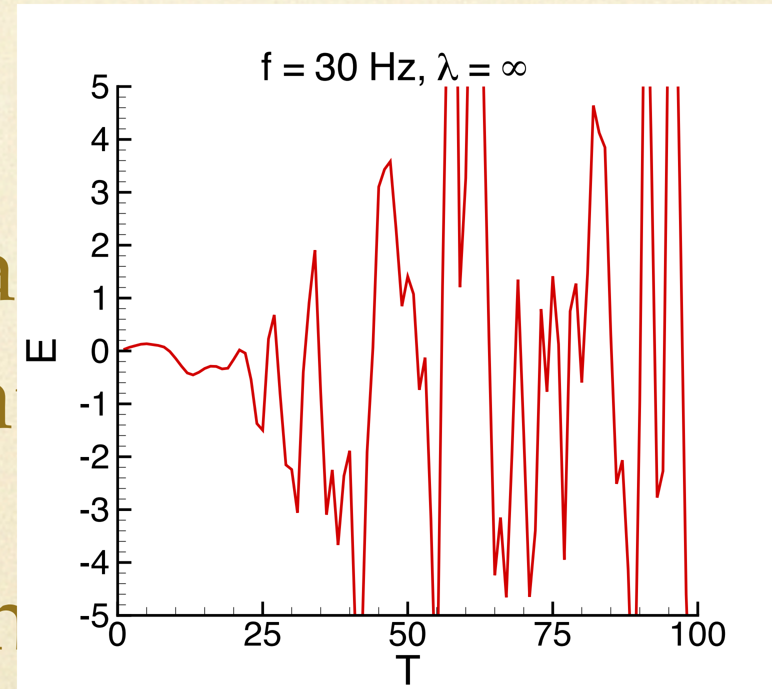

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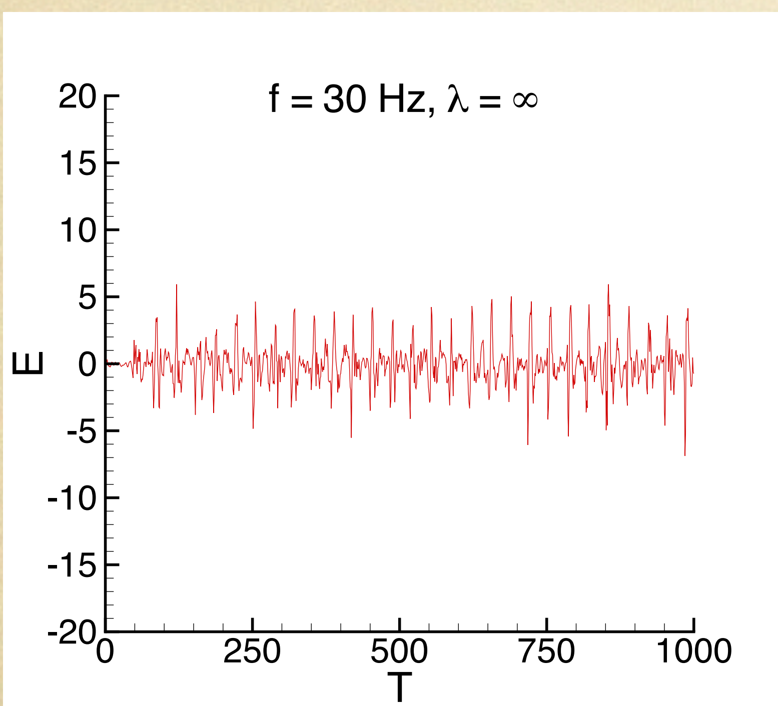
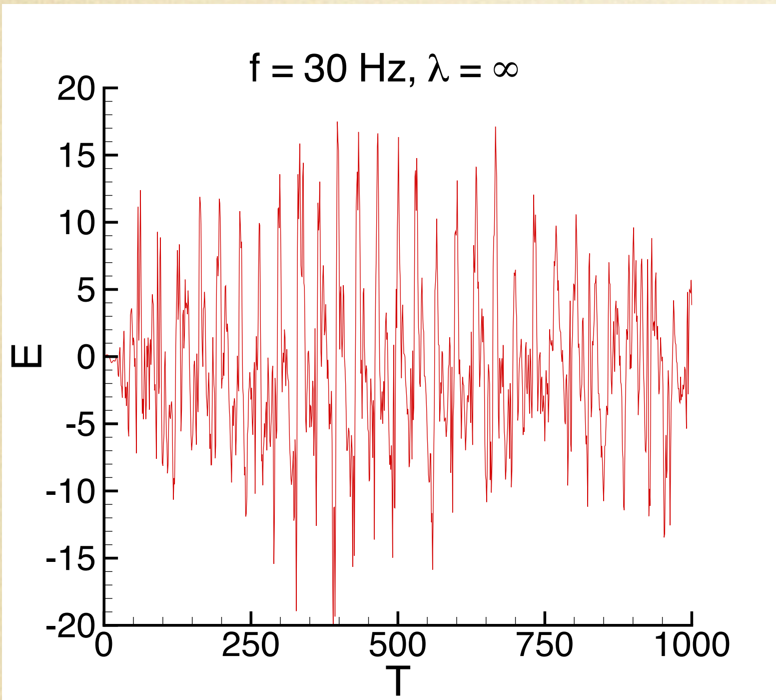
n linear dispersion

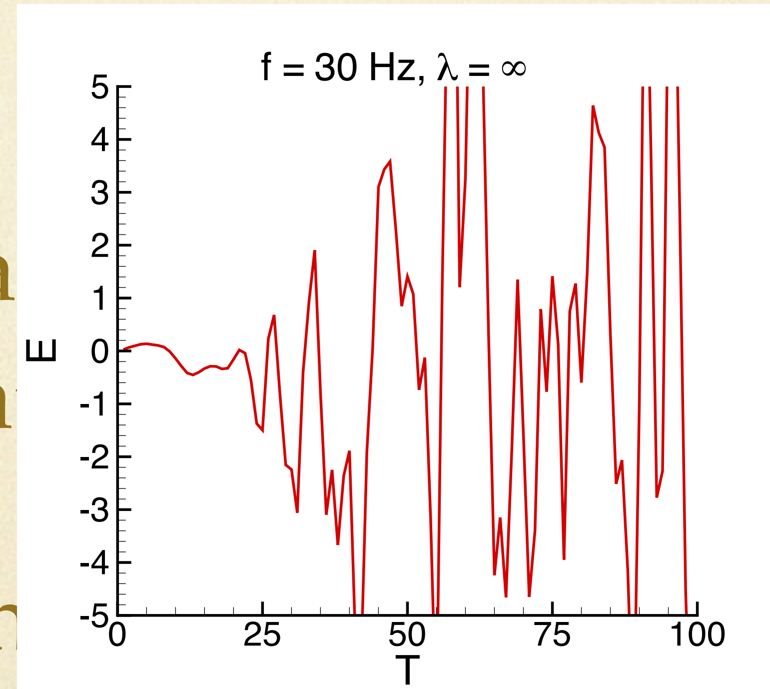


agation consistent with the results

ture

Infinite wavelength - main results





pa ... for low

ta

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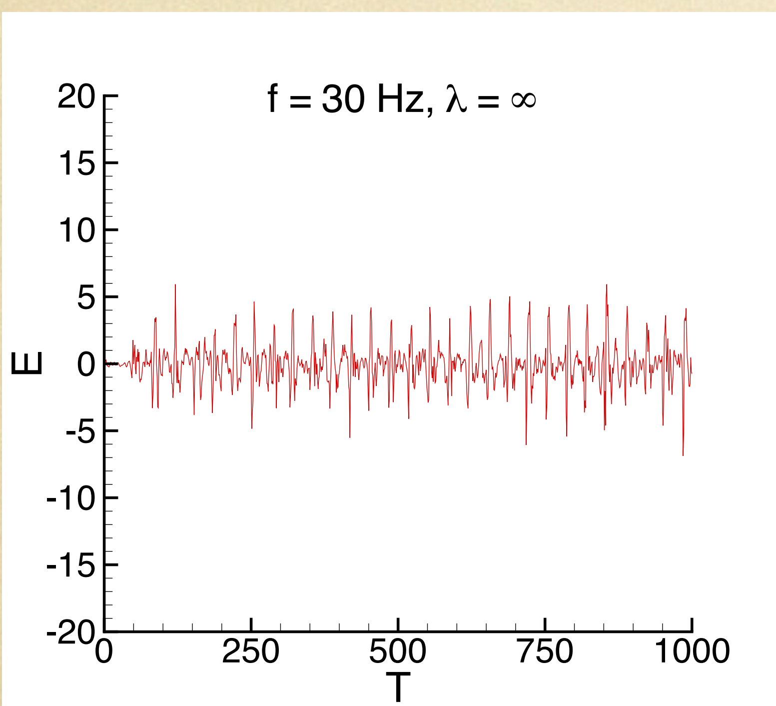
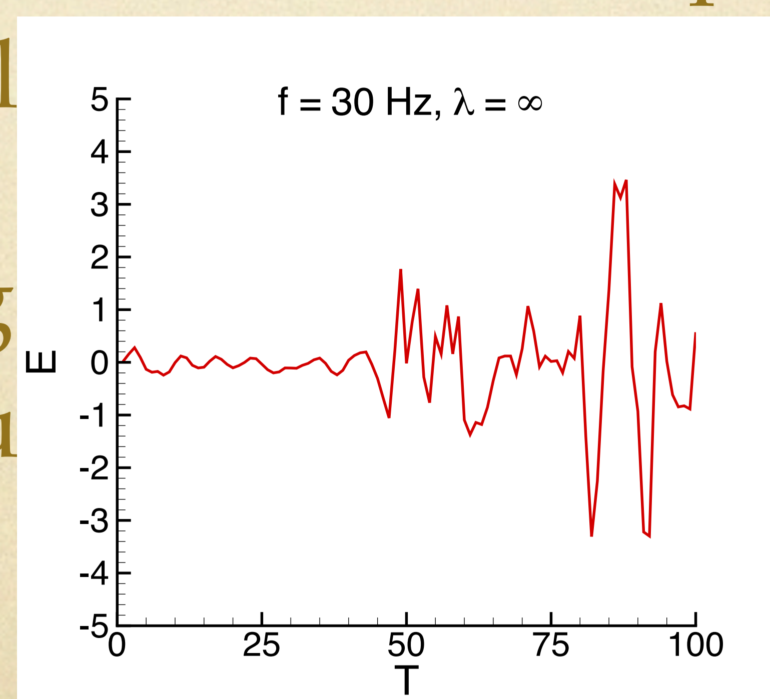
wavenumber in the direction of propagation

n l

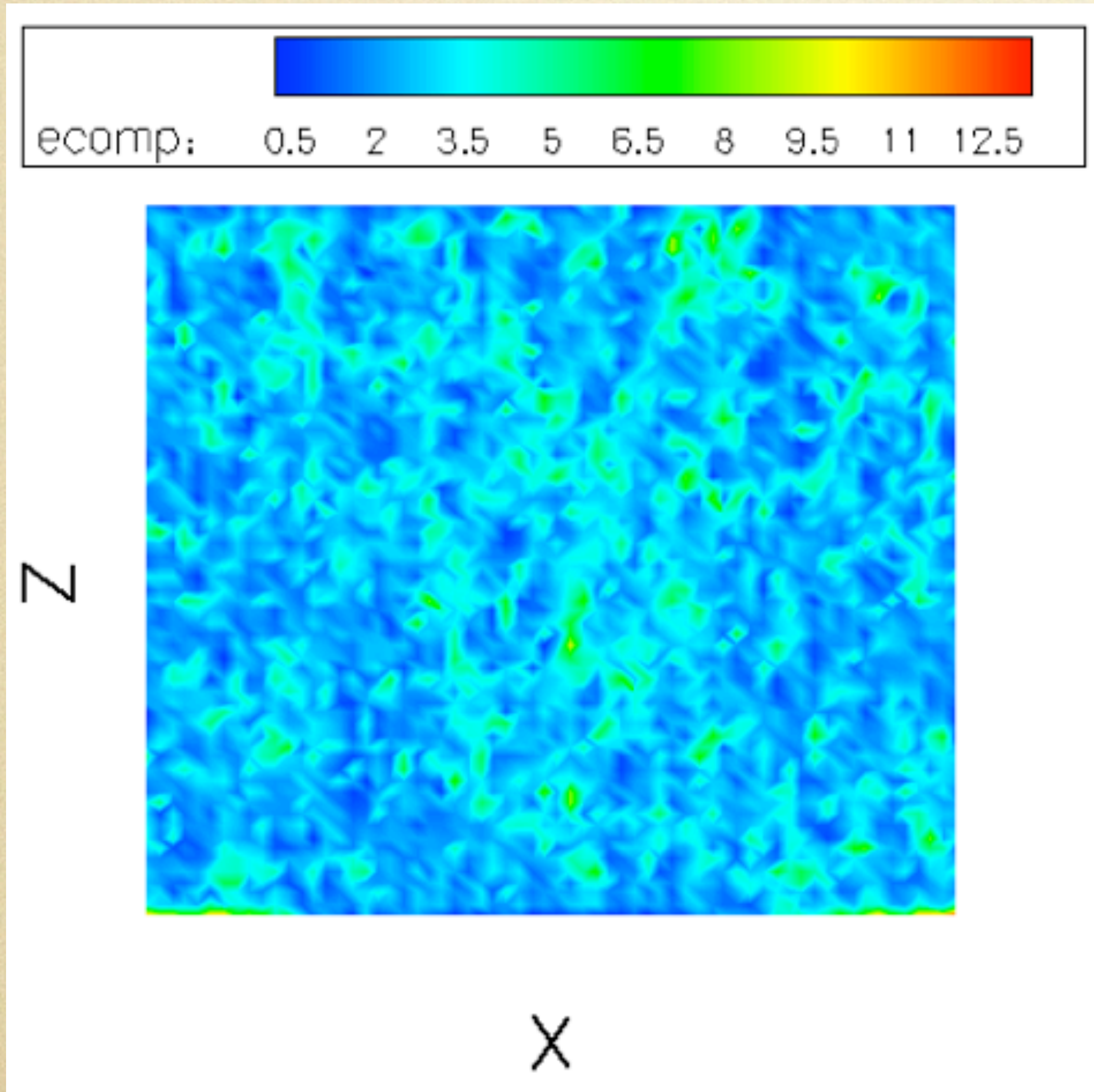




ag ... ith the results

tu

Finite wavelength perturbation - basic case (elastic energy)

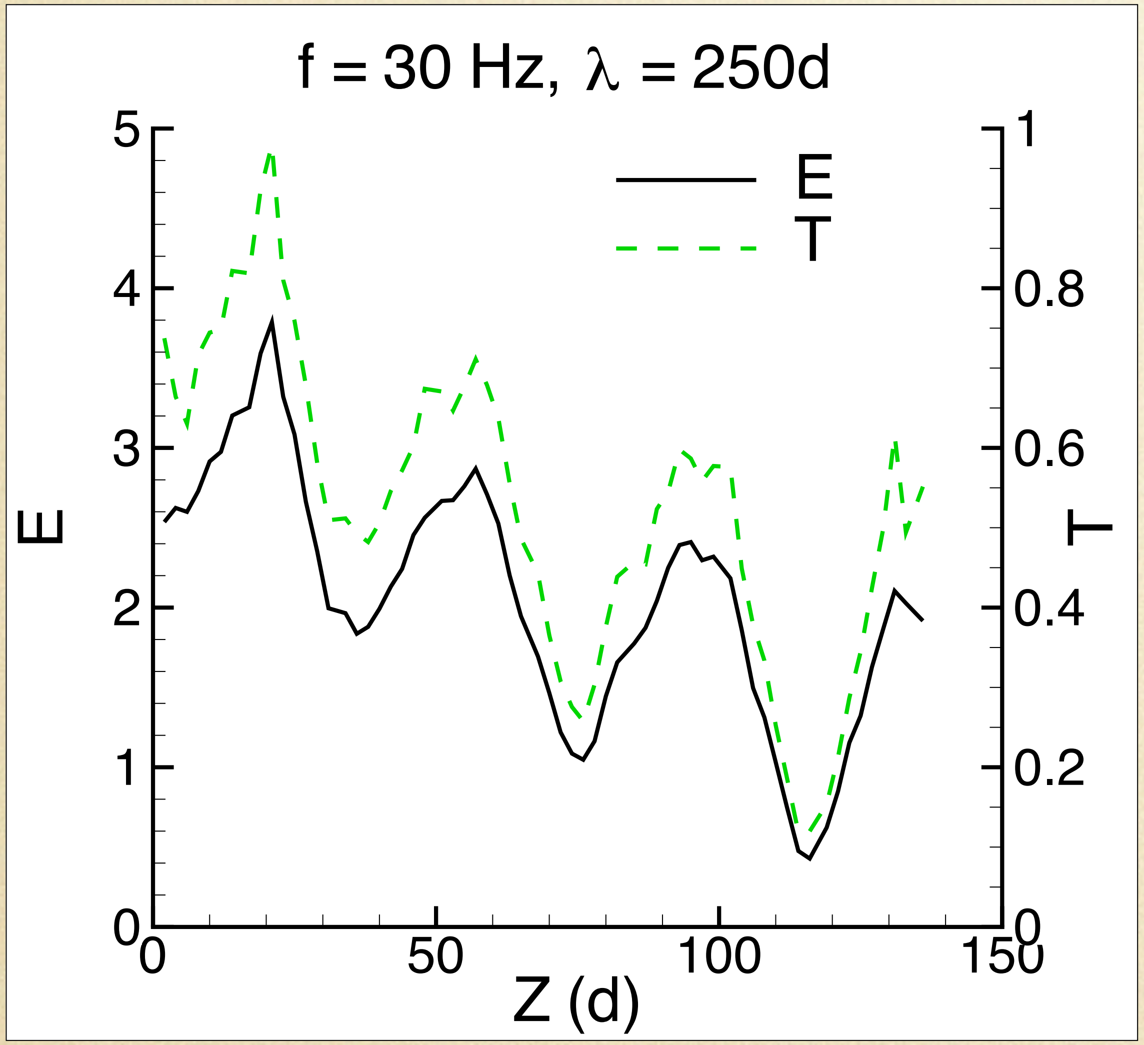


$$\lambda = 250\ d$$

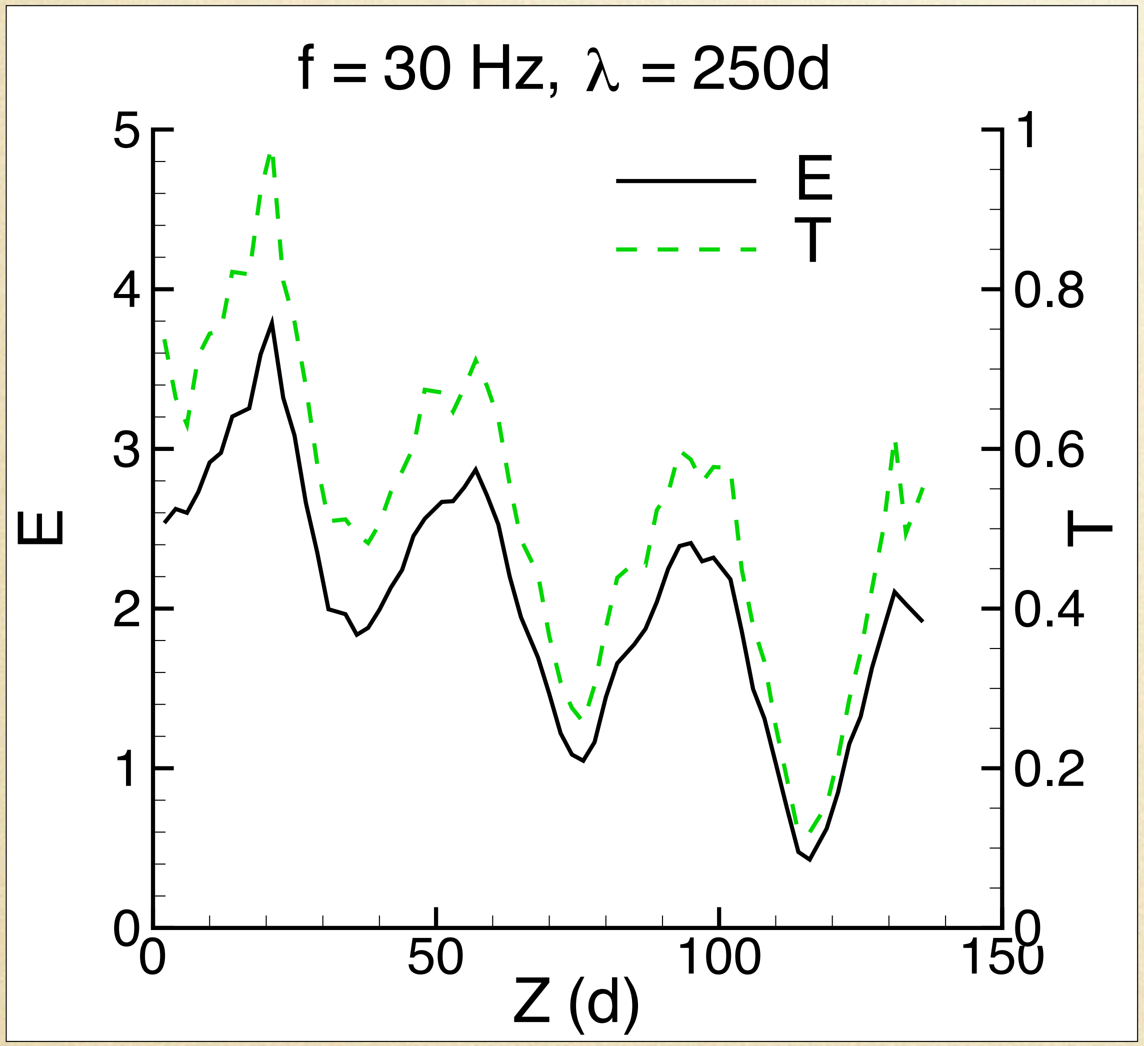
Still wave-like propagation, similarly to infinite wavelength perturbation

Look into FT of the propagating signal

FT - basic case

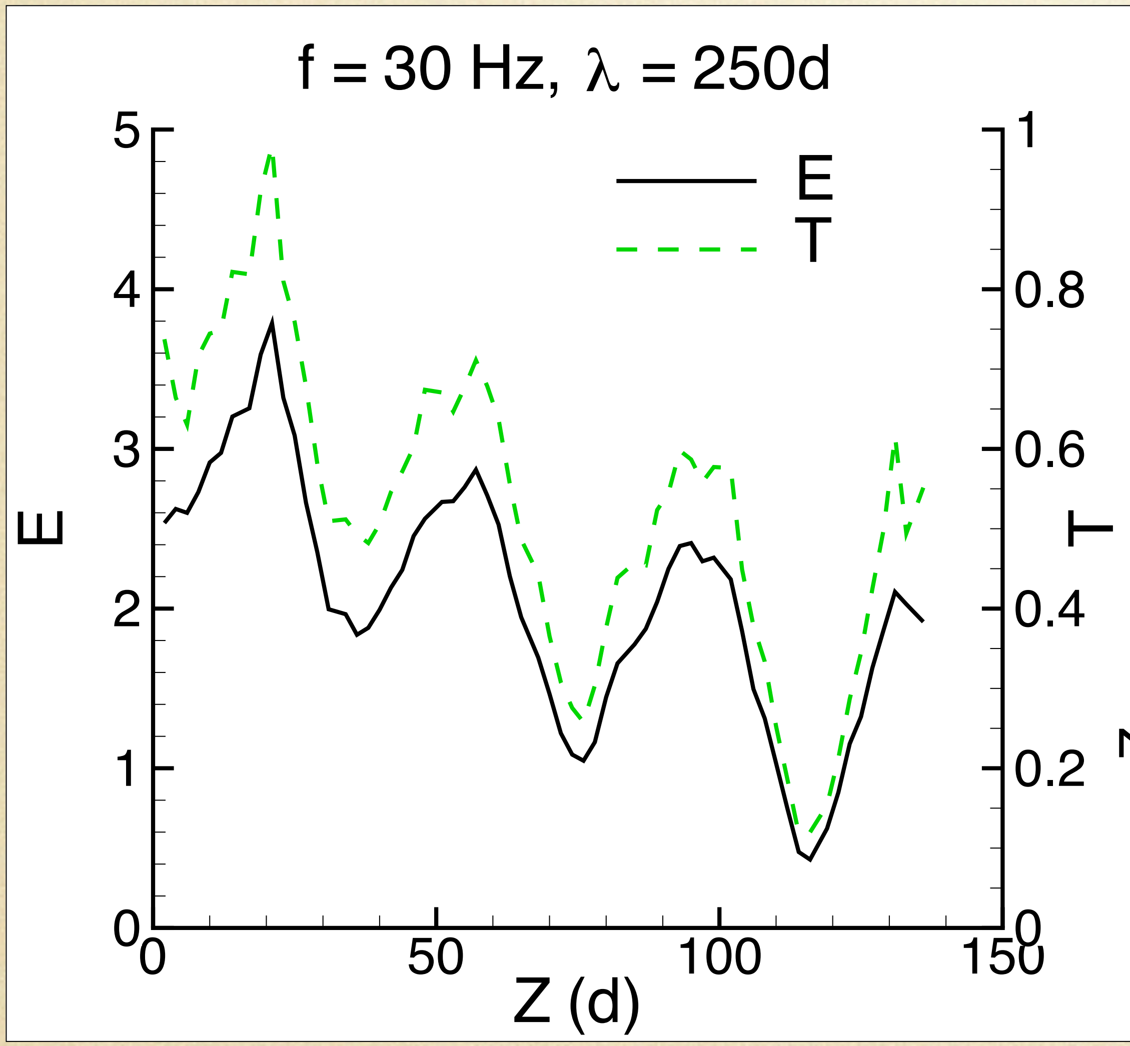


FT - basic case

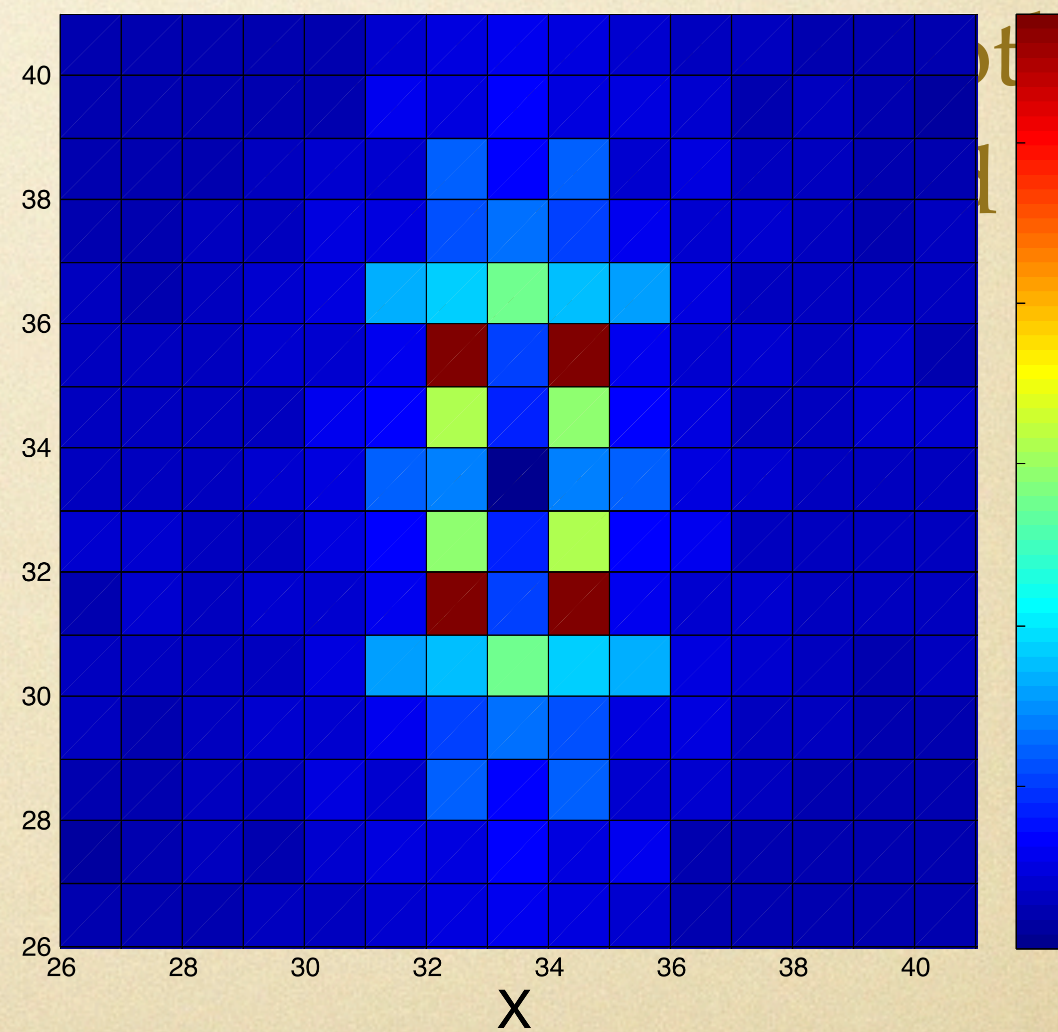


- Well defined wavelike propagation of both elastic energy and temperature
- Are these results robust?

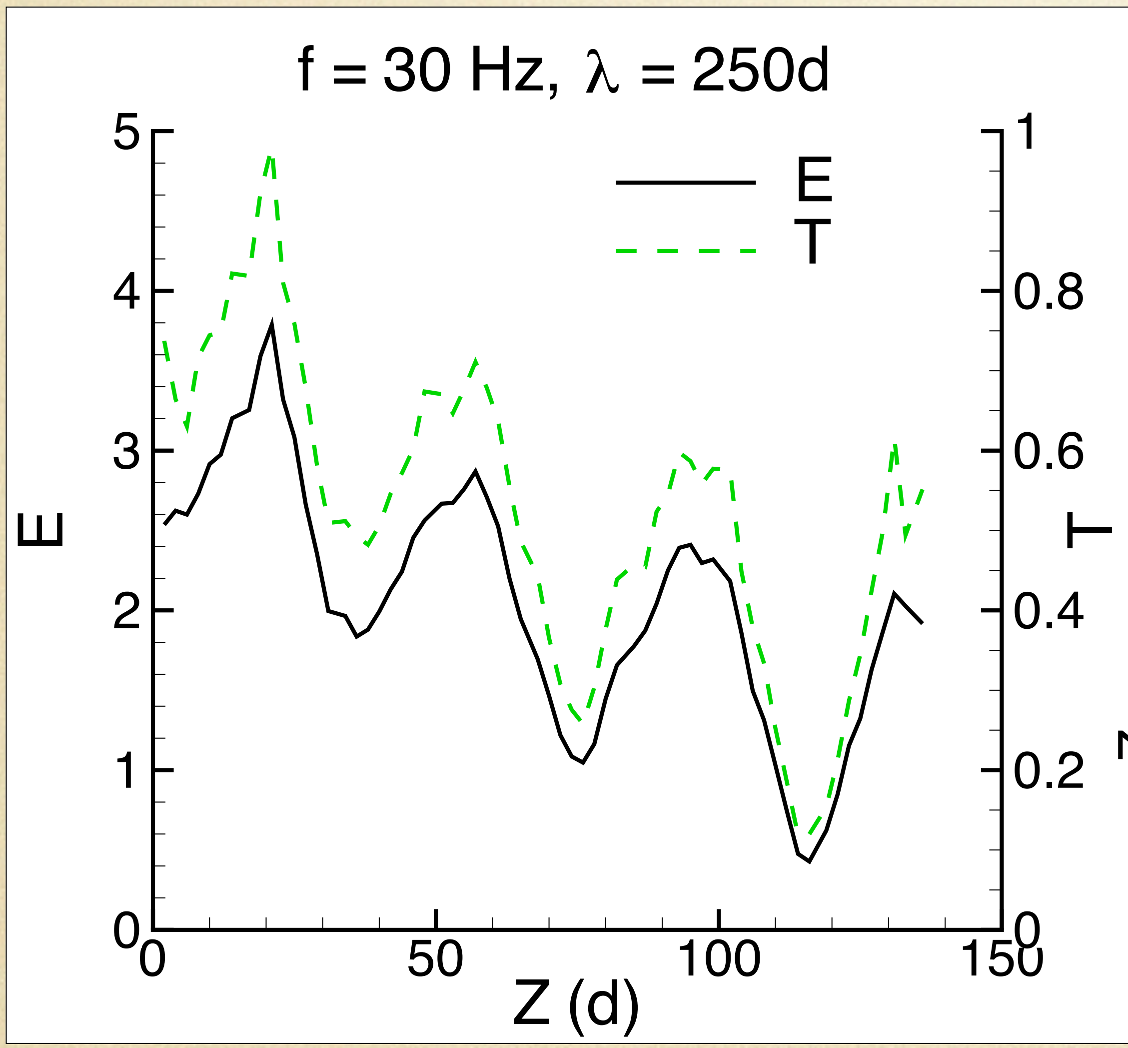
FT - basic case



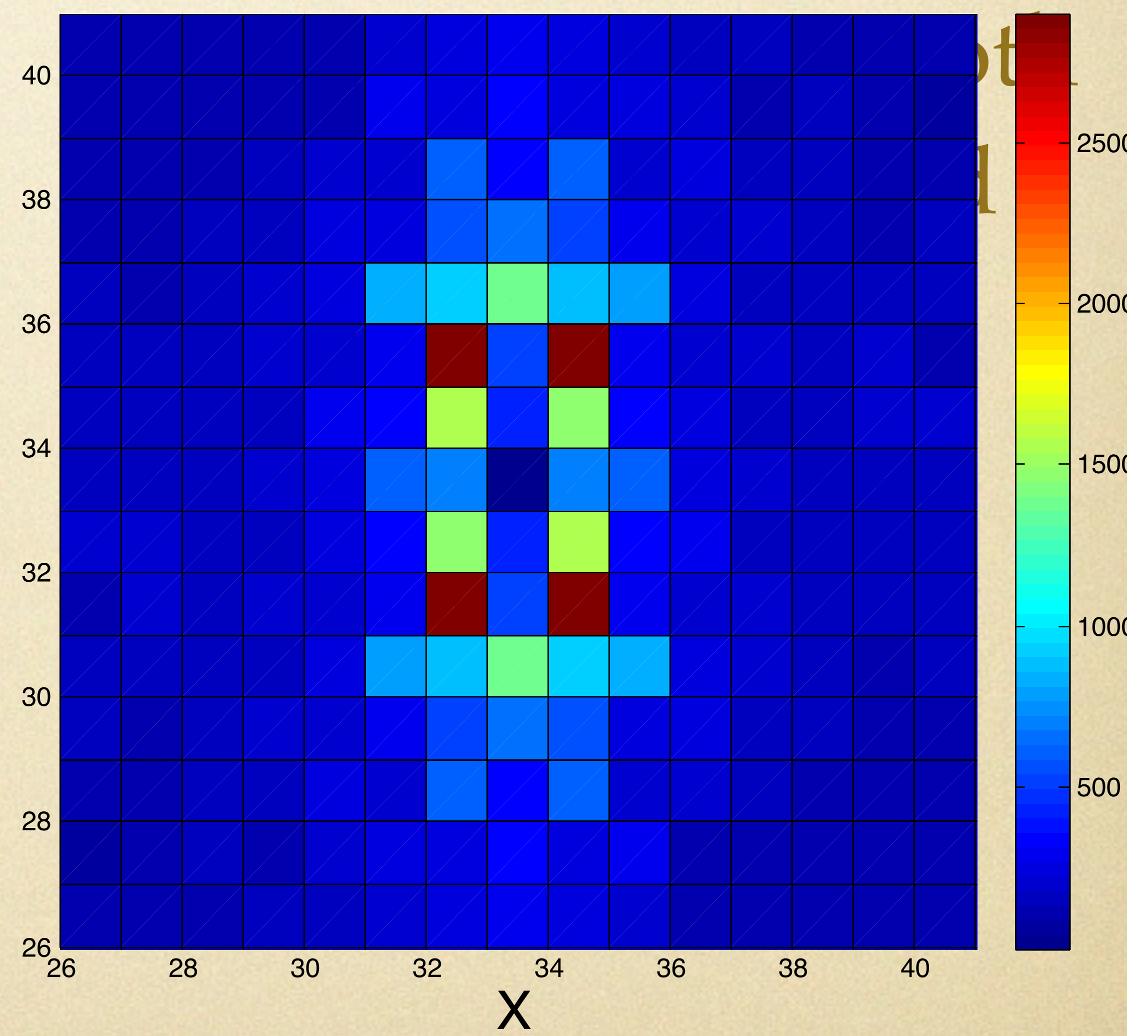
- Well defined wavelike



FT - basic case

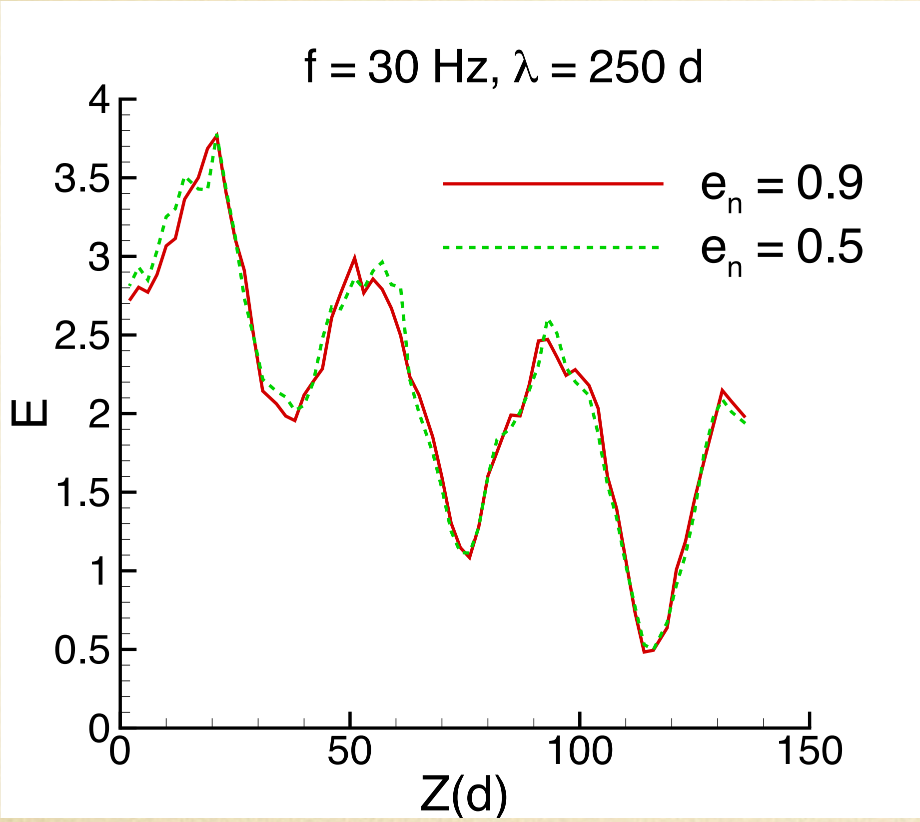


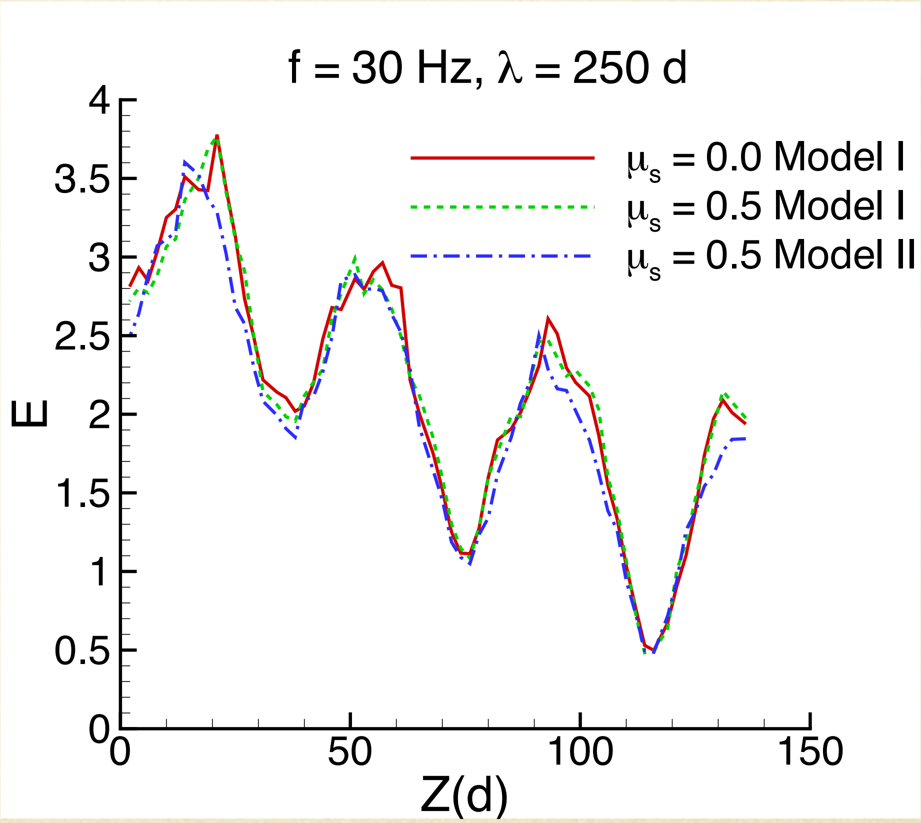
- Well defined wavelike



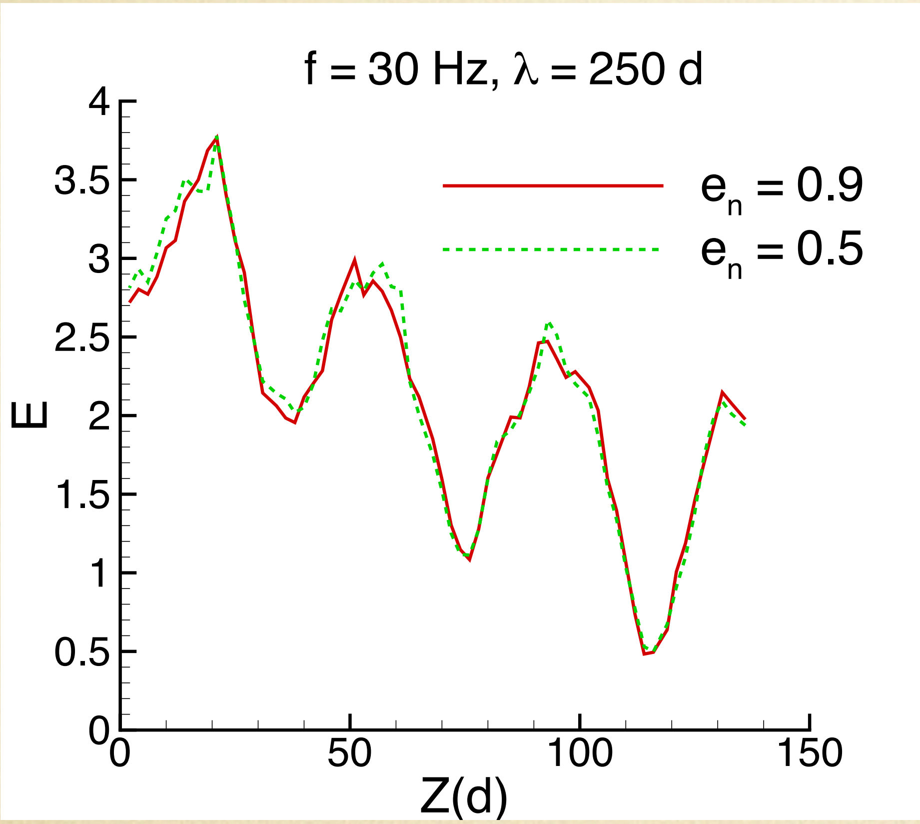
2D FT

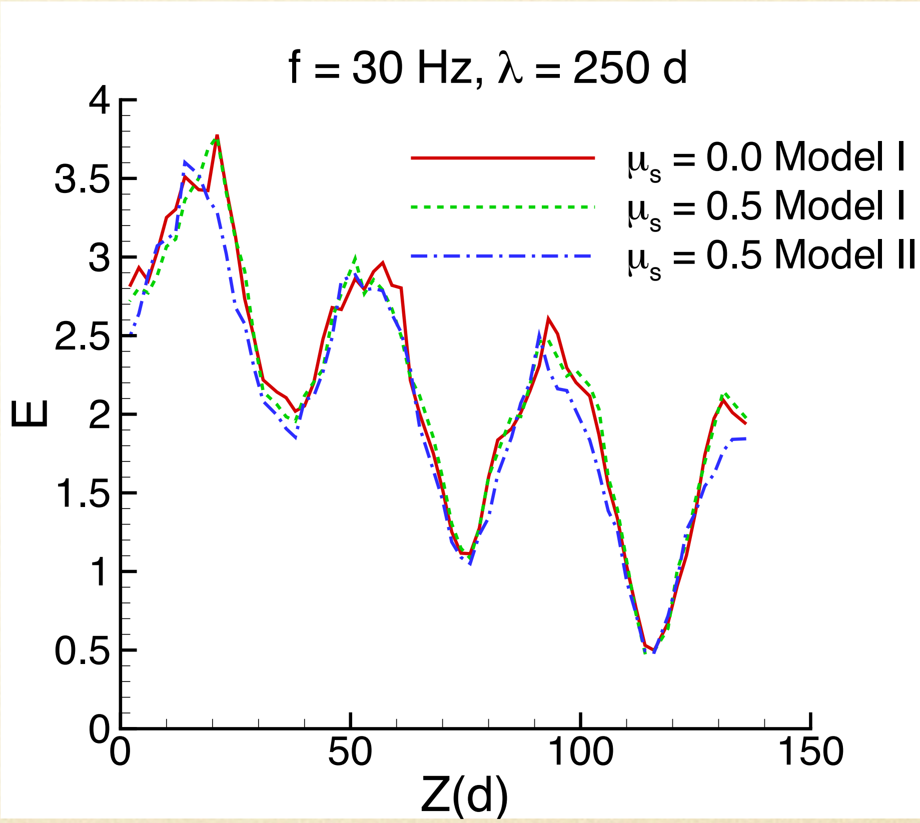
Particle properties: elasticity, friction





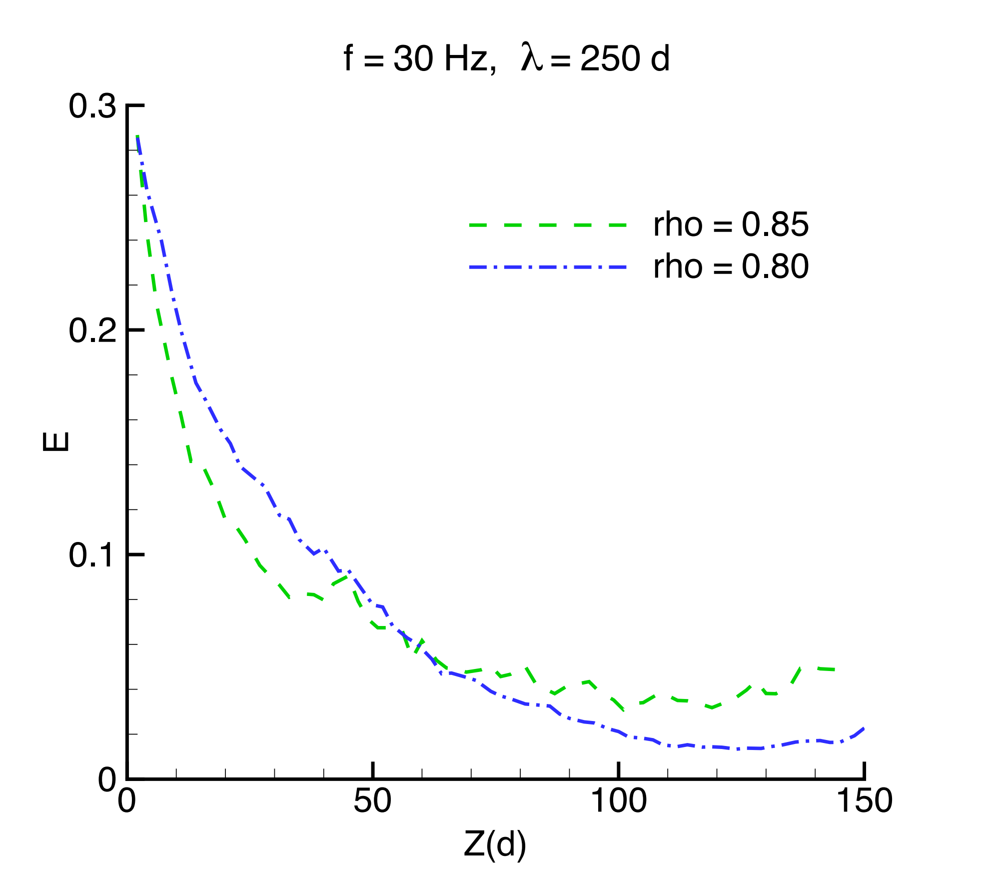
Particle properties: elasticity, friction



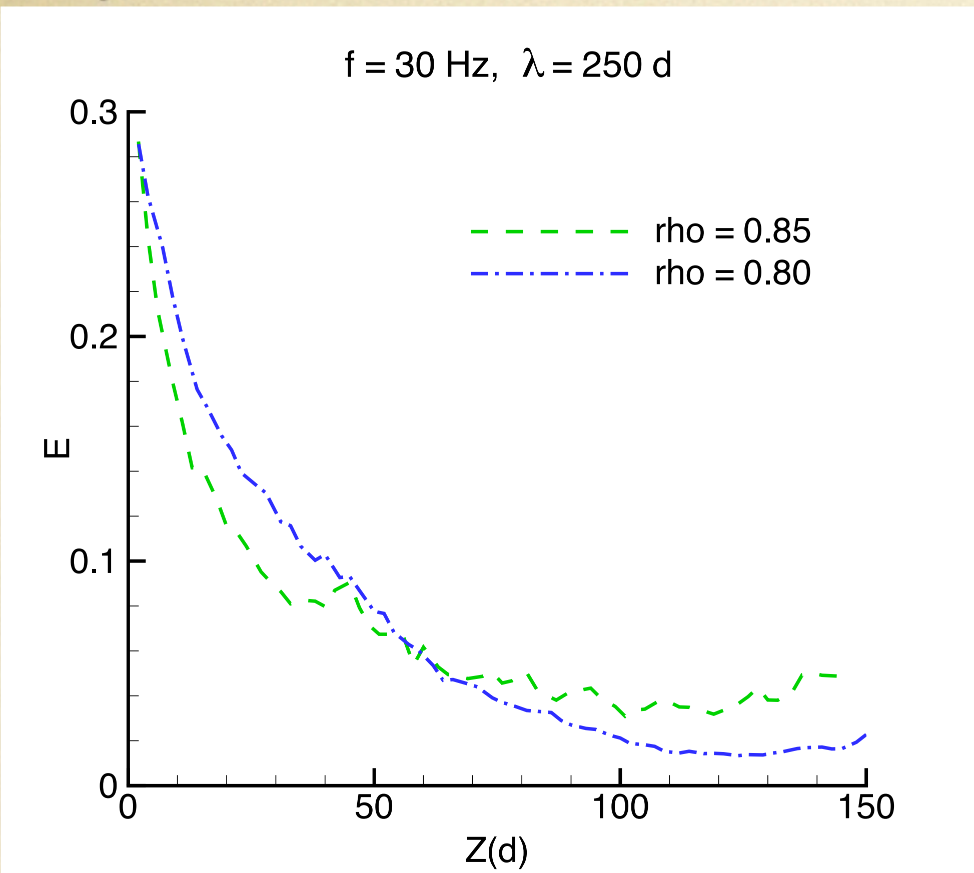


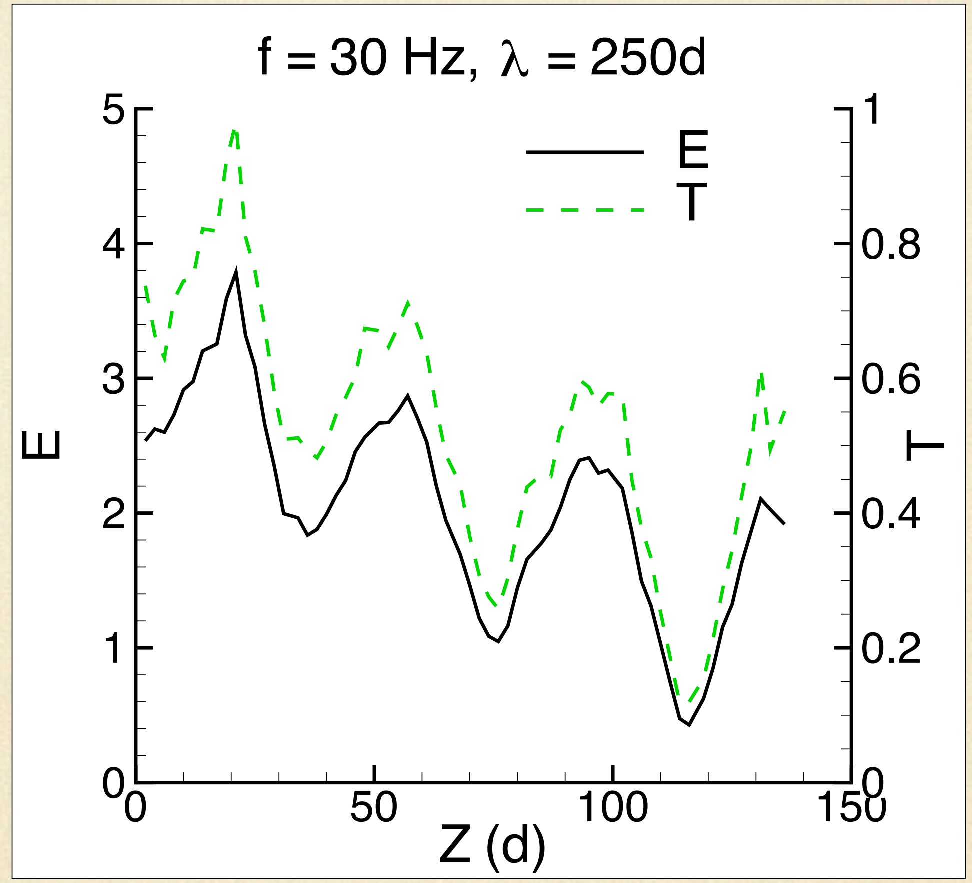
- no influence of elasticity
- no influence of friction
- no influence of force model

Volume fraction

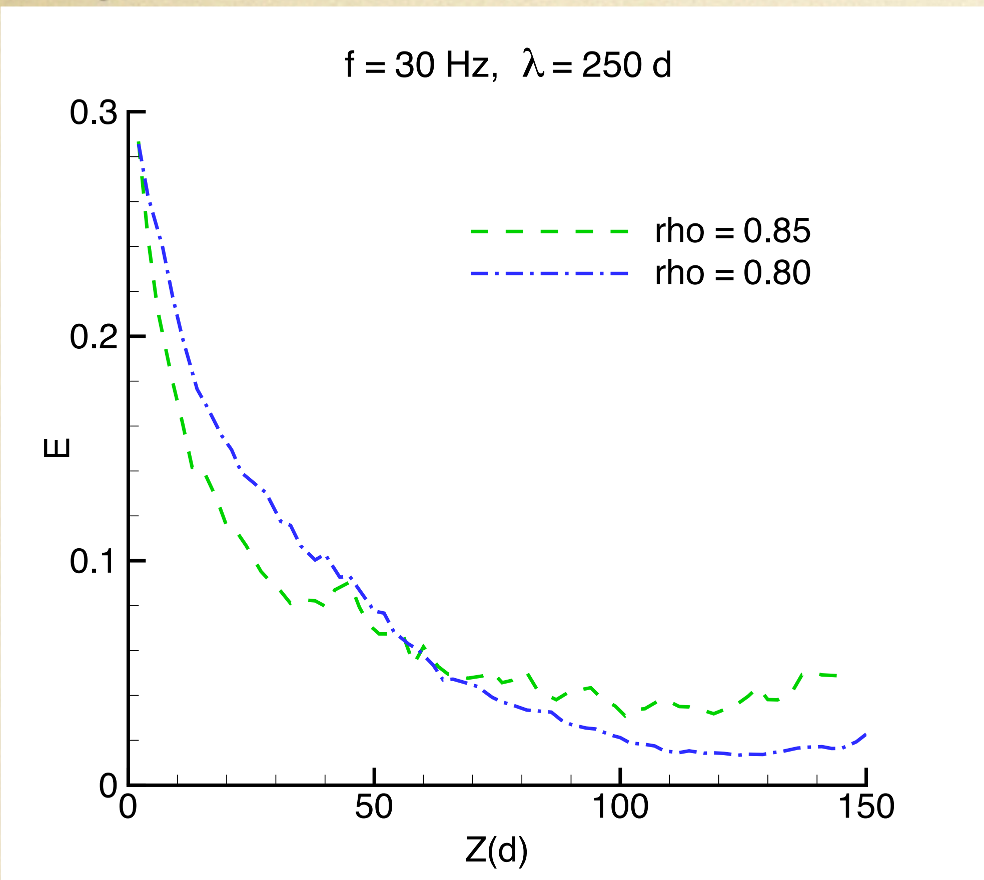


Volume fraction

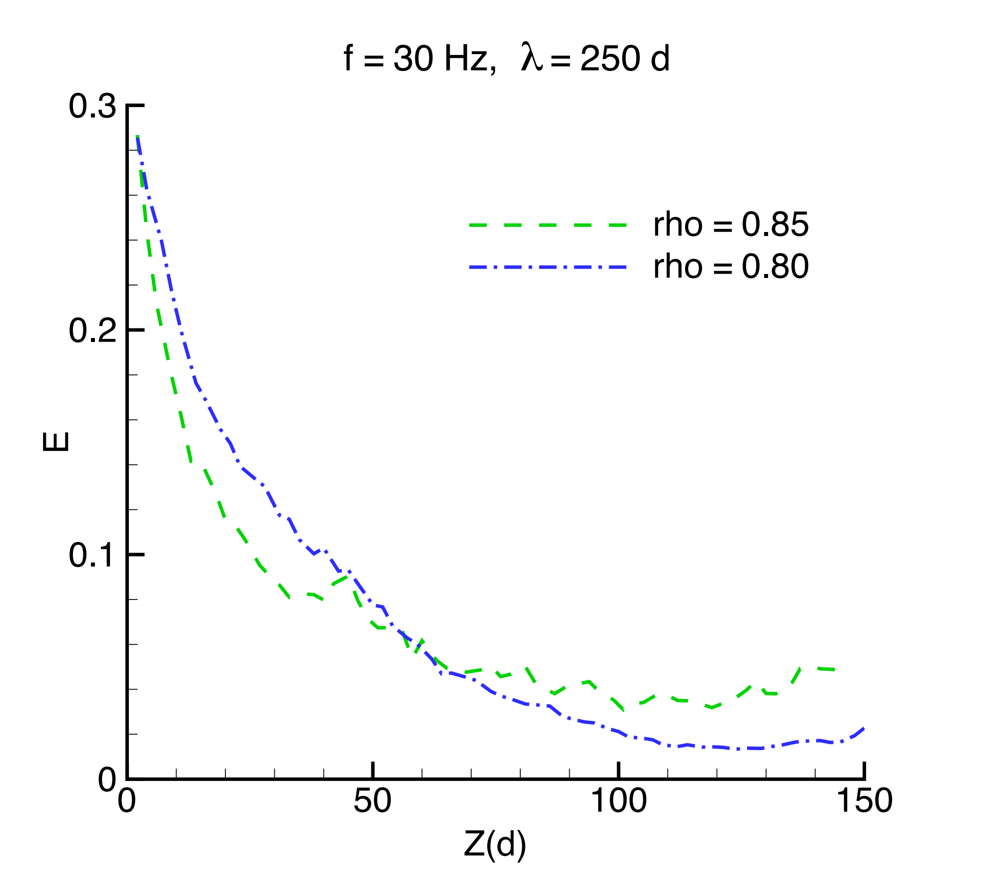




Volume fraction

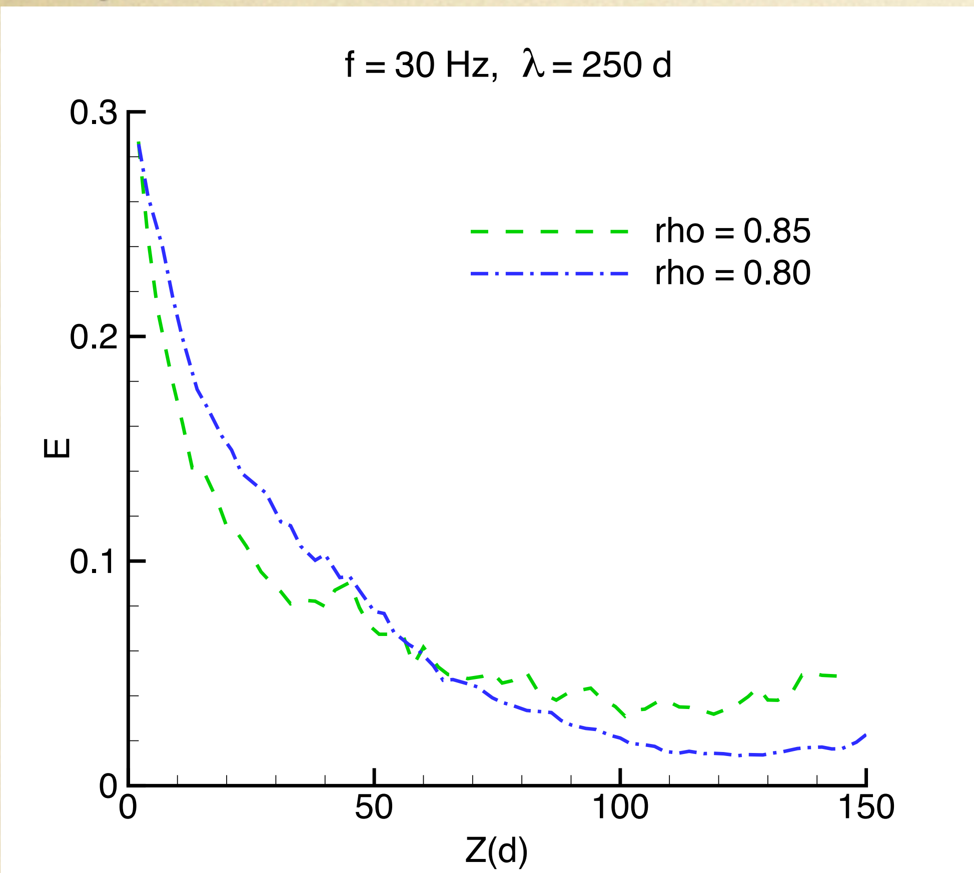


Volume fraction



- strong influence of volume fraction
- jammed / unjammed transition?

Volume fraction



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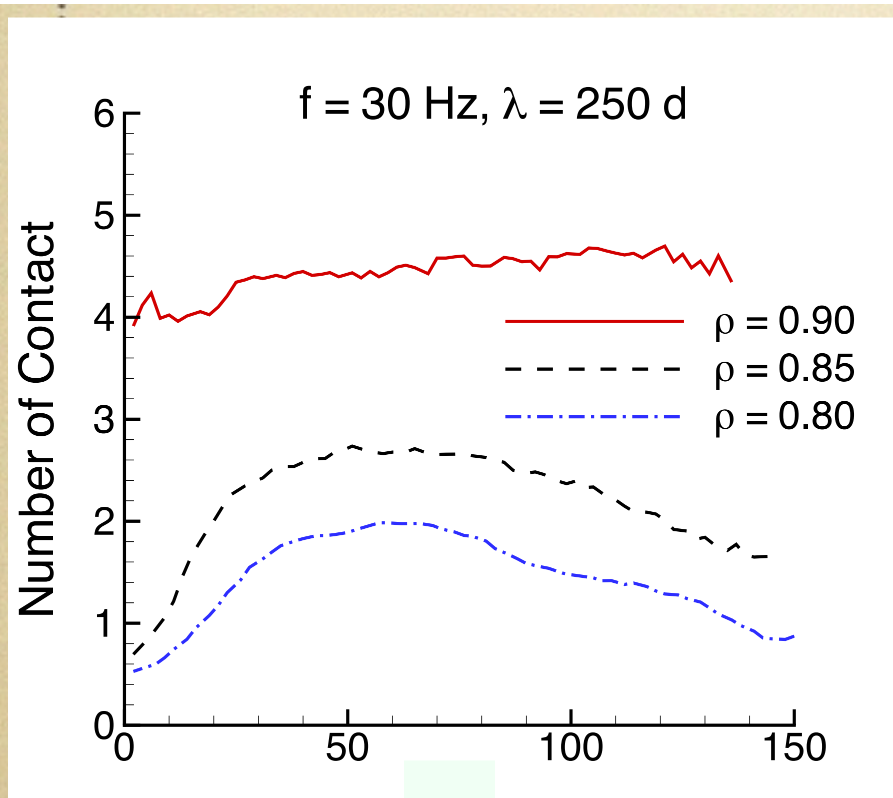
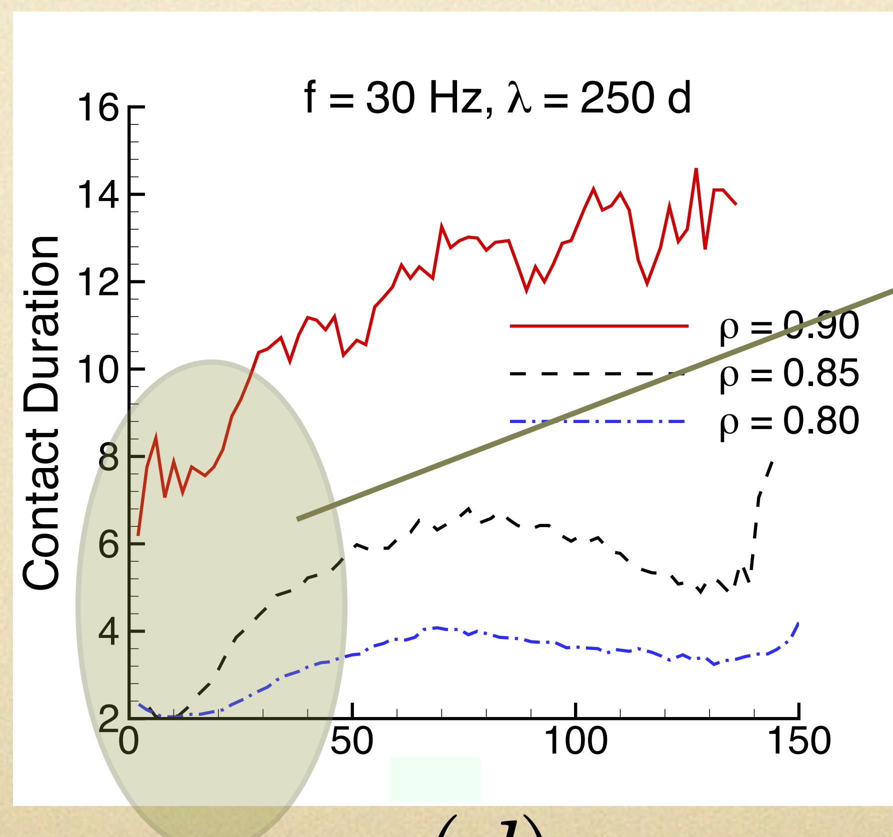


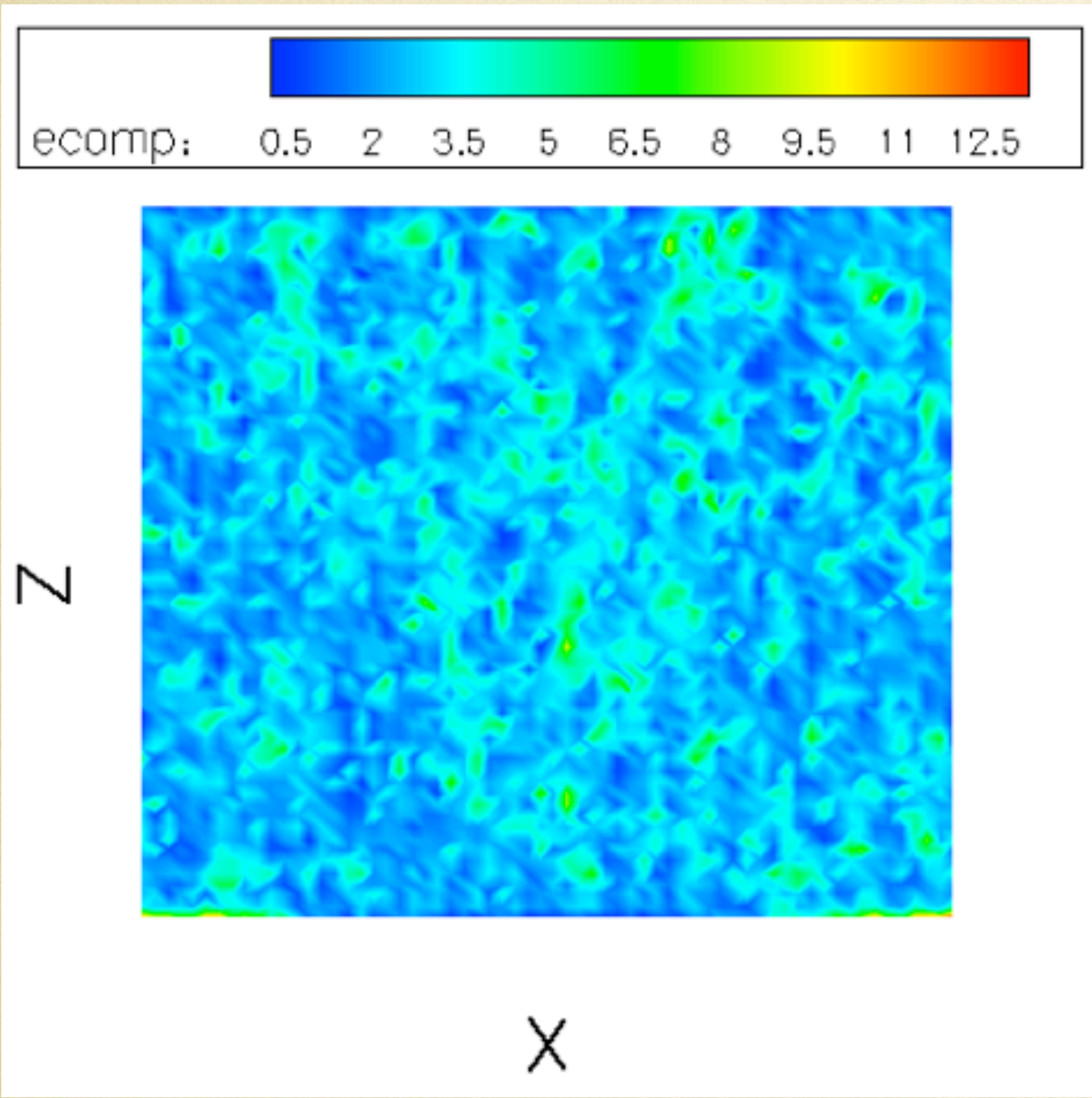
$z(d)$



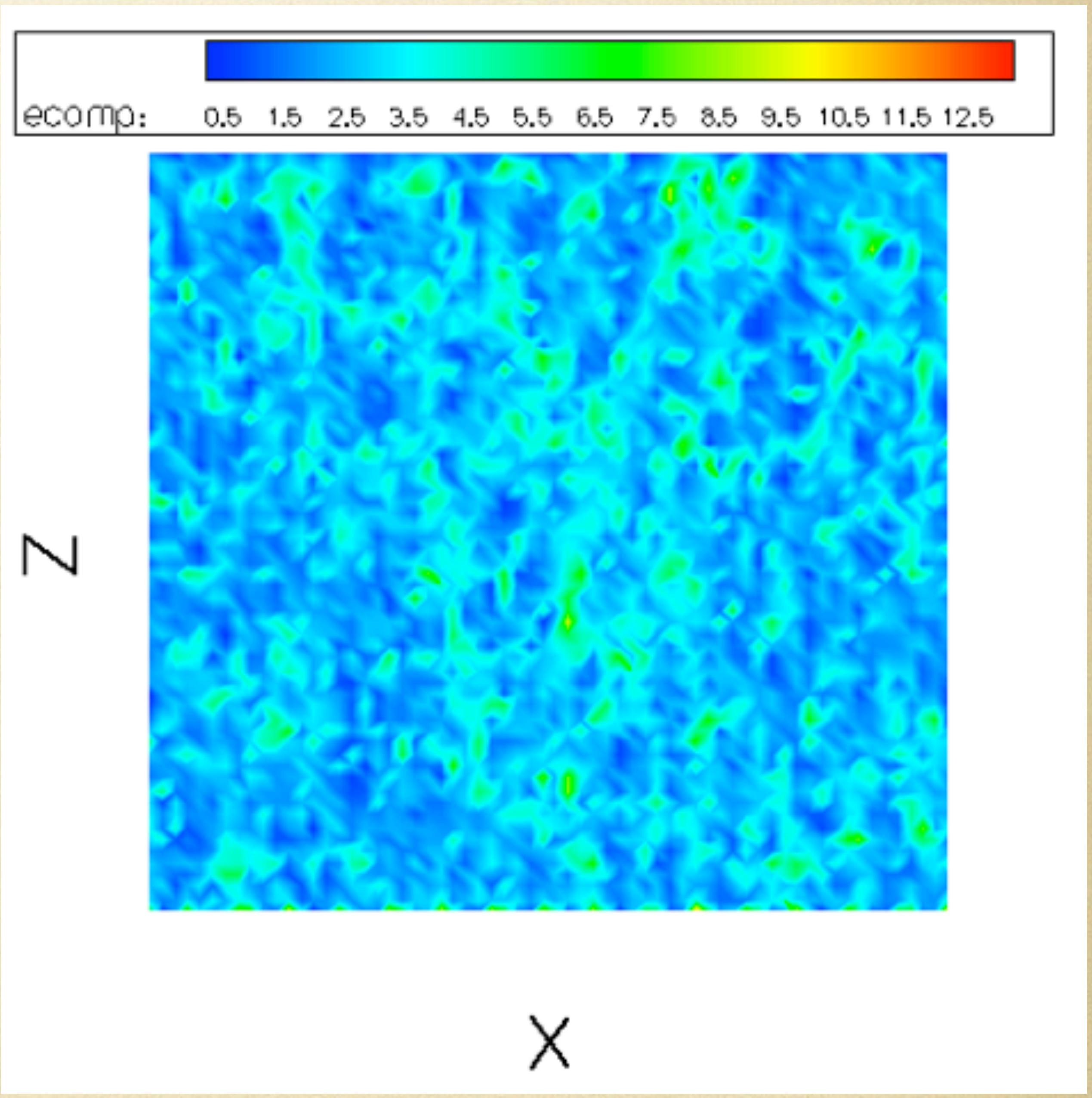
$z(d)$

excitations next to the perturbing boundary

Perturbation wavelength: example

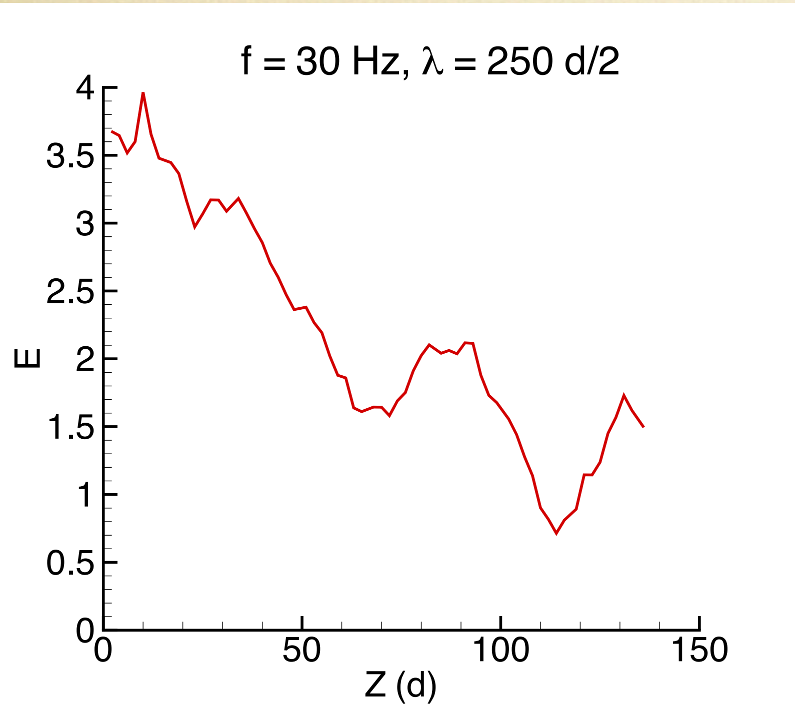


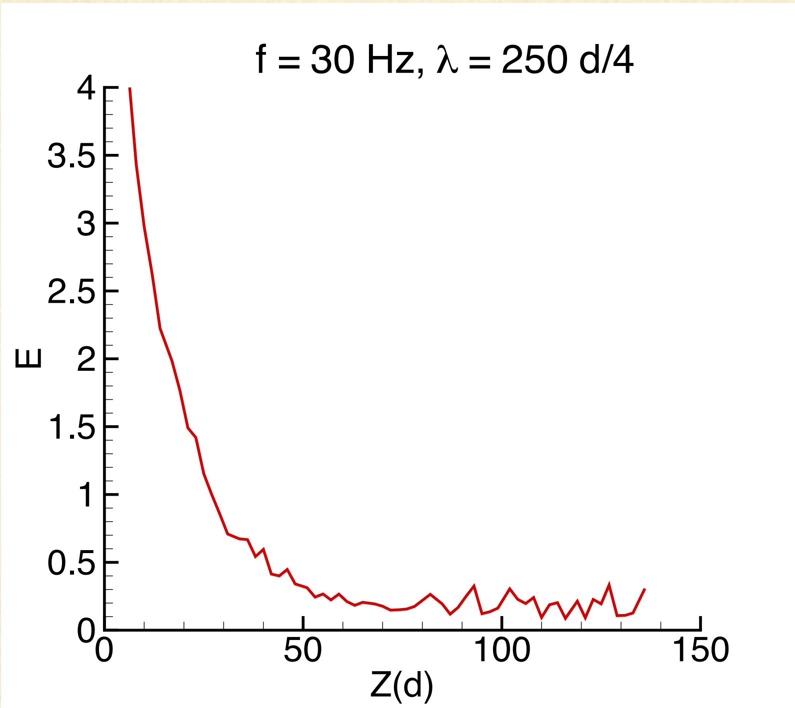
$\lambda = 250\ d$

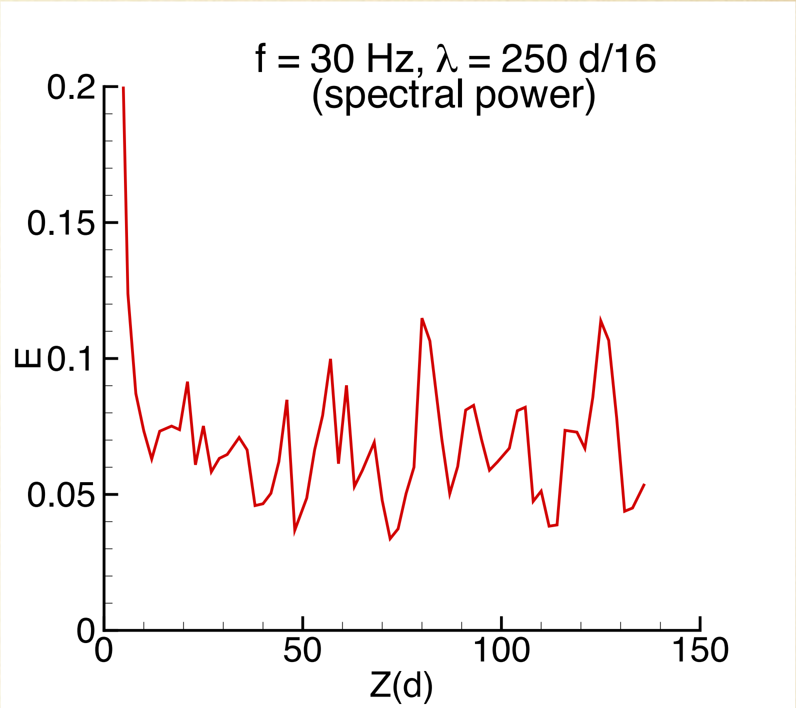


$\lambda \approx 16\ d$

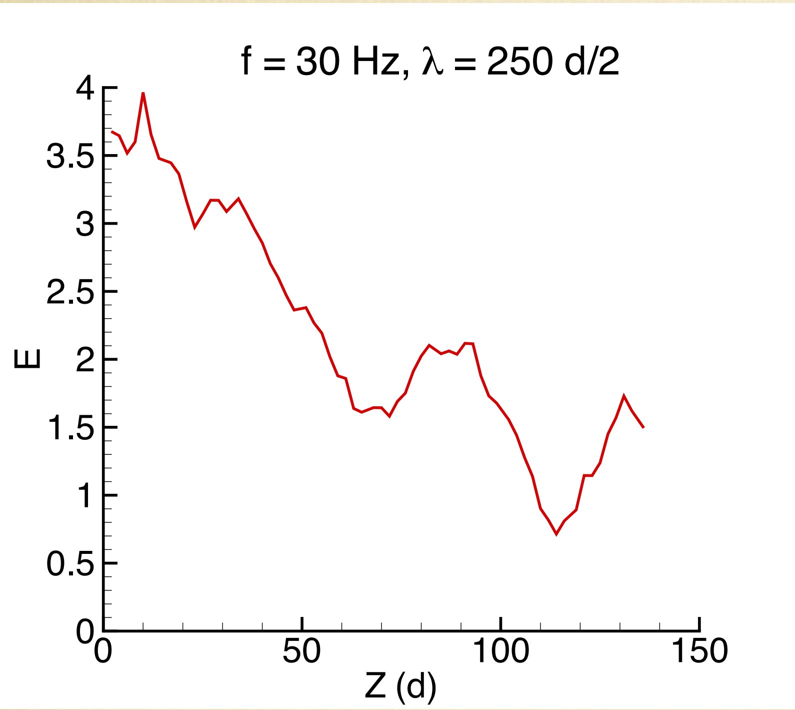
Perturbation wavelength

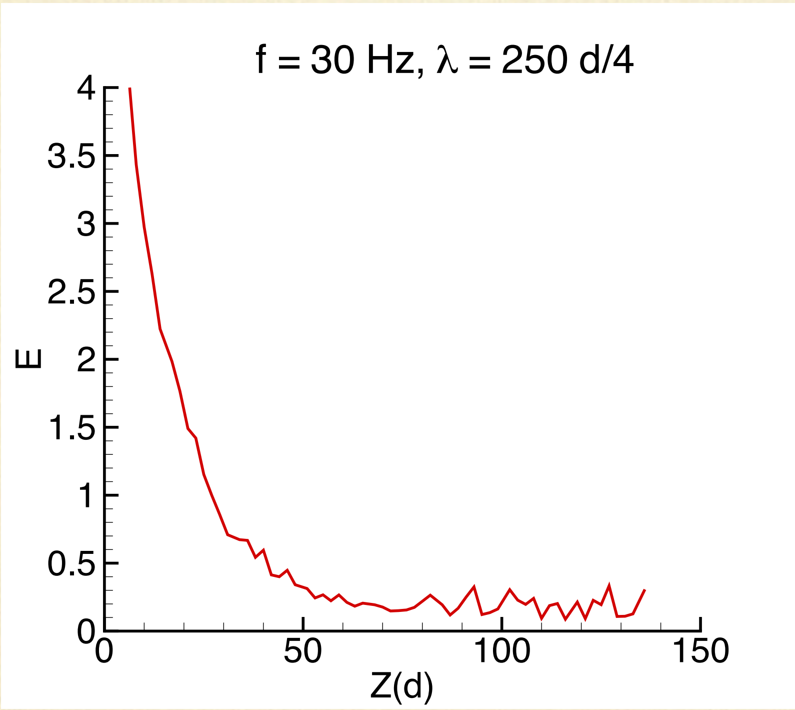


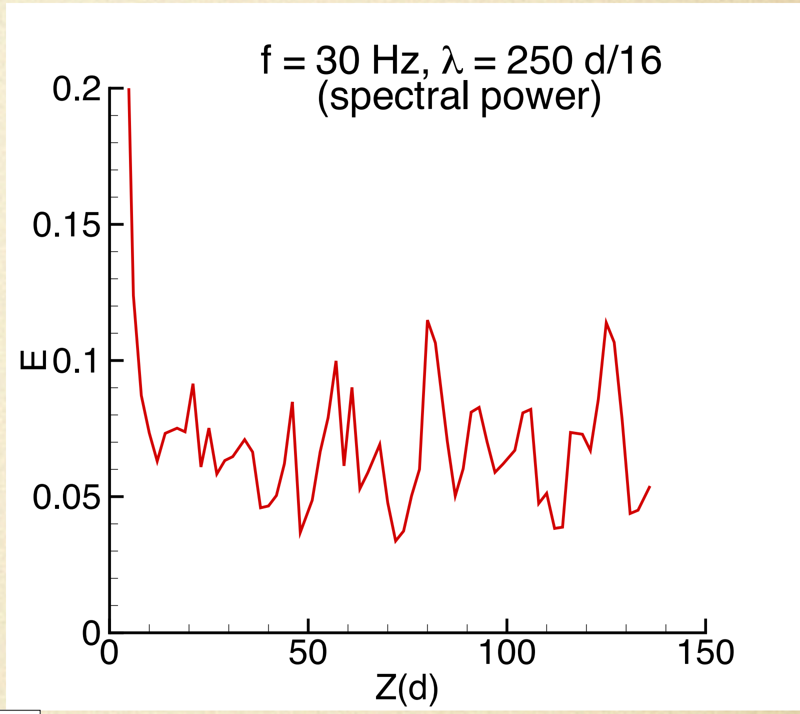


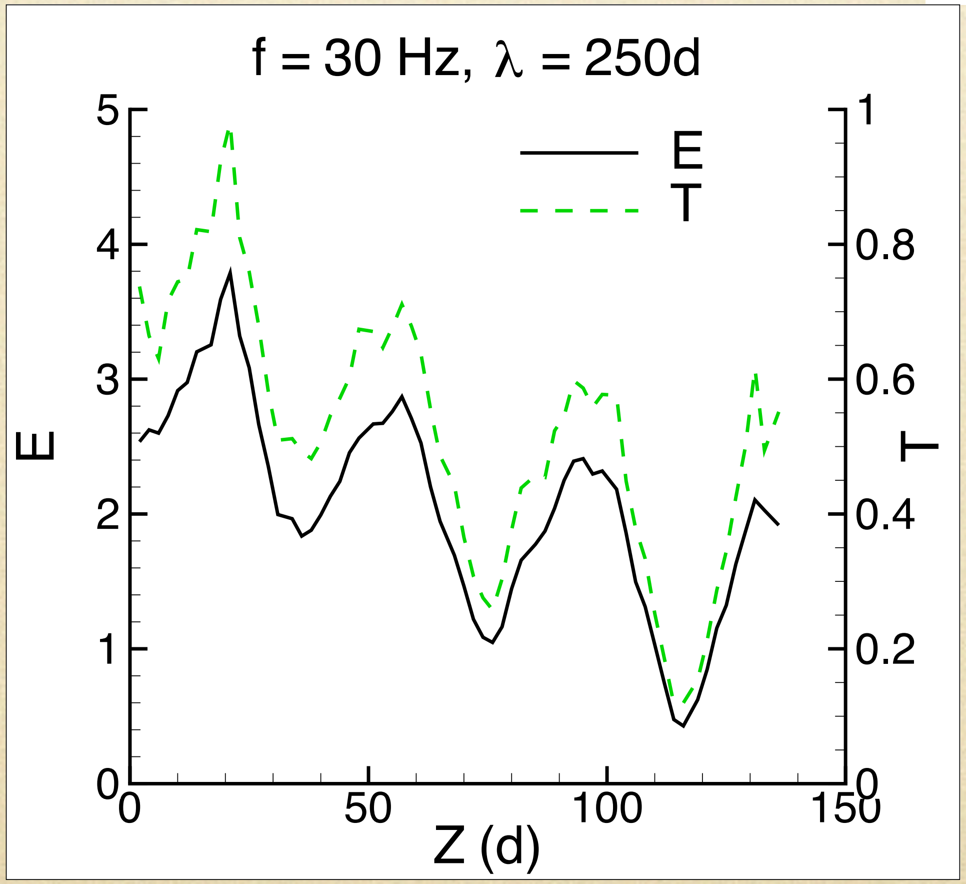


Perturbation wavelength

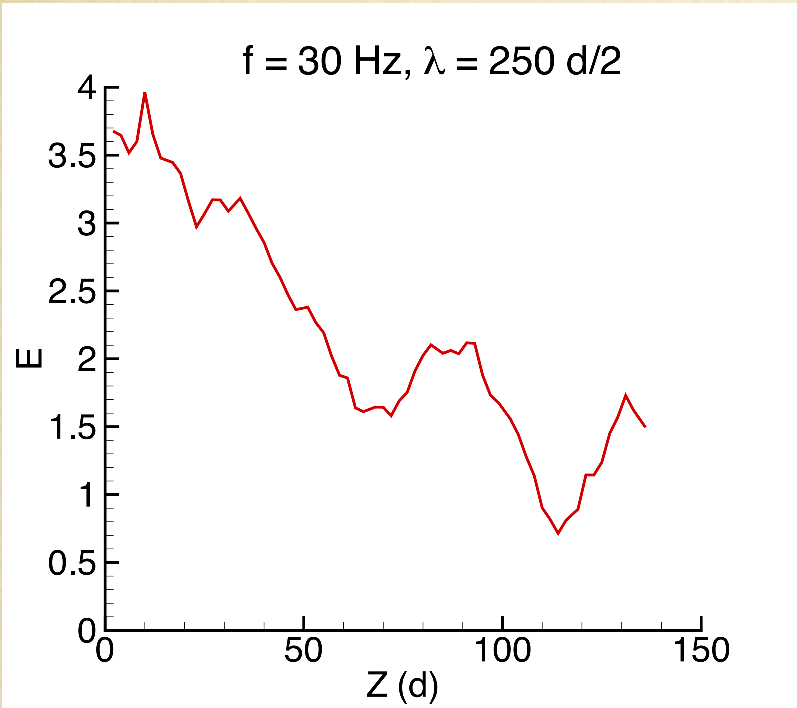


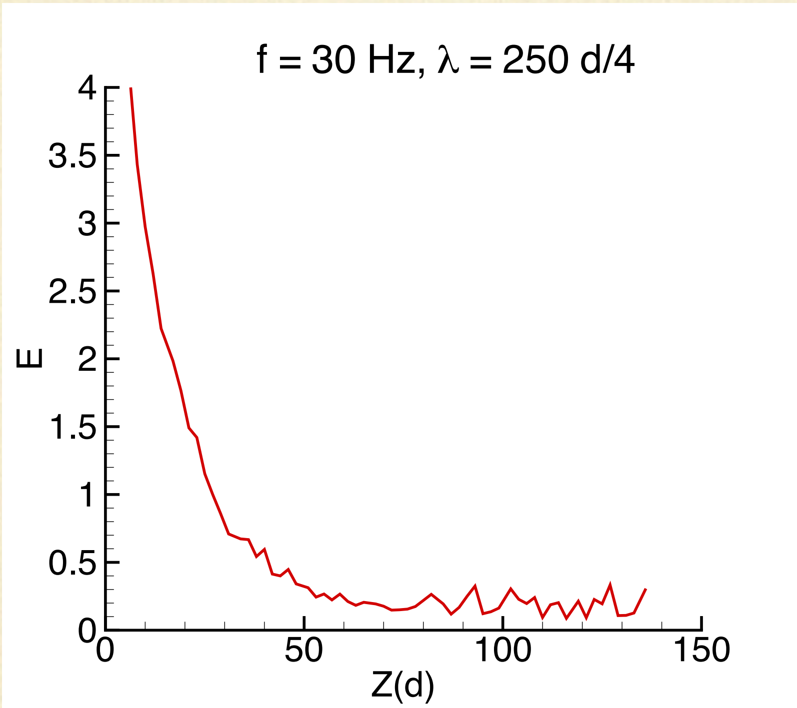


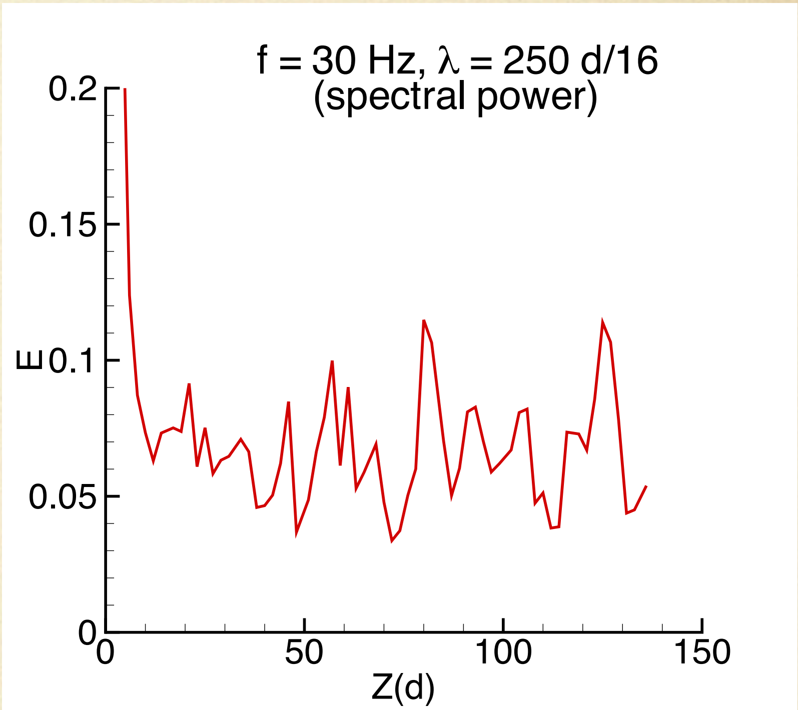




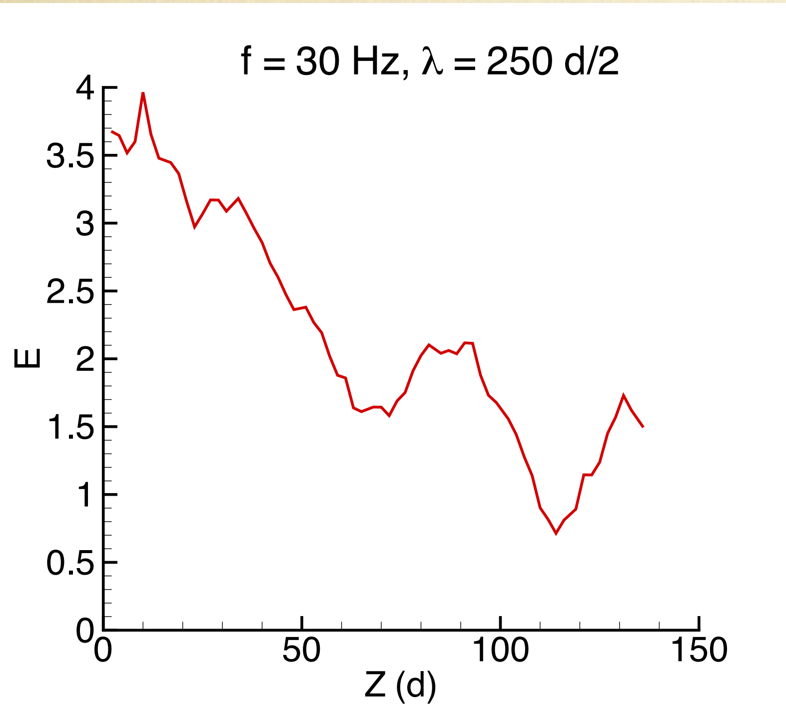
Perturbation wavelength

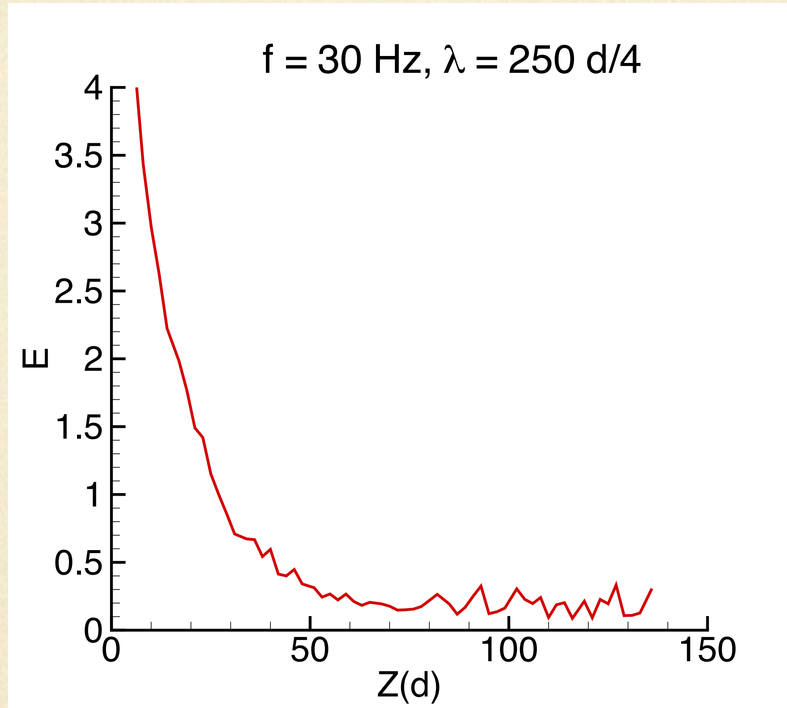


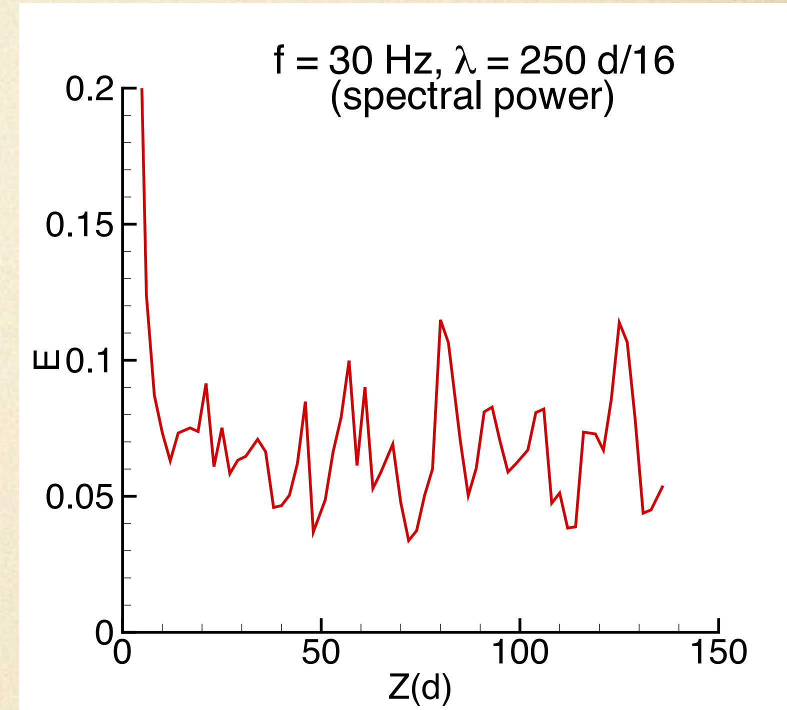




Perturbation wavelength

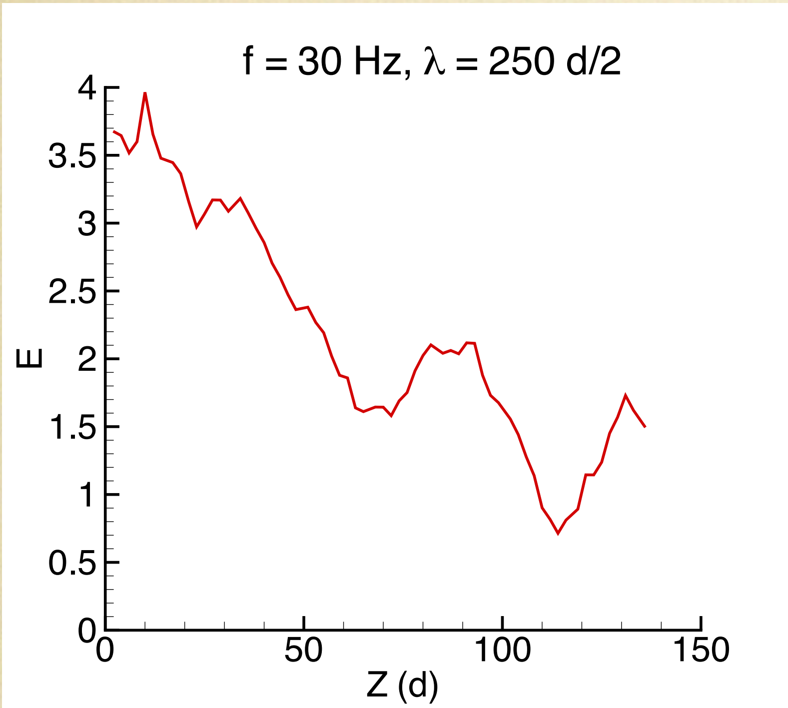


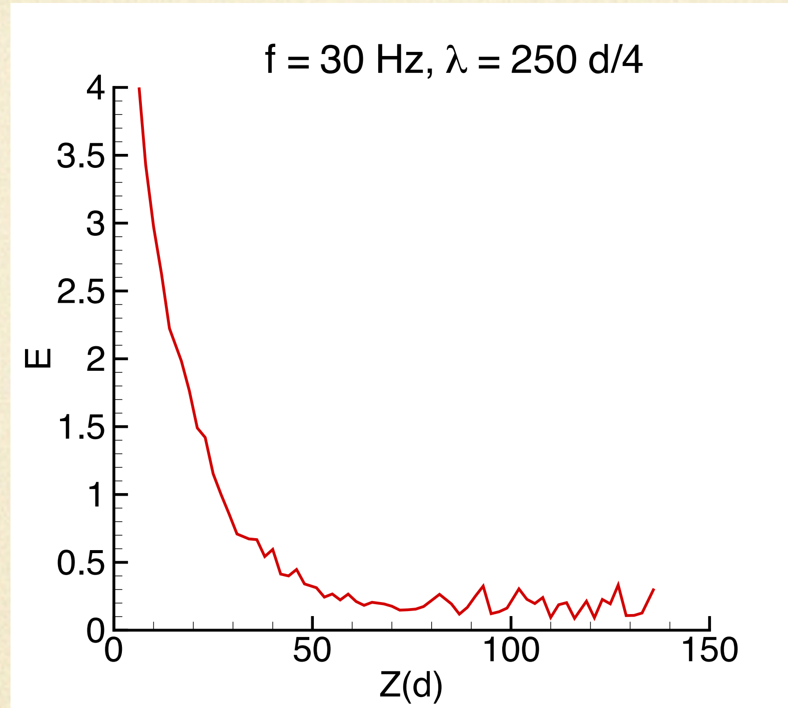


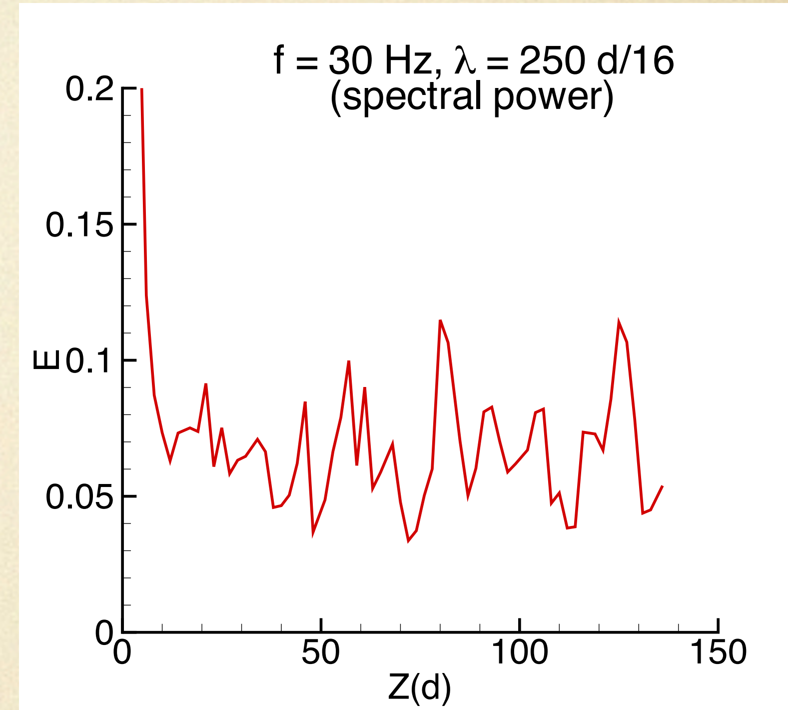


- strong influence of the wavelength of the imposed perturbation, although it is much larger than particle diameter

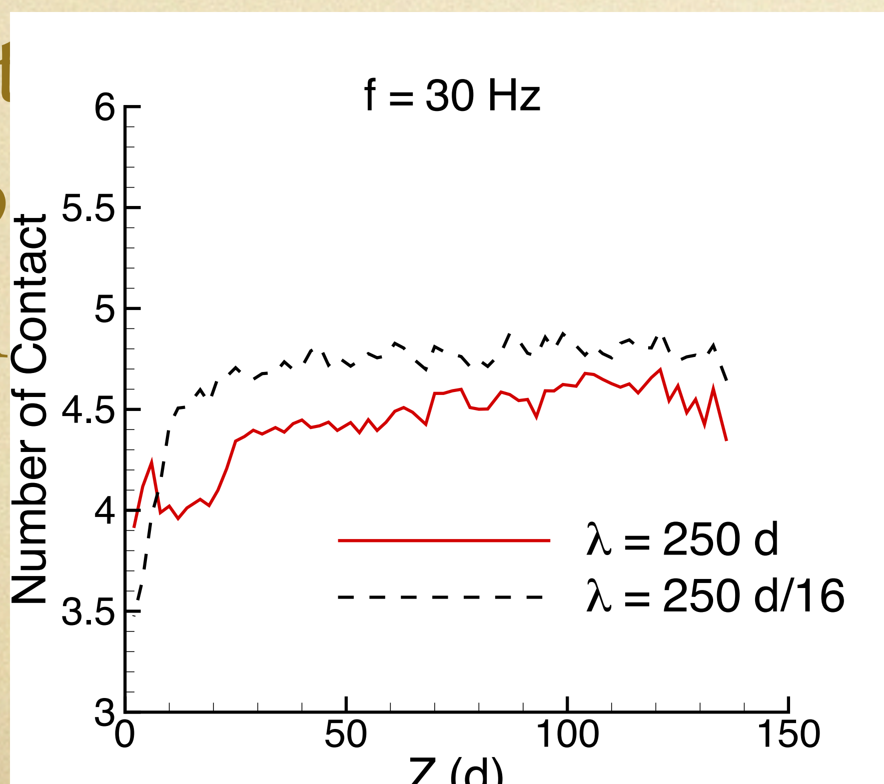
Perturbation wavelength

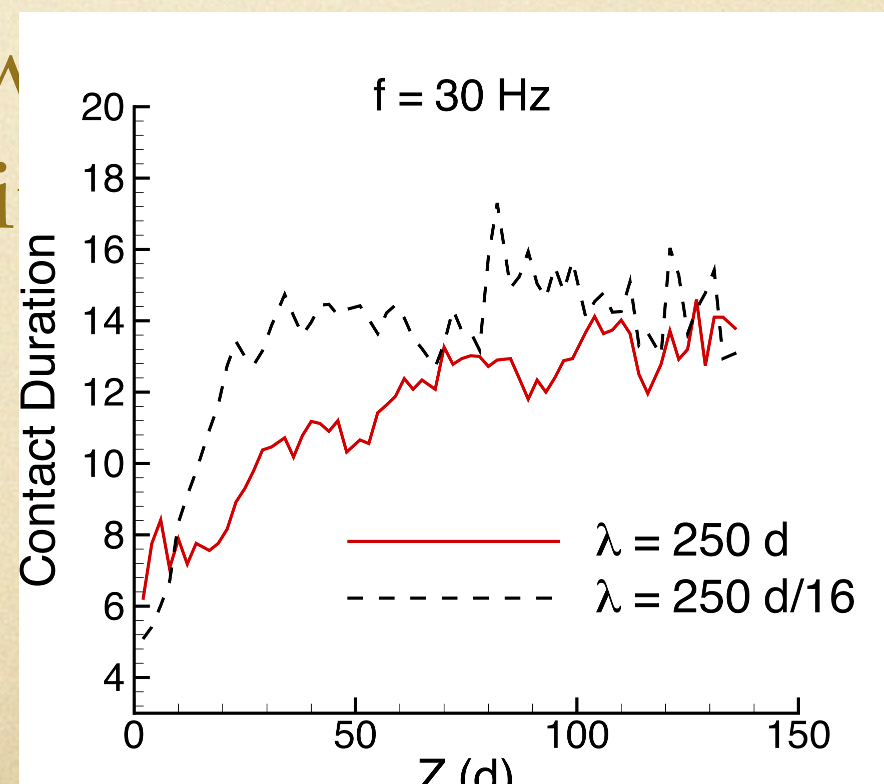




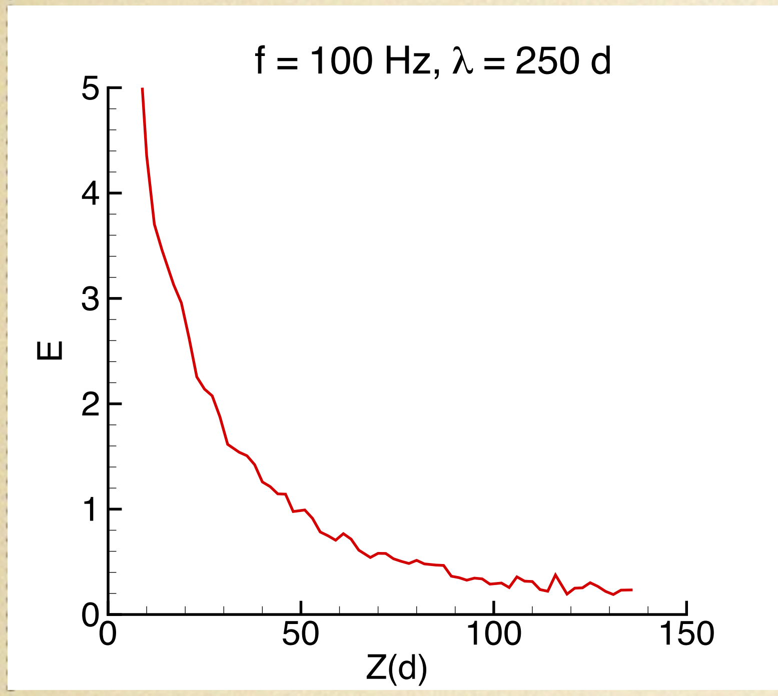


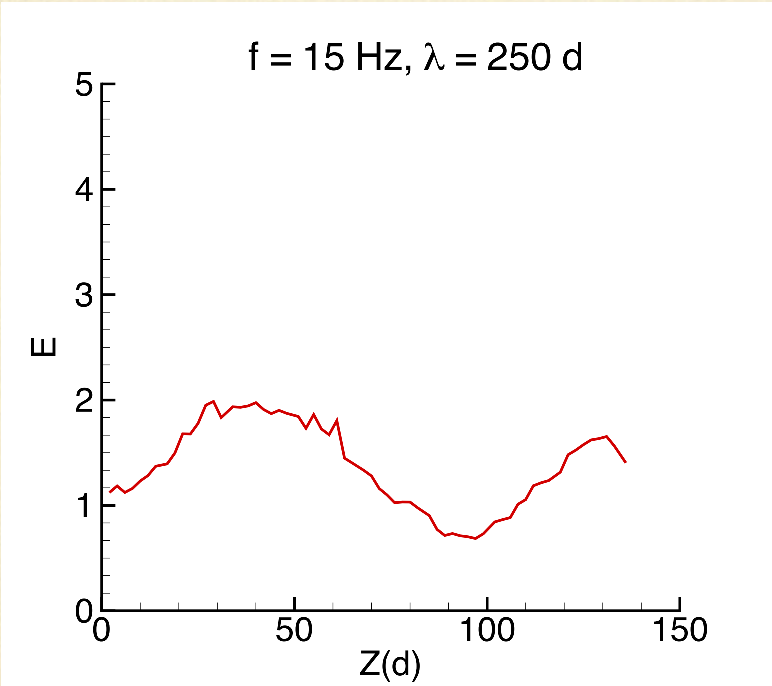
- st … w … posed p … n i … particle d

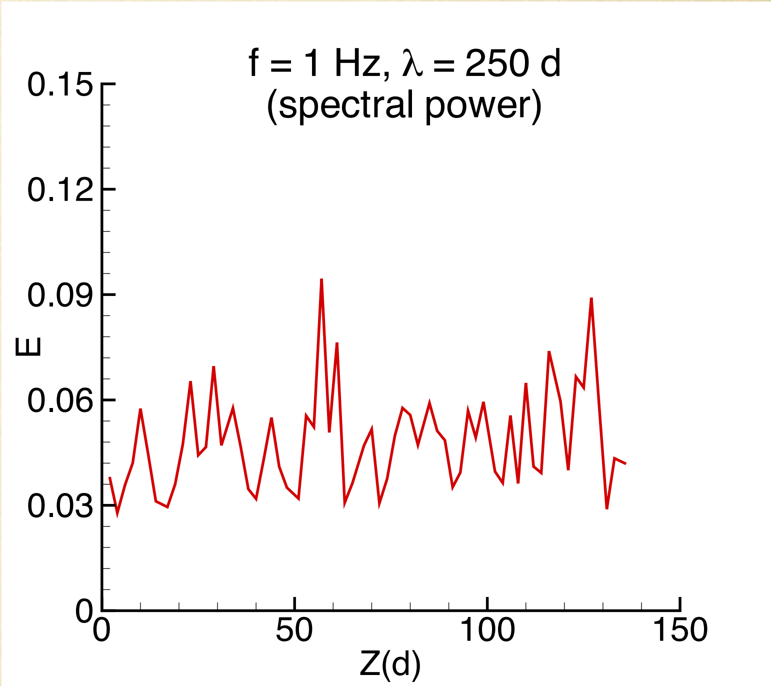




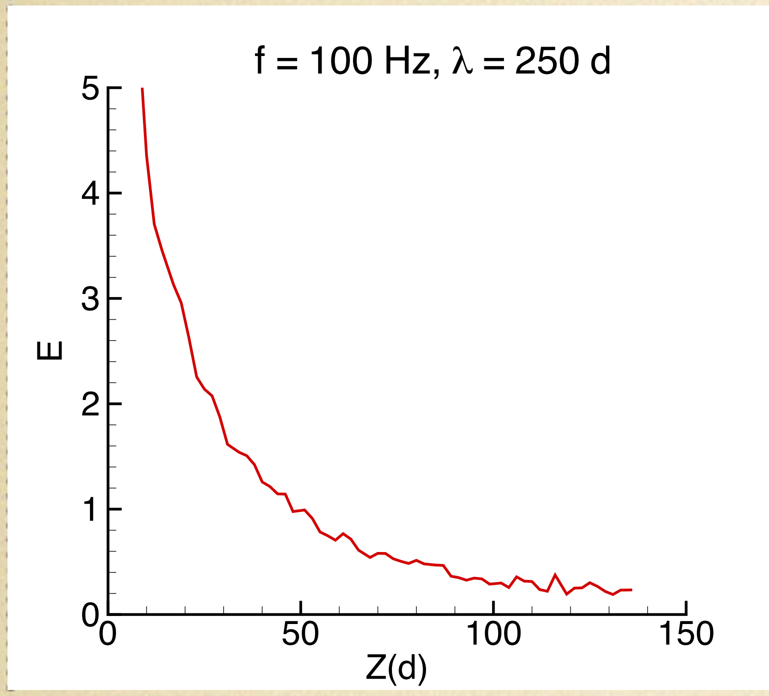
Perturbation frequency

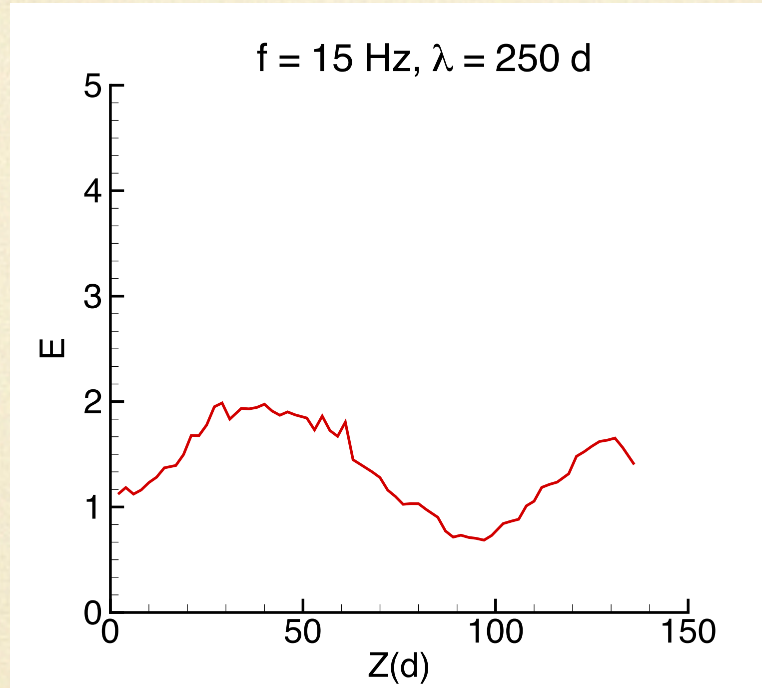


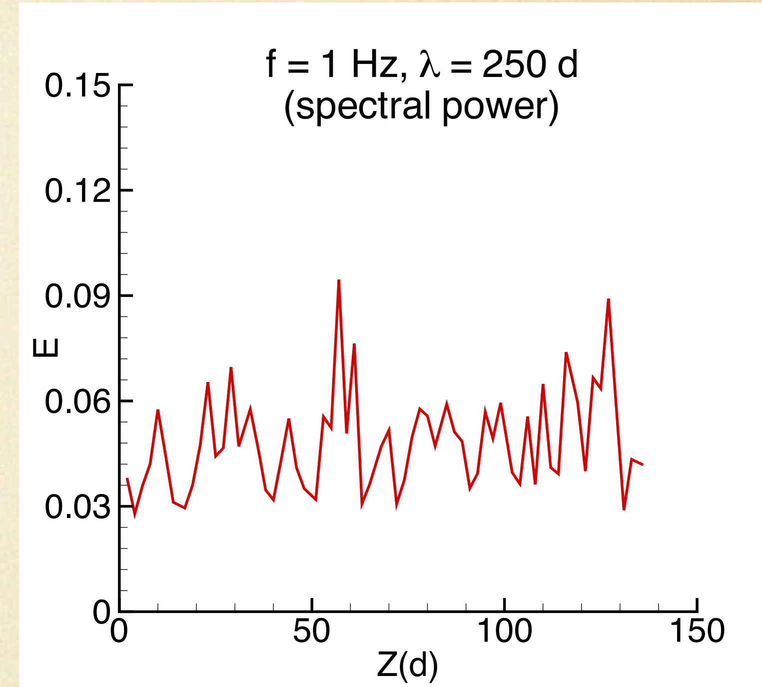


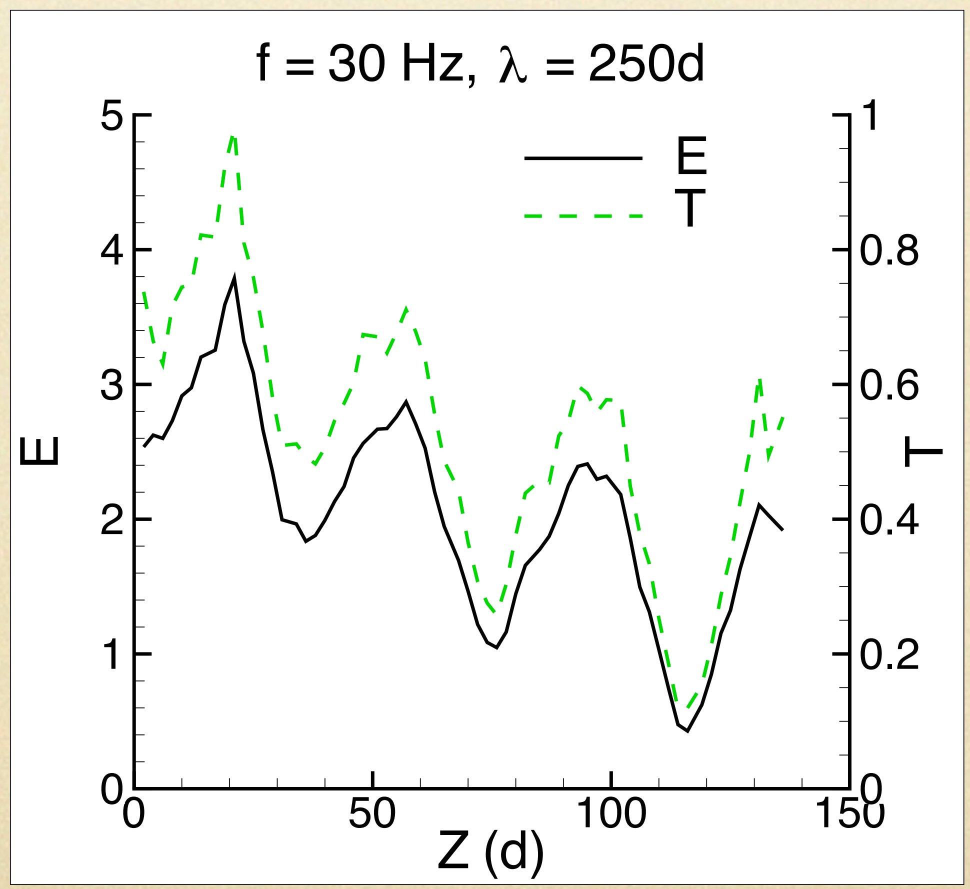


Perturbation frequency

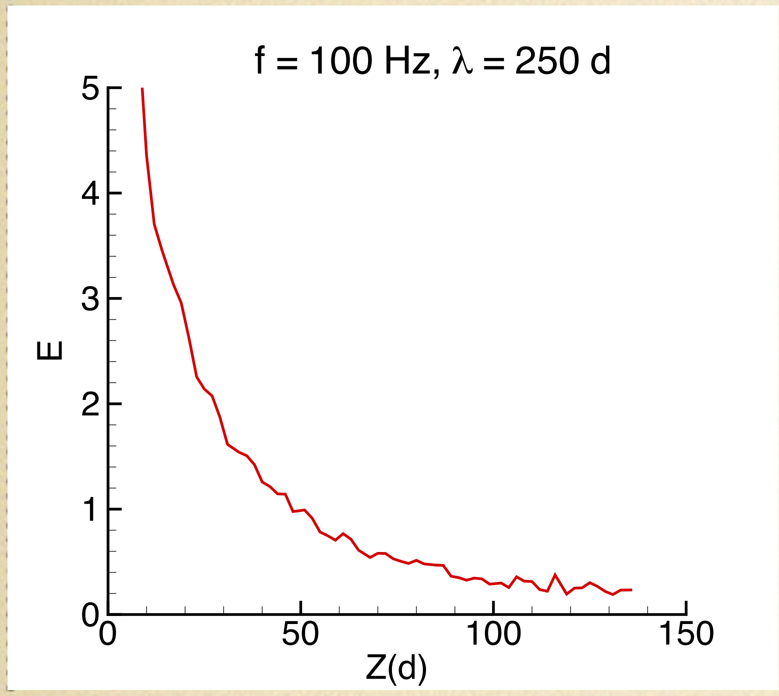


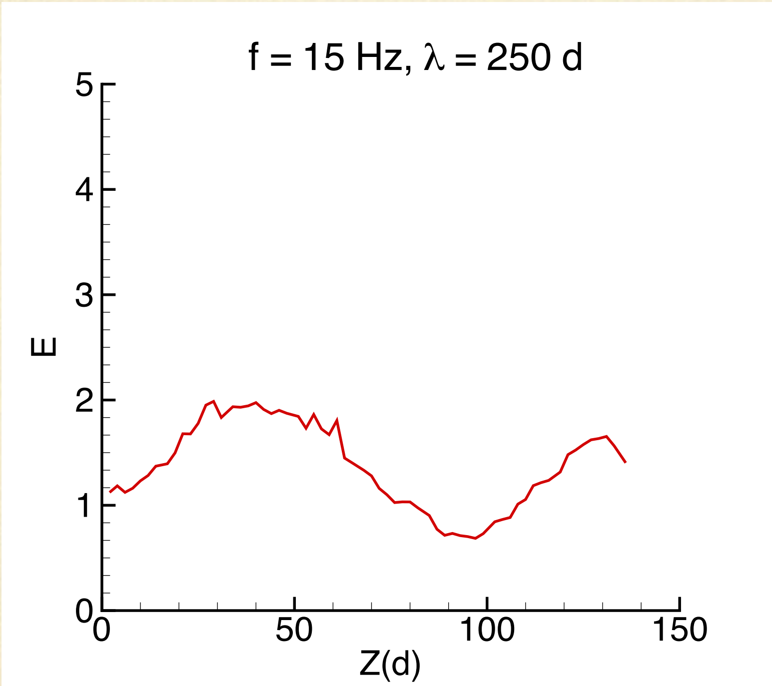


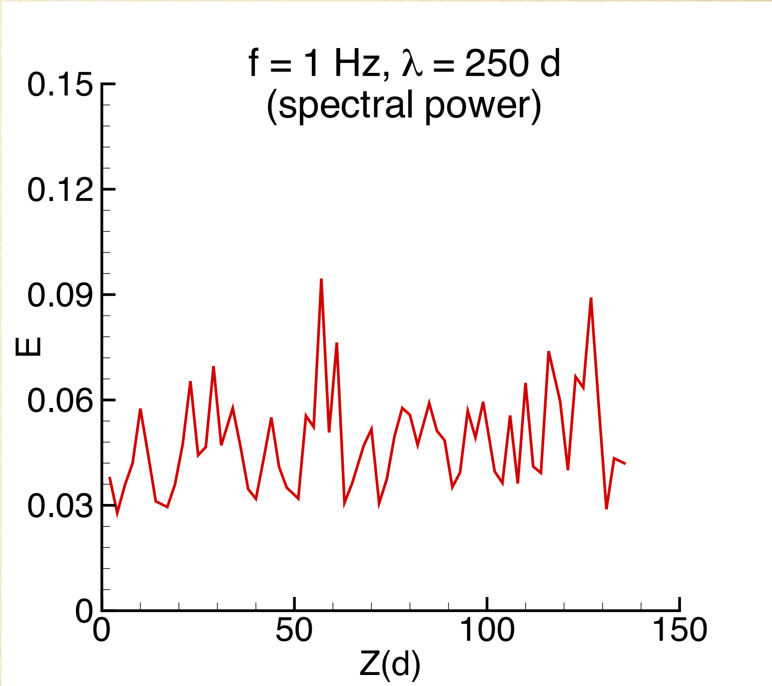




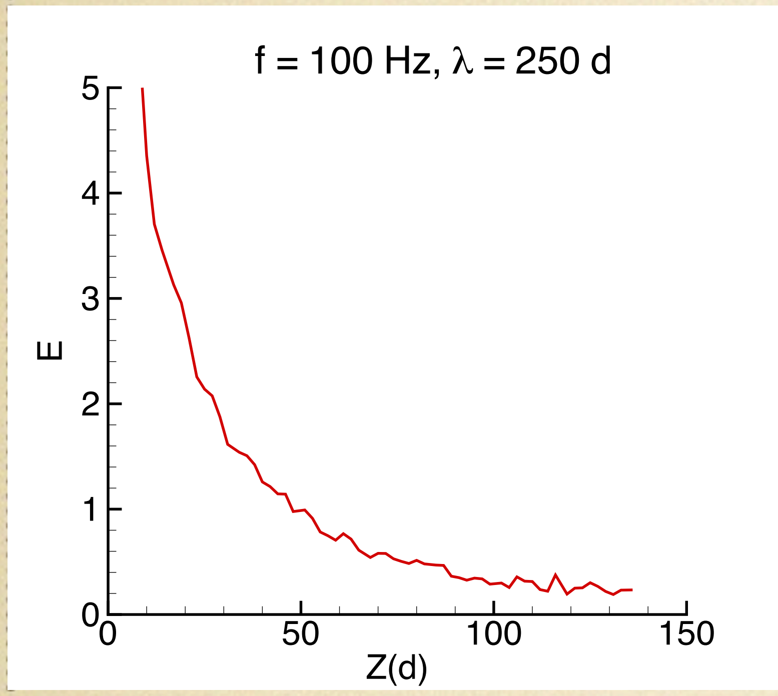
Perturbation frequency

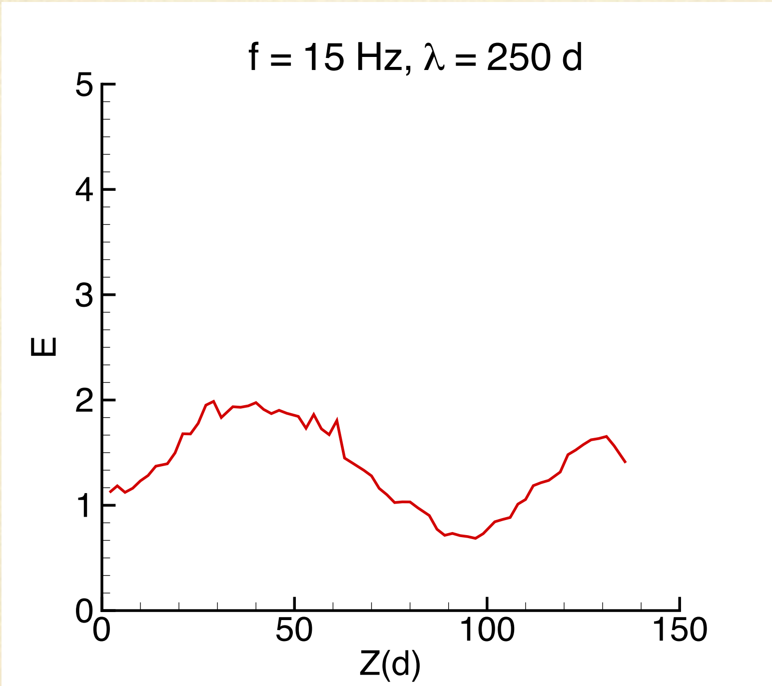


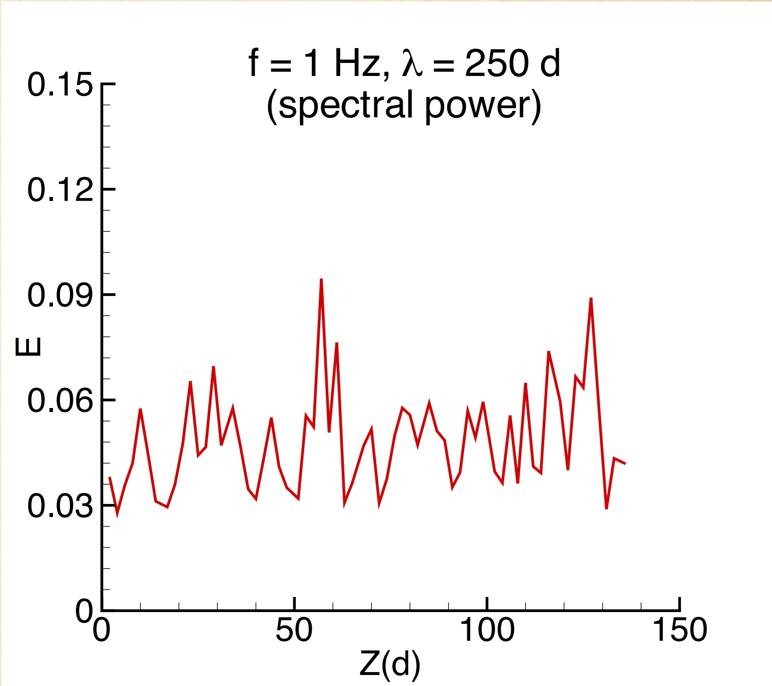




Perturbation frequency

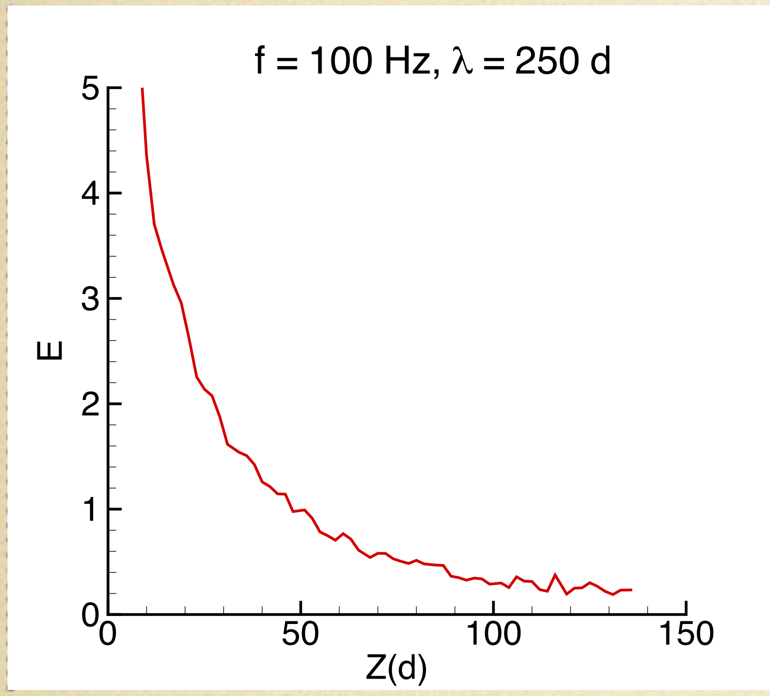


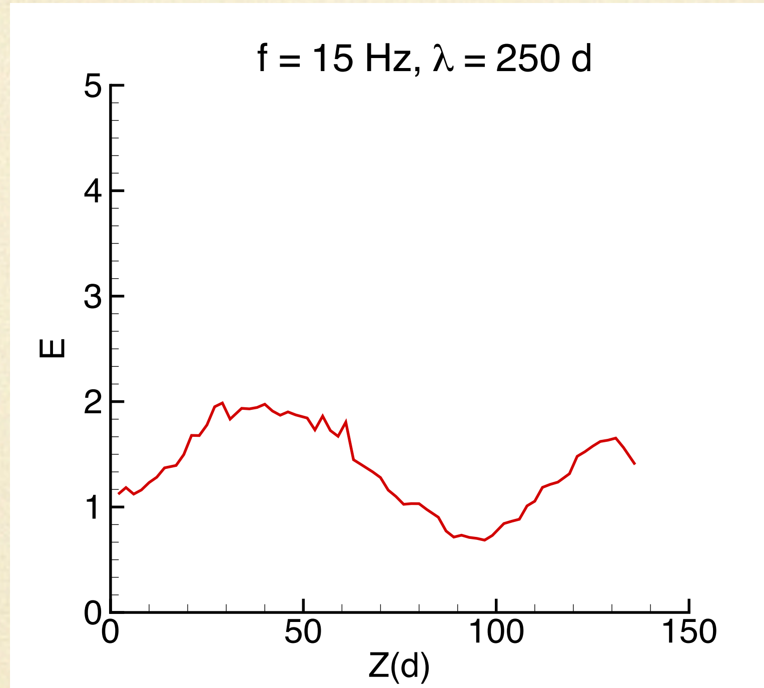


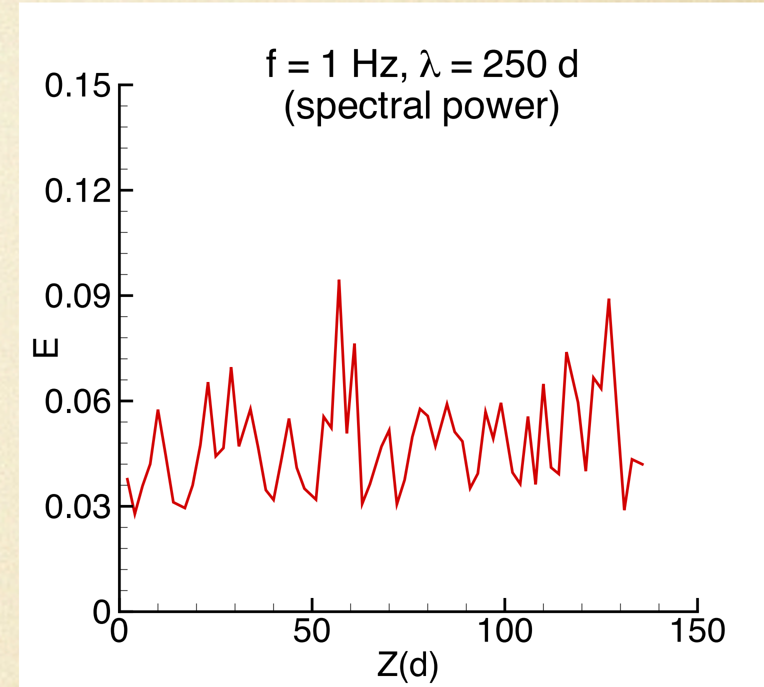


- there is a frequency window where well defined signal propagation is observed

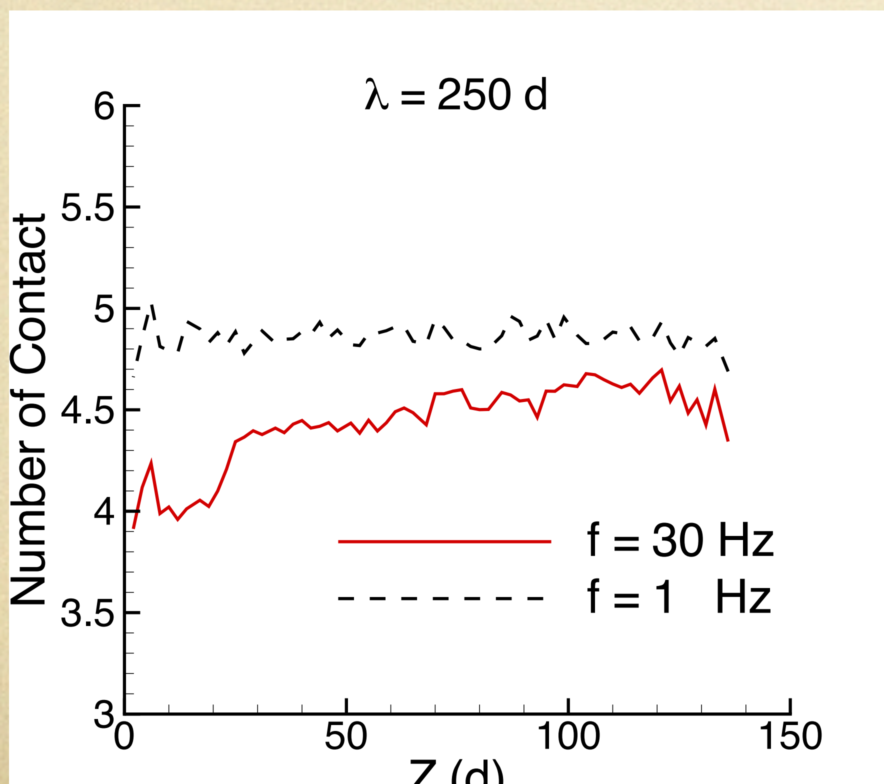
Perturbation frequency

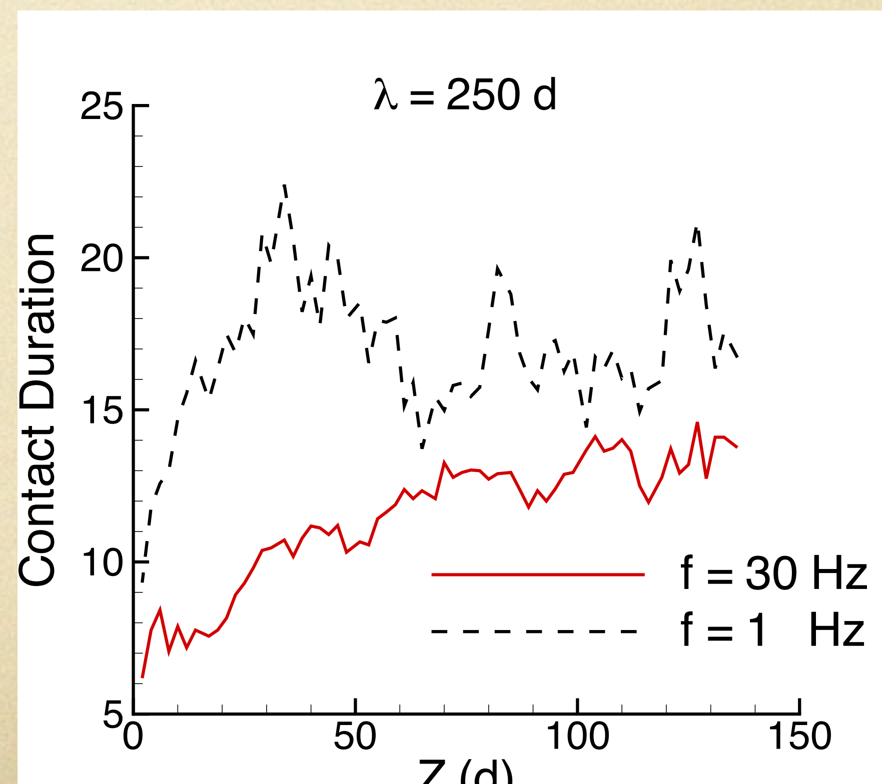






- there is a frequency window where well defined signal propagation is observed





Continuum model

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} - \frac{1}{D}\frac{\partial E}{\partial t} = 0$$

$D, c :$ constants (?)

$k = \frac{2\pi}{\lambda}$

$q :$ wave number in z - direction

$$E(x,z,t) = E_0 e^{\imath\omega t} e^{\imath k x} e^{\imath q z}$$

$$q = -|q| e^{\imath\phi/2}, \quad |q^2| = \mathcal{X}^2 + (\omega/D)^2, \quad \tan\phi = -\frac{\omega}{(D\mathcal{X})}$$

$$\mathcal{X} = (\omega/c)^2 - k^2$$

Can we explain signal properties based on this simple model?

more precise approach using Green's function solution with similar conclusions

Kondic, Dybenko, Behringer, PRE '09

Continuum model - predictions

Continuum model - predictions

f fixed, k increases $\rightarrow$ q decreases

Continuum model - predictions

f fixed, k increases $\rightarrow$ q decreases ✓

Continuum model - predictions

f fixed, k increases $\rightarrow q$ decreases ✓

f fixed, k increases $\rightarrow$ stronger attenuation

Continuum model - predictions

f fixed, k increases $\rightarrow q$ decreases ✓

f fixed, k increases $\rightarrow$ stronger attenuation ✓

Continuum model - predictions

f fixed, k increases $\rightarrow$ q decreases ✓

f fixed, k increases $\rightarrow$ stronger attenuation ✓

k fixed, f decreases $\rightarrow$ q decreases

Continuum model - predictions

f fixed, k increases $\rightarrow q$ decreases ✓

f fixed, k increases $\rightarrow$ stronger attenuation ✓

k fixed, f decreases $\rightarrow q$ decreases ✓

Continuum model - predictions

f fixed, k increases $\rightarrow q$ decreases ✓

f fixed, k increases $\rightarrow$ stronger attenuation ✓

k fixed, f decreases $\rightarrow q$ decreases ✓

f increases $\rightarrow q$ increases, stronger attenuation

Continuum model - predictions

f fixed, k increases $\rightarrow q$ decreases ✓

f fixed, k increases $\rightarrow$ stronger attenuation ✓

k fixed, f decreases $\rightarrow q$ decreases ✓

f increases $\rightarrow q$ increases, stronger attenuation ?

Continuum model - predictions

f fixed, k increases $\rightarrow q$ decreases ✓

f fixed, k increases $\rightarrow$ stronger attenuation ✓

k fixed, f decreases $\rightarrow q$ decreases ✓

f increases $\rightarrow q$ increases, stronger attenuation ?

$$\left[\frac{2\pi}{q} \sim d \rightarrow \text{model breaks down?}\right]$$

Continuum Model

- Parameters (D, c) assumed to be perturbation - independent
- Values used:

$$\frac{c}{c*} \approx 0.025; \quad c* = \sqrt{\frac{E}{\rho(1-\sigma^2)}}$$ consistent with time-of-flight

$$D = v_e l/3; \quad v_e \approx c* \rightarrow l \approx 30 - 40d$$

Sheng '95
Jia '04

- Large correlation length! Related to force chain structure?
- Correlation length diverging close to jamming?
- More work to be done to understand this issue

Energy and stress propagation during impact on a granular system

- Discuss main features of impact
- Illustrate the effect of material properties on the impact dynamics
- Concentrate on the granular material:
  - force and energy fields in the material itself: topological and other measures

Literature

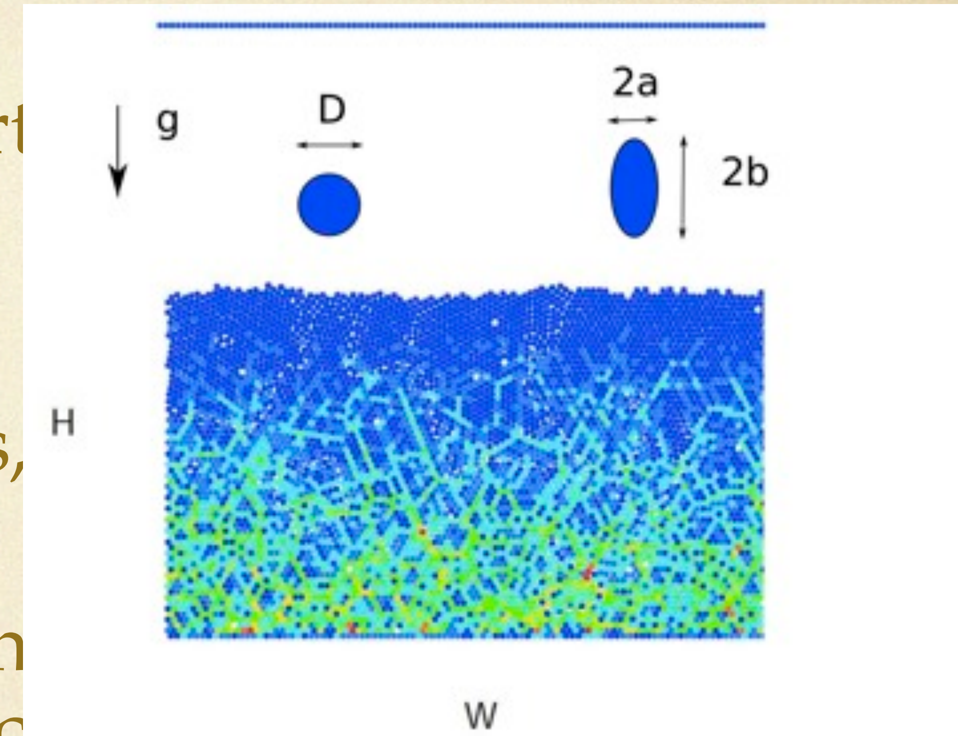
- Large number of works considering impact on granular systems
- Scaling of the penetration depth with impact velocity, size, ...

  Debouf etal, PRE'09; Walsh etal PRL'03; Uehara etal PRL'03; Ciamarra etal PRL'04;...
- Development of effective models based on
  - collisional effects (Seguin etal EPL'09)
  - fluid-like models (de Bruyn Can. J. Phys'04)
  - ballistic approach (Katsuragi&Durian Nat. Phys.'07, Tsimring&Volfson P&G'05); ...
- very few works considering influence of microstructure (Toya etal PRL'04)

Setup

- modeling inelastic, frictional spherical particles (soft particle simulations)
- details:
  - typically polydisperse granular particles, with the sizes randomly sampled in the range $1 \pm r$
  - 6,000 - 90,000 particles in 2D are used; linear springs are implemented, and both static and kinetic friction are considered; gravity is included
  - initial granular configuration: typically particles given random initial velocities and left to settle under gravity; in addition regular packings are considered
  - system: periodic boundaries left-right; absorbing walls top/bottom
  - impacting object large compared to a particle size (currently 5 - 15 average diameters), positioned at a fixed height above the granular particles and given initial velocity downwards
  - in this talk: concentrate (mostly) on shallow impacts

Setup



- modeling inelastic, frictional spherical part simulations)
- details:
  - typically polydisperse granular particles, sampled in the range $1 \pm r$
  - 6,000 - 90,000 particles in 2D are used; lin implemented, and both static and kinetic friction are considered; gravity is included
  - initial granular configuration: typically particles given random initial velocities and left to settle under gravity; in addition regular packings are considered
  - system: periodic boundaries left-right; absorbing walls top/bottom
  - impacting object large compared to a particle size (currently 5 - 15 average diameters), positioned at a fixed height above the granular particles and given initial velocity downwards
  - in this talk: concentrate (mostly) on shallow impacts

Simulations

- Animations: normal force which particles and the intruder experience during impact
- Significant differences between disordered, polydisperse and ordered, monodisperse systems
- structure of the material seem to be playing an important role in determining the intruder’s dynamics

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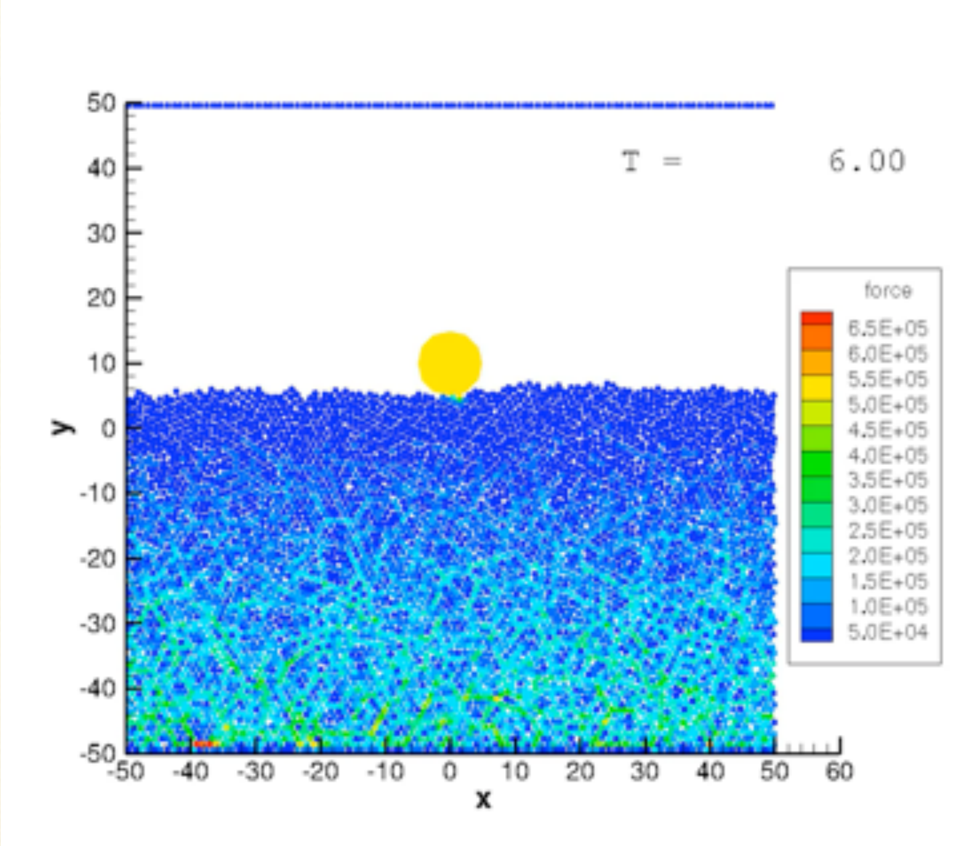

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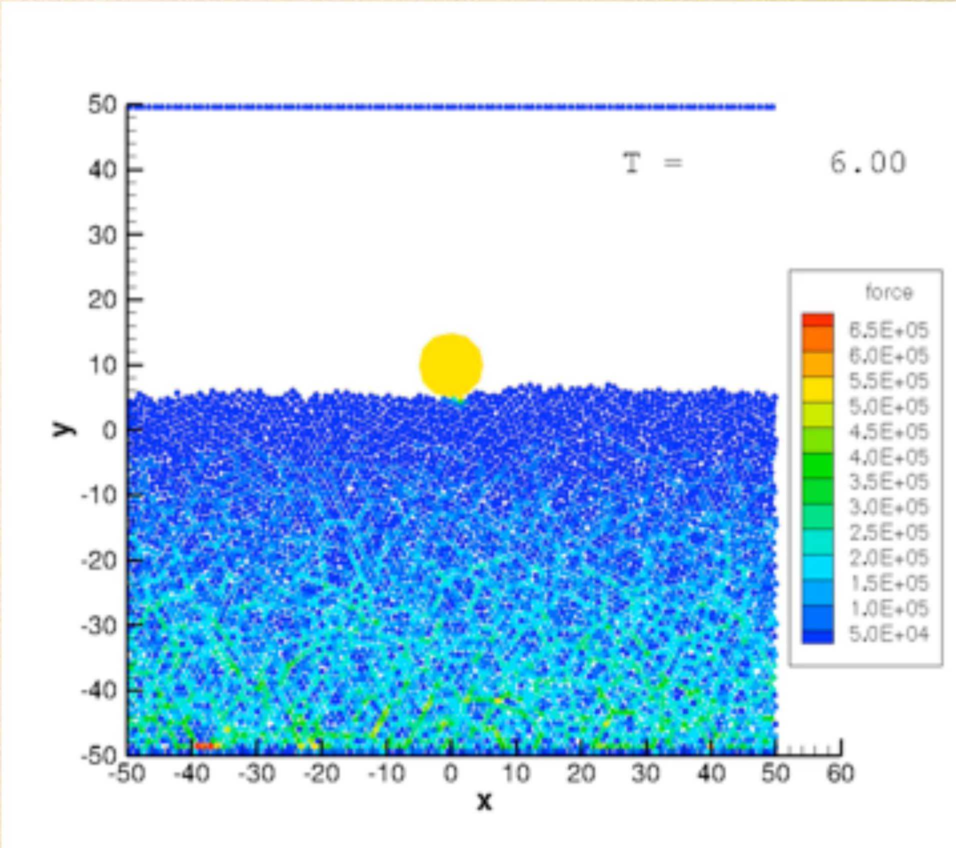
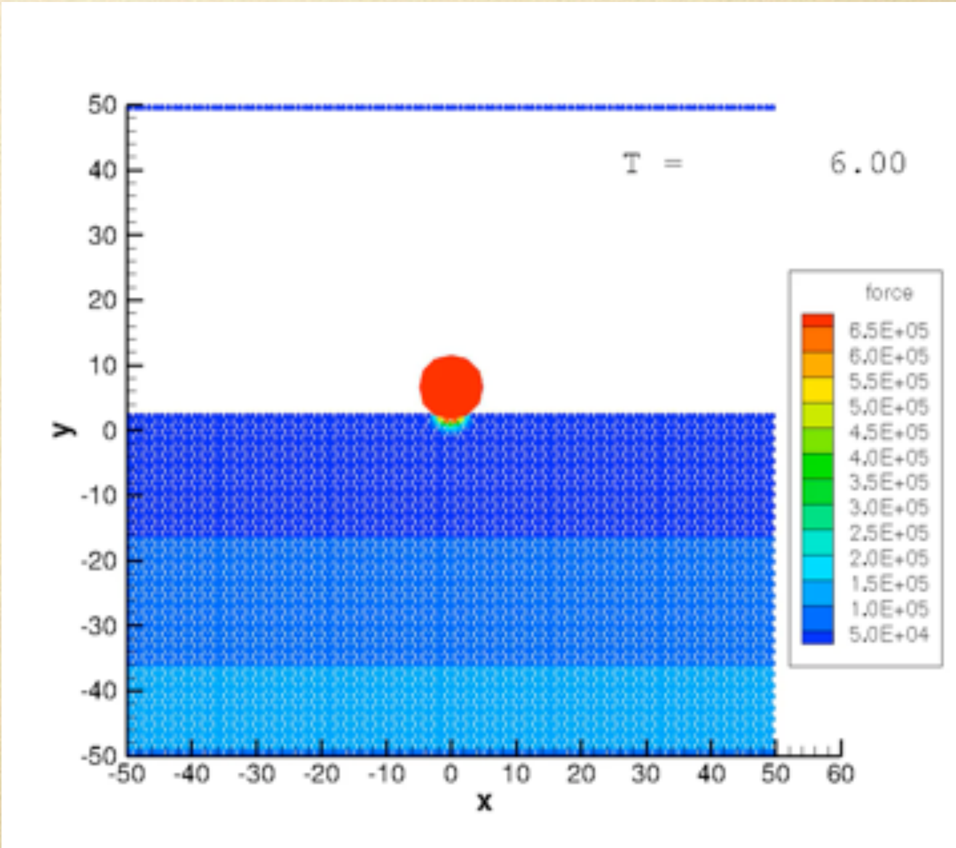
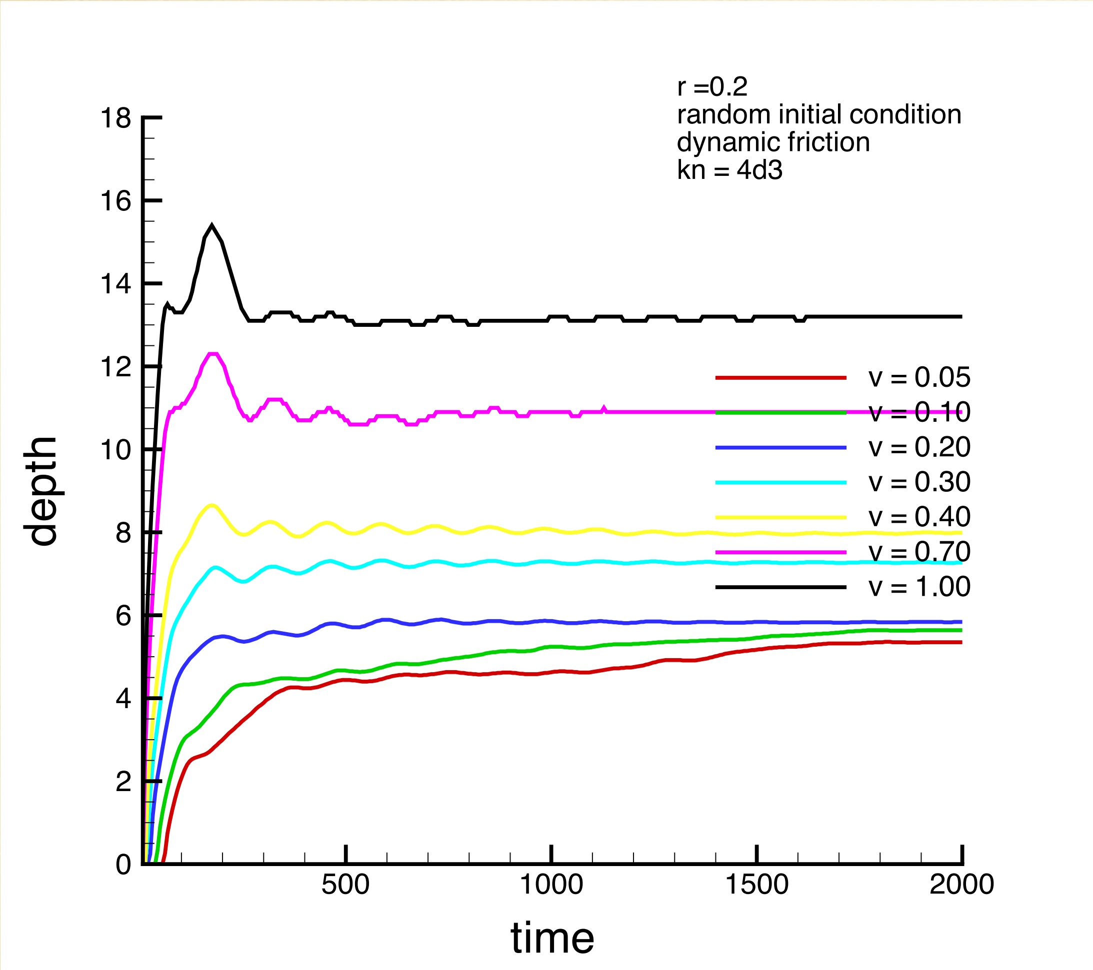




Impactor velocity (basic system)



- depth: the position of the bottom part of impactor with respect to upper boundary of the system particles at the point of impact
- basic system: increase of penetration depth with increased velocity; initial overshoot for larger velocities; elastic damping of oscillations
- considered velocities comparable to the average speed of propagation of elastic waves (speed of sound) $c \approx 2$

$$\tau_c = \pi \sqrt{\frac{d}{2gk_n}}$$

$$k_n = 4 \cdot 10^3; d \approx 4mm; \tau_c \approx 7 \cdot 10^{-4}; d/\tau_c \approx 5m/s$$

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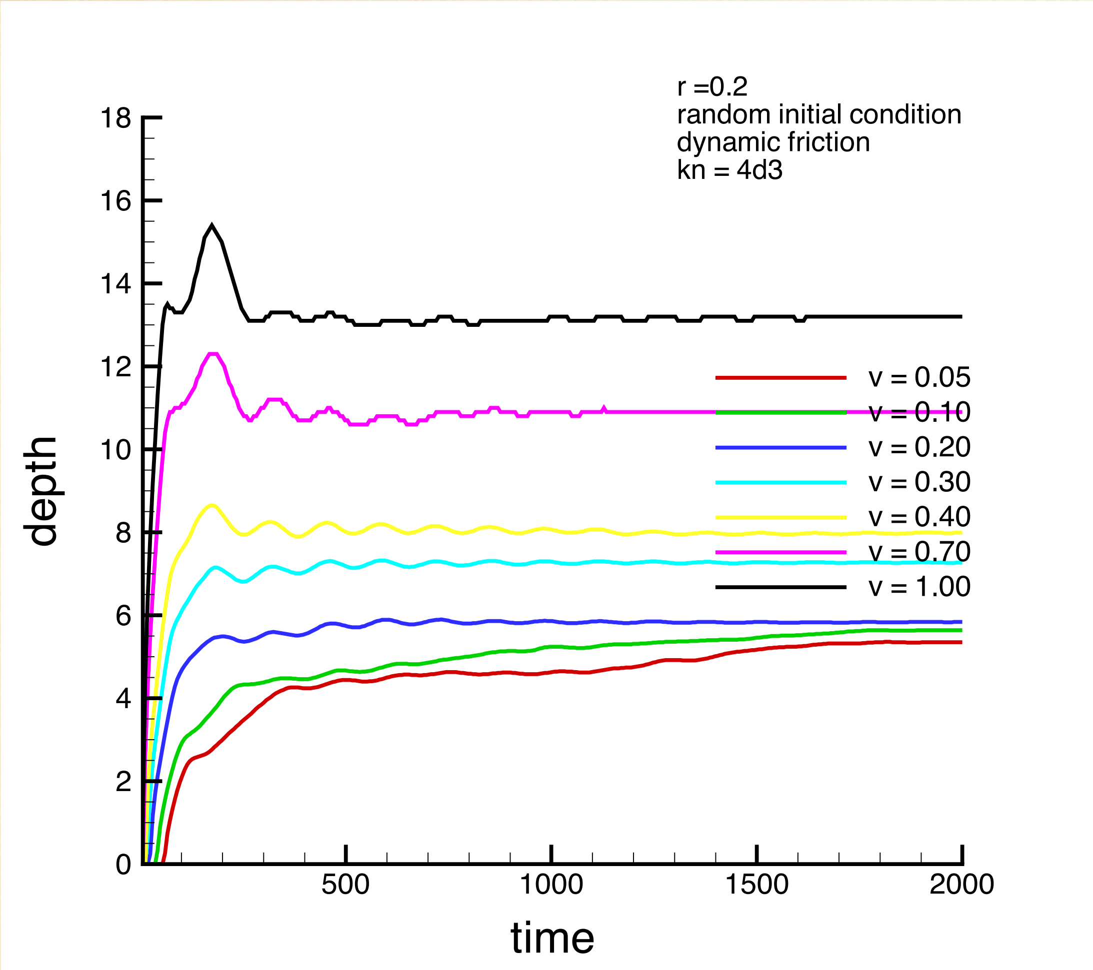

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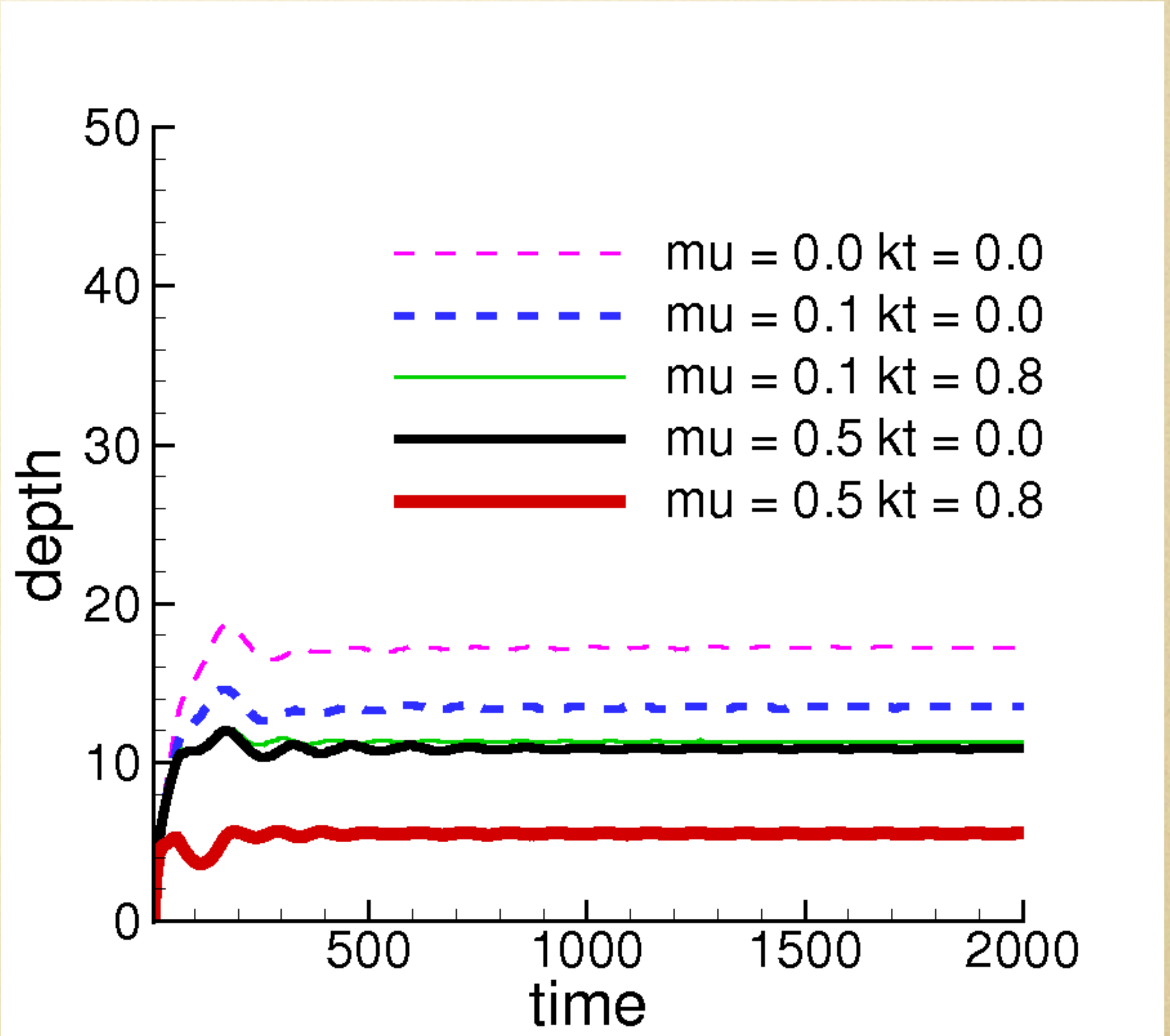
Scales

- length scale: particle diameter d
- time scale: binary collision time $\tau_c = \pi\sqrt{\frac{d}{2gk_n}}$
- velocity scale d/τ_c

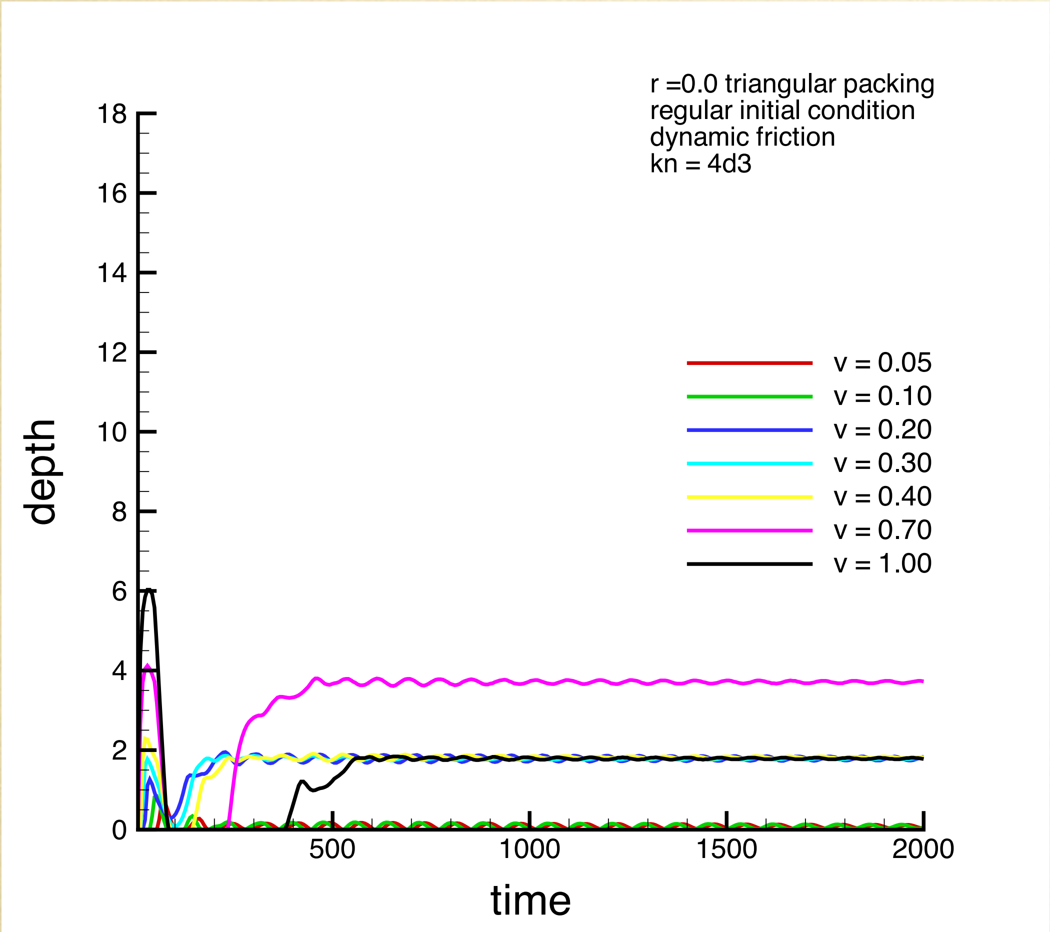
$$k_n = 4 \cdot 10^3; d \approx 4mm; \tau_c \approx 7 \cdot 10^{-4}; d/\tau_c \approx 5m/s$$

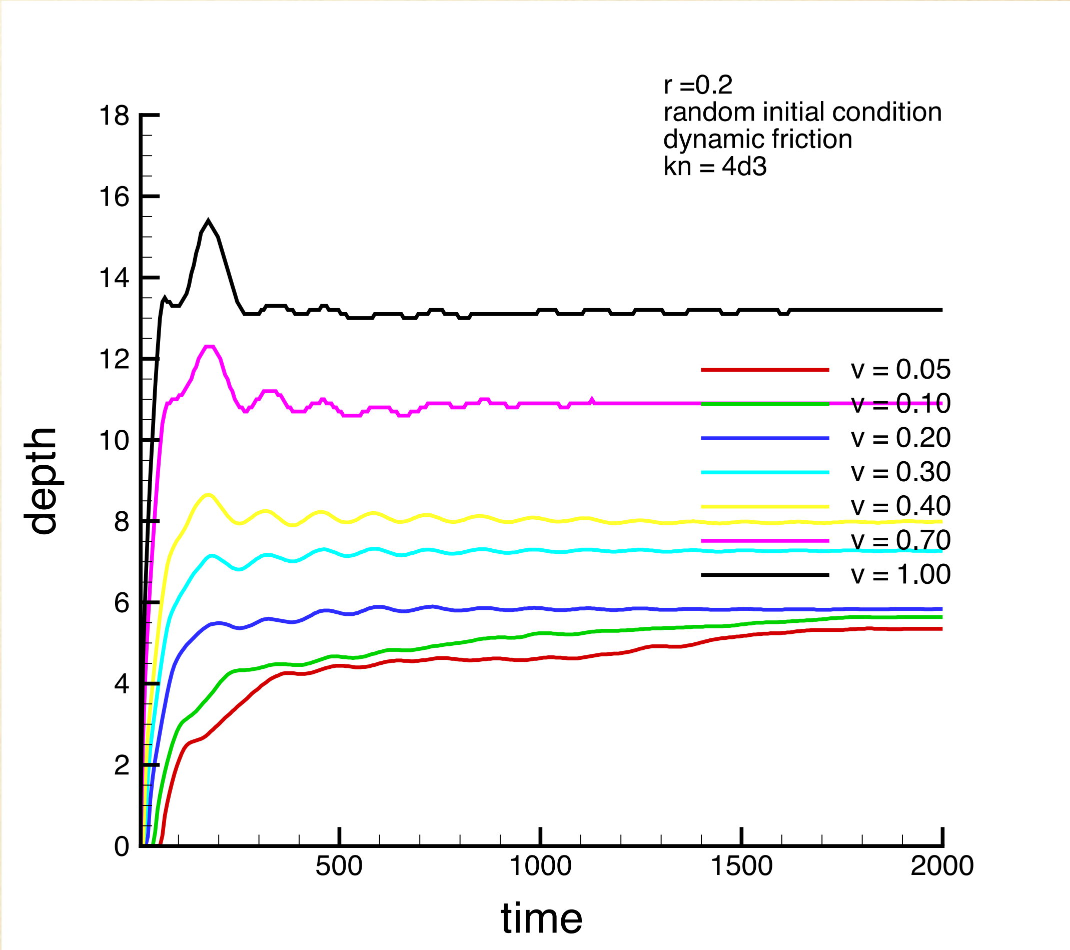
Material parameters

- simulations allow for precise analysis of the influence of system (size, particle properties) and intruder properties (size, density, shape) on penetration
- here: concentrate on the influence of
  - friction
  - packing and polydispersity



System preparation: order versus disorder



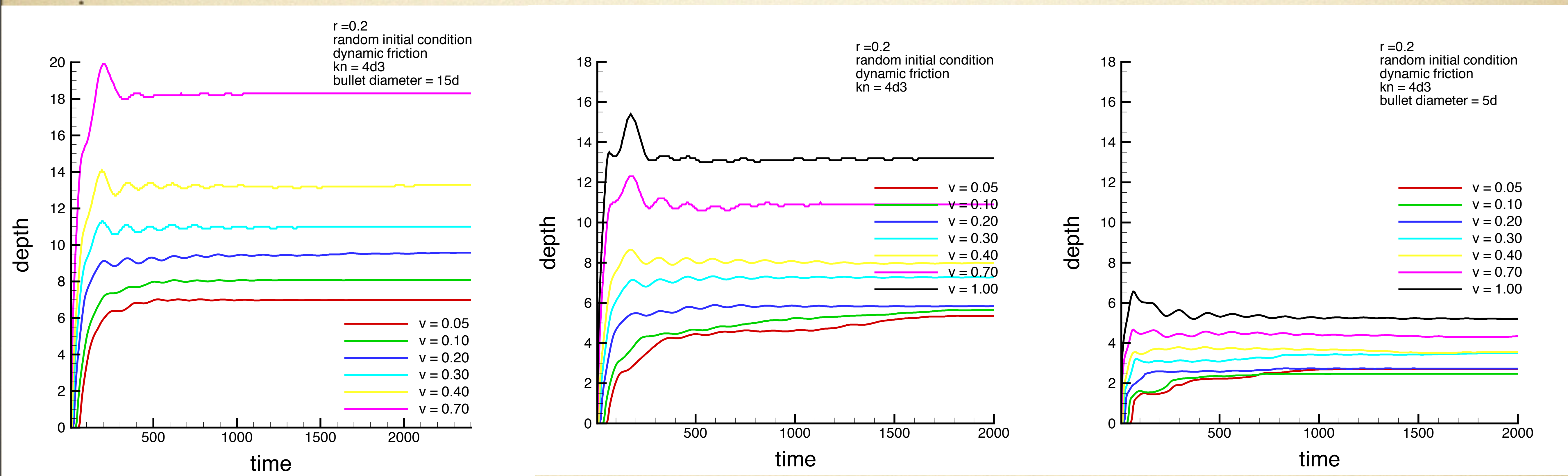


triangular/hexagonal

random

- deeper penetration into disordered polydisperse systems
- concentrate on force field structure and its influence on the dynamics of the impact

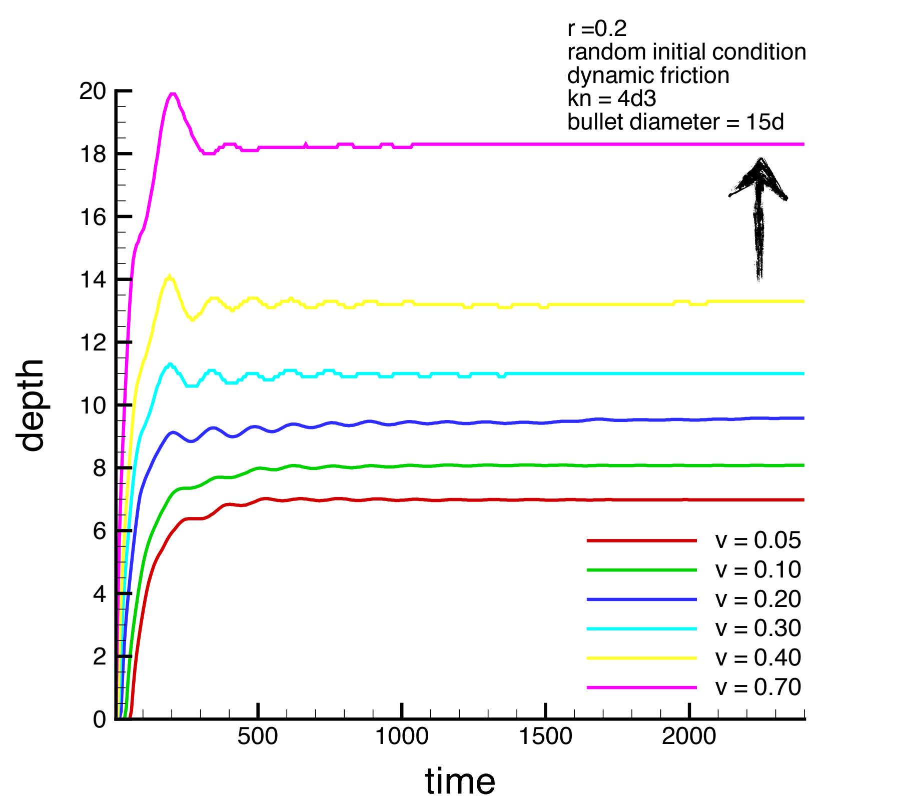
Impactor size (basic system)



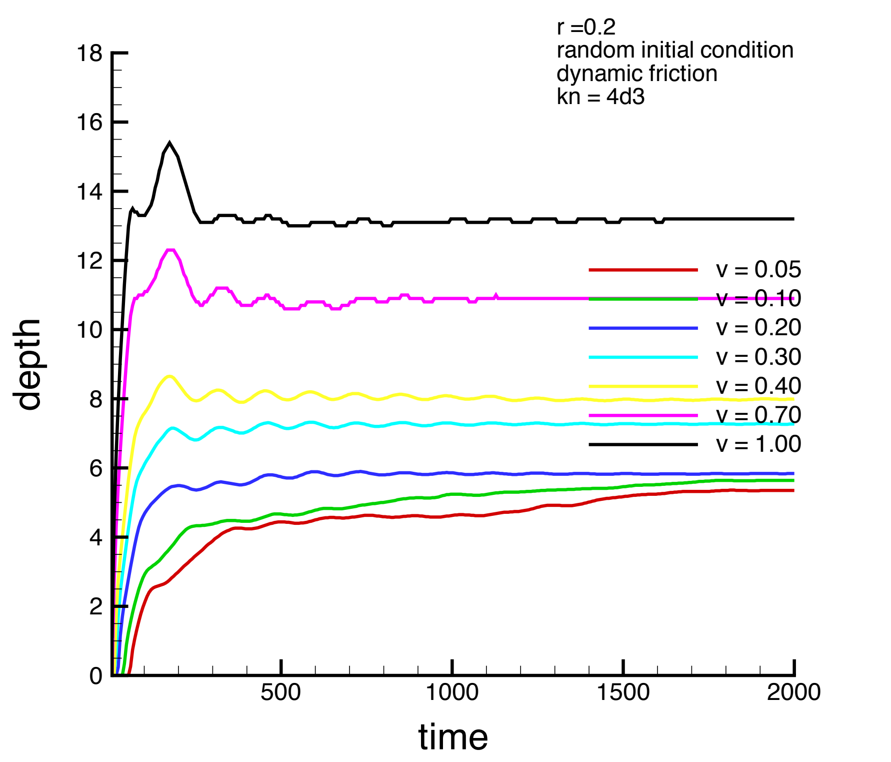
large medium small

- deeper penetration for larger impactors of same initial velocity

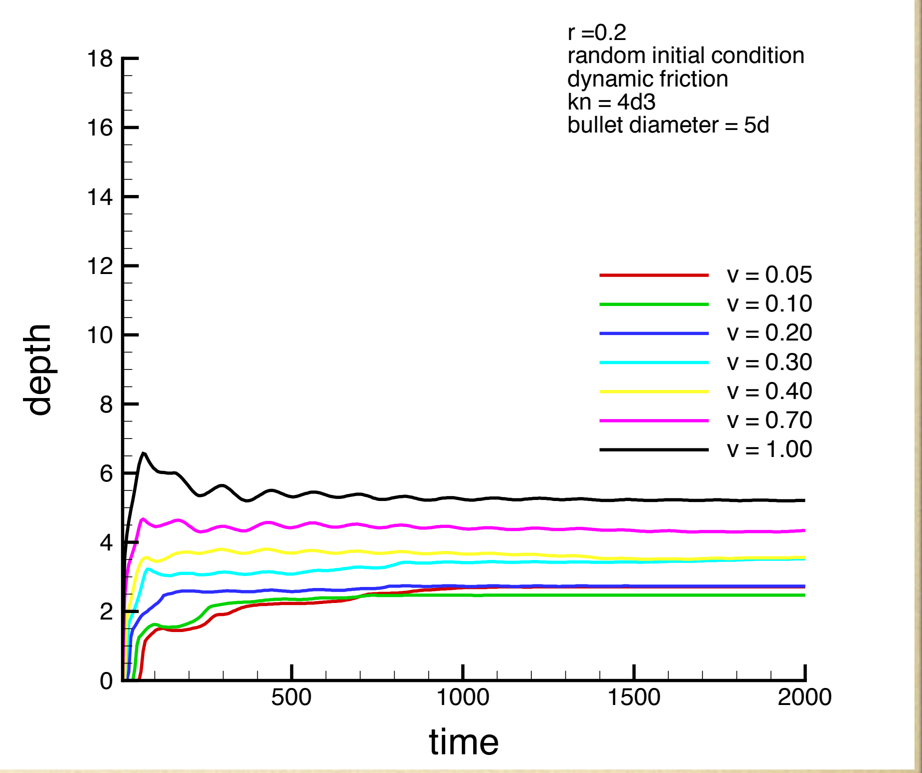
Impactor size (basic system)



large



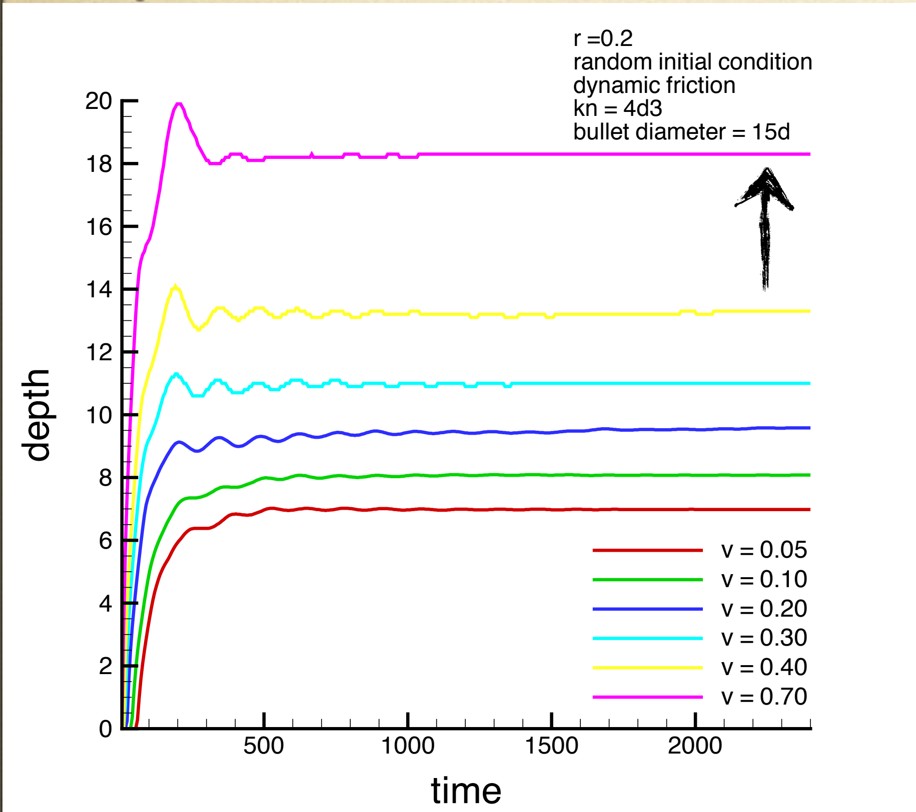
medium

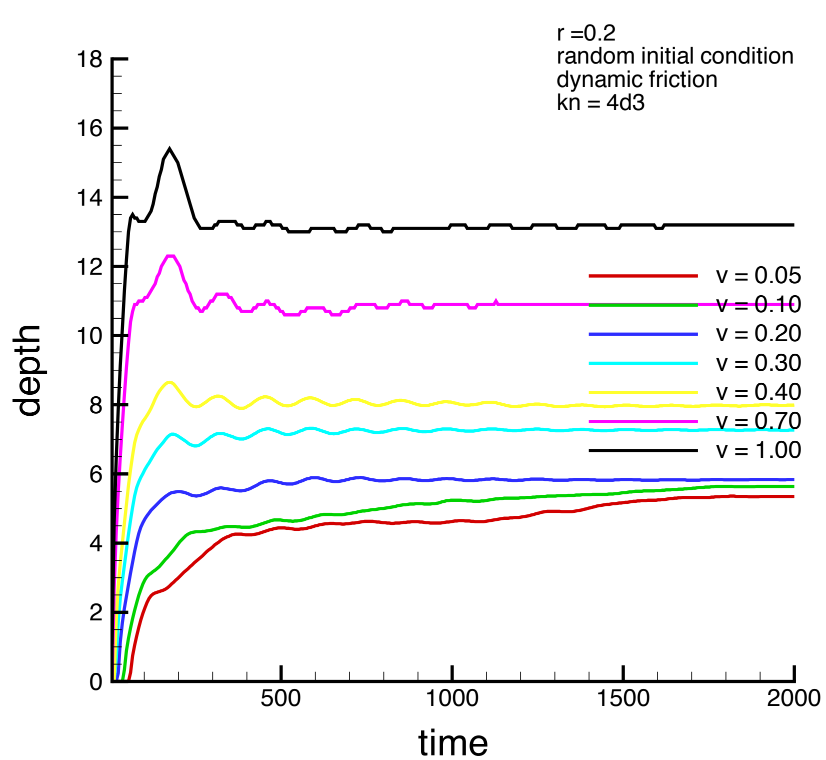


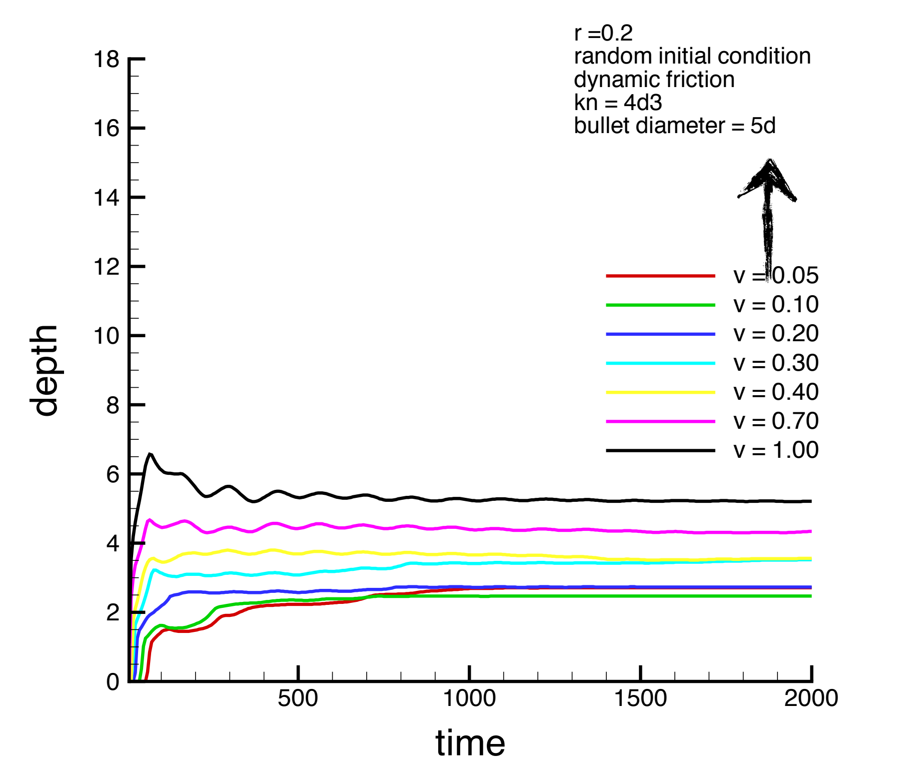
small

- deeper penetration for larger impactors of same initial velocity

Impactor size (basic system)







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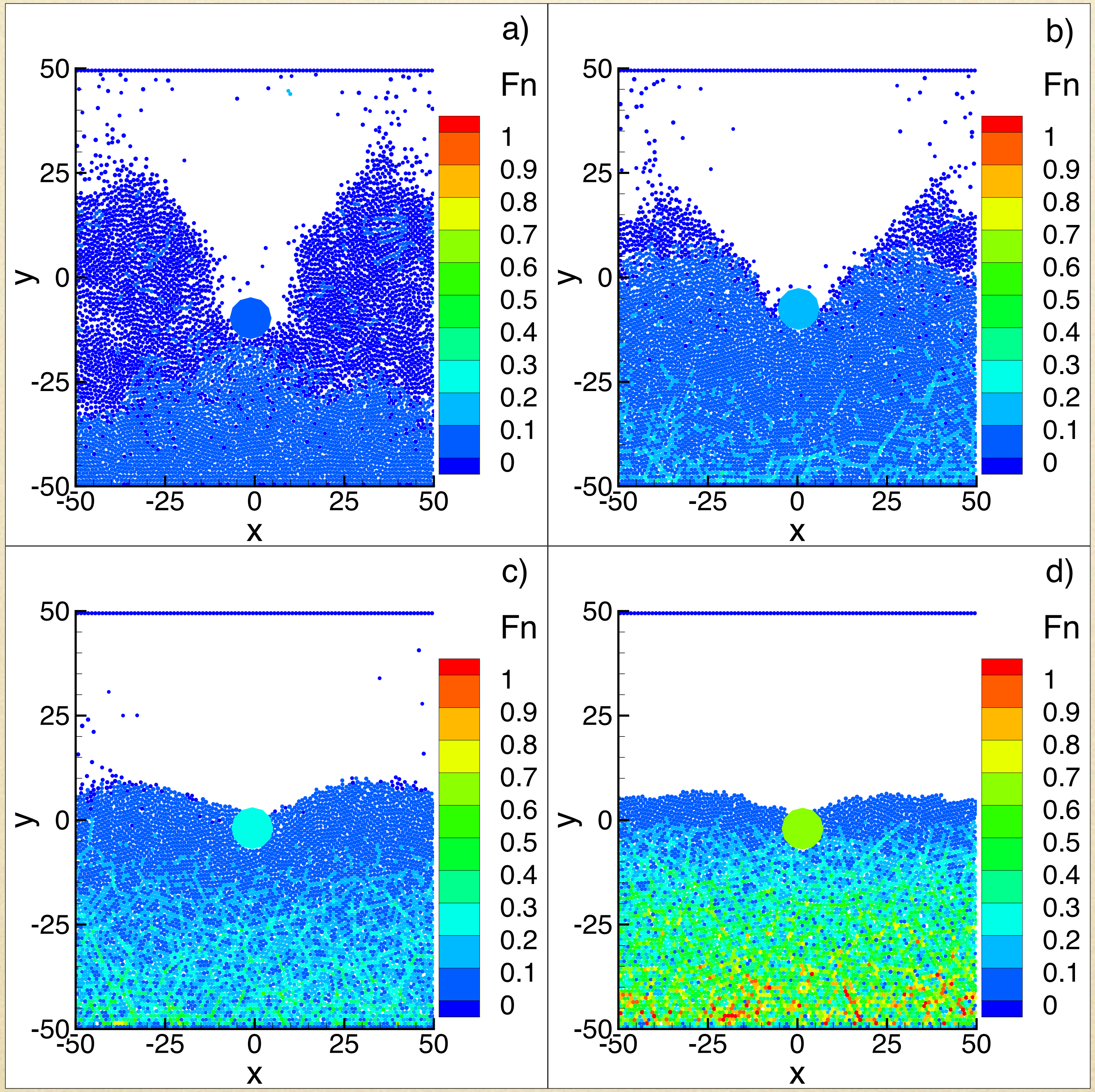
Influence of gravity



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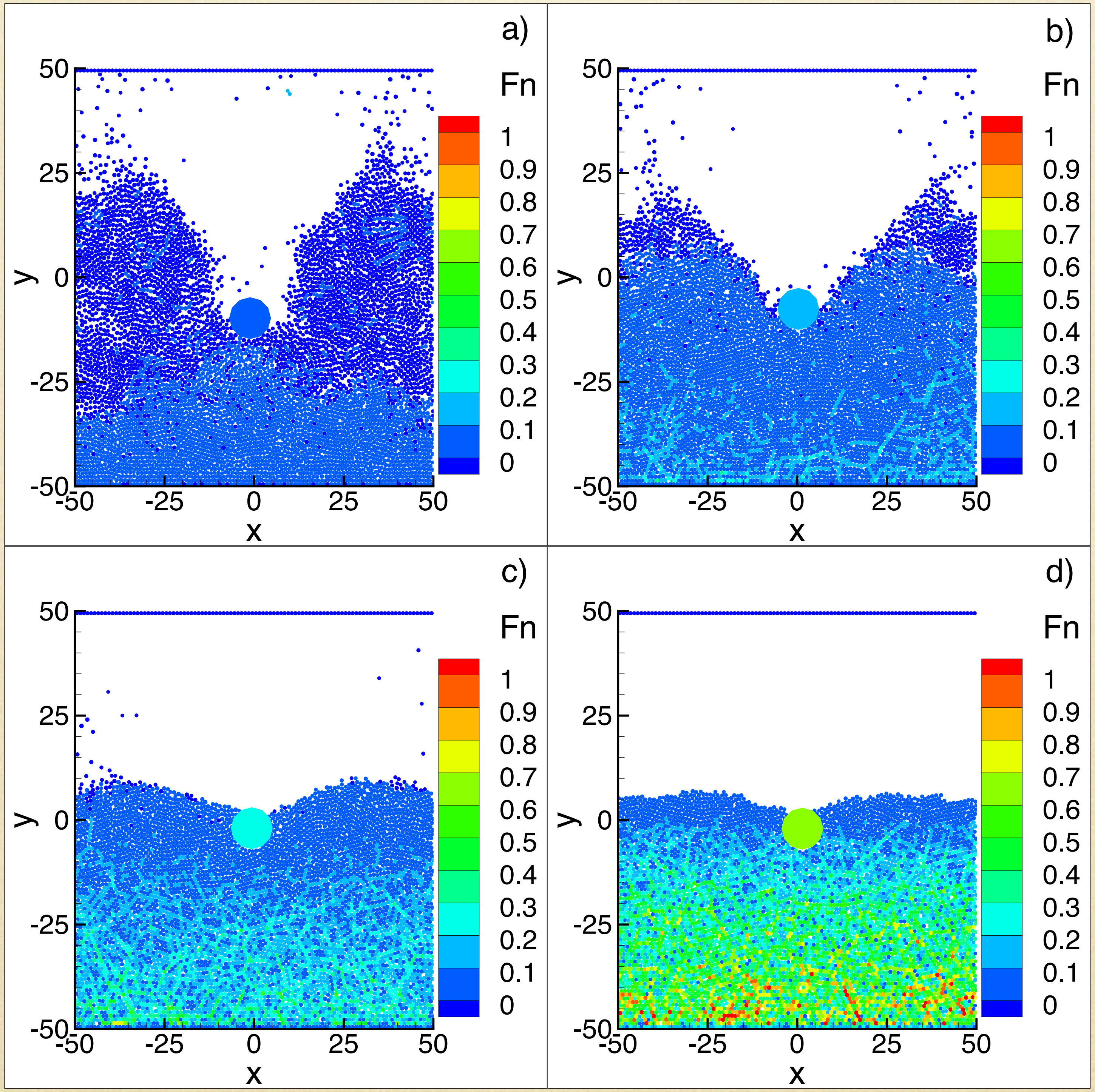

- strong influence of gravity on impact (keeping all other parameters the same)

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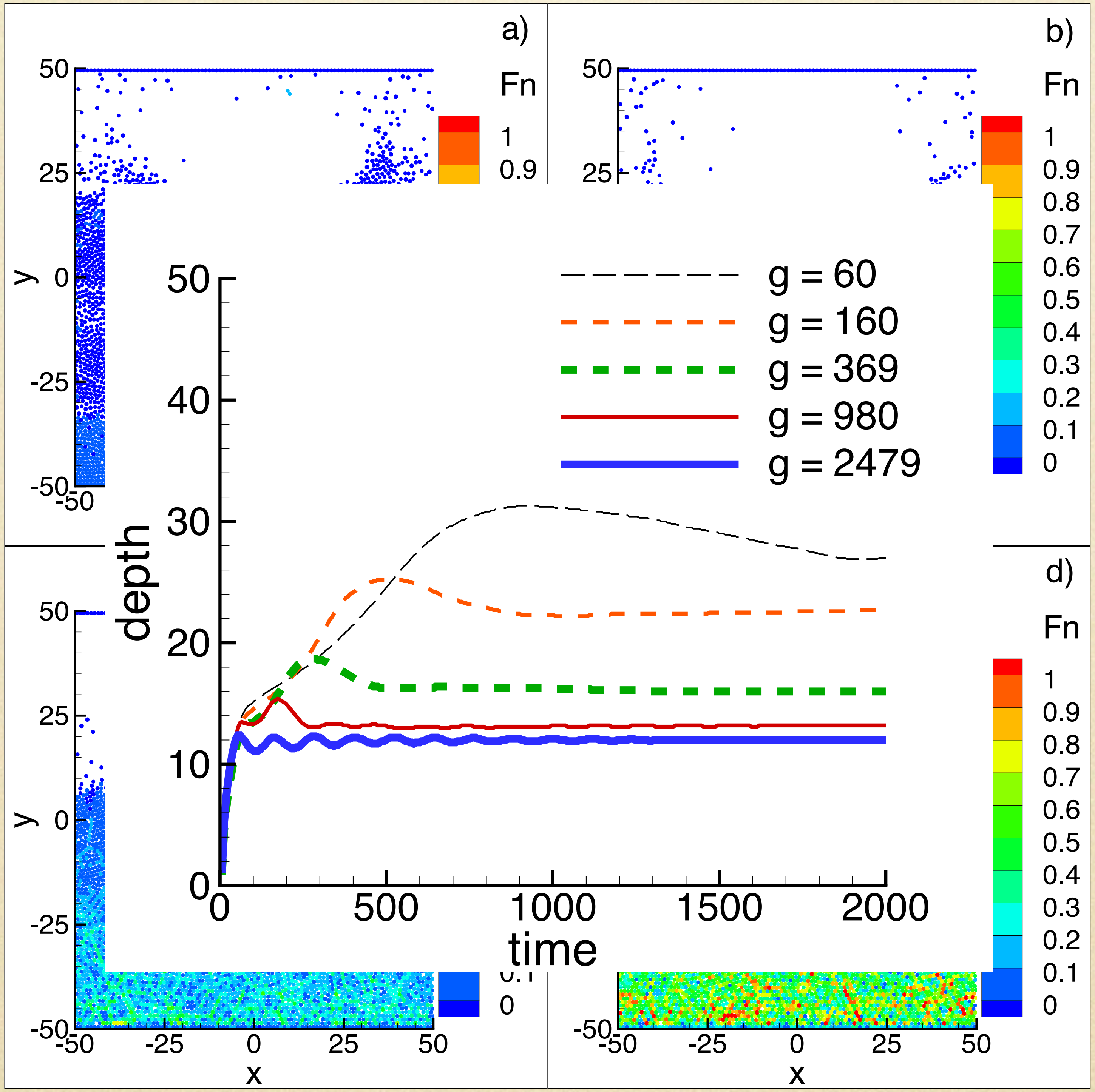

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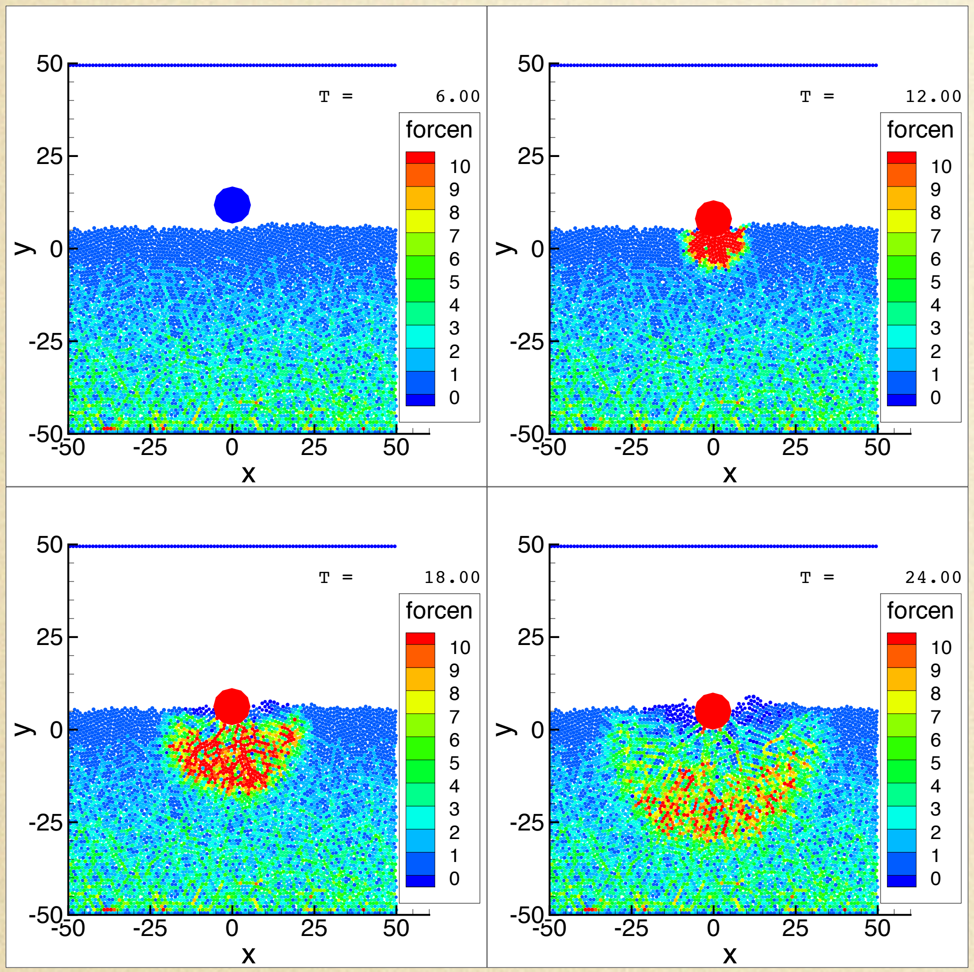
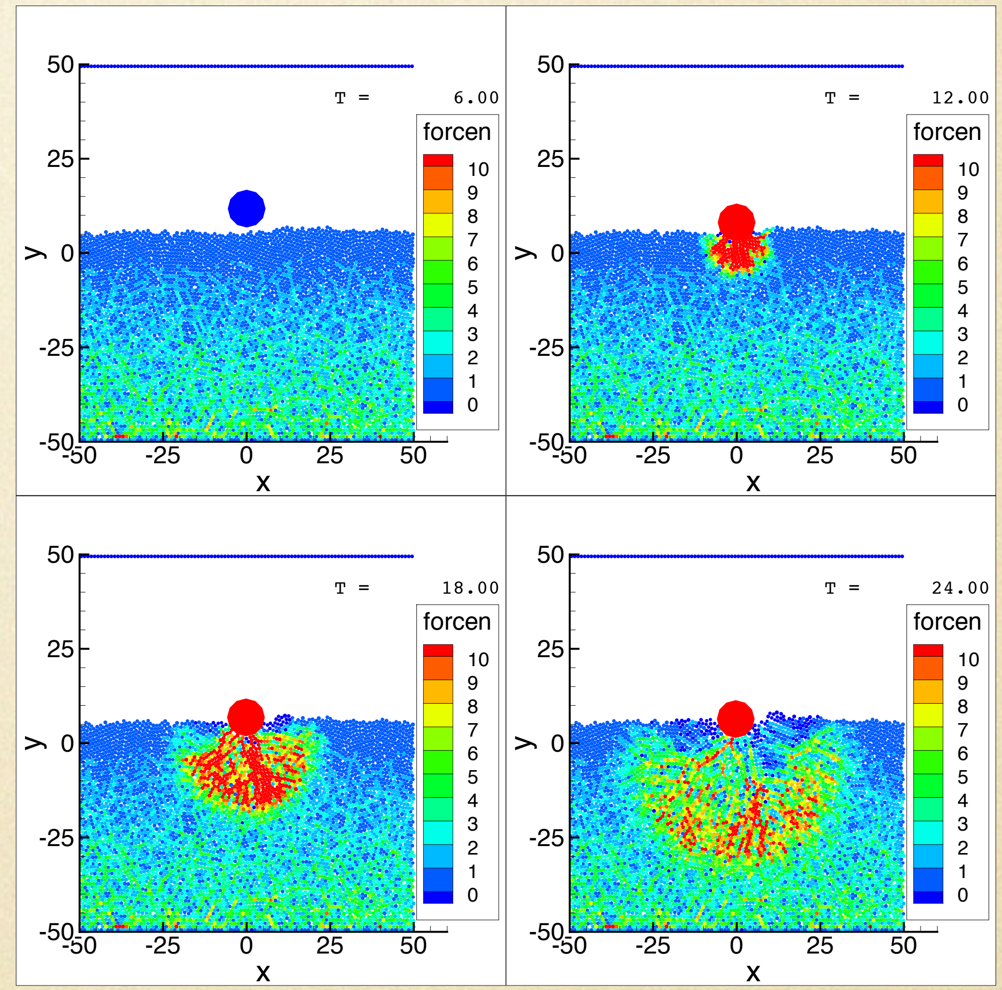
Force networks for different friction models: Normal force

polydisperse system



kinetic friction

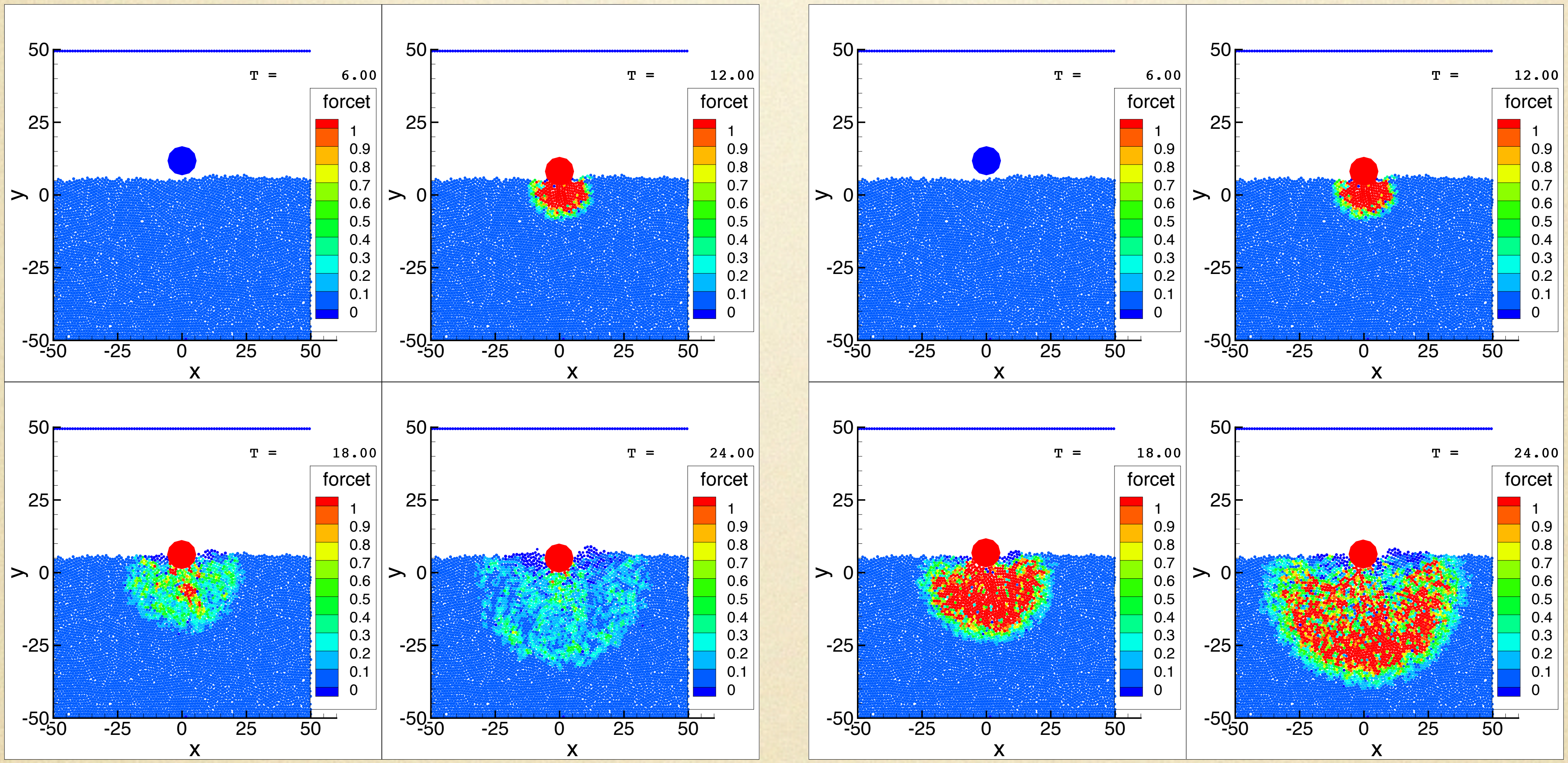


static friction

- force networks are complicated and difficult to analyze
- we need clear and objective measure of force field for different systems

Force networks for different friction models: Tangential force

polydisperse system



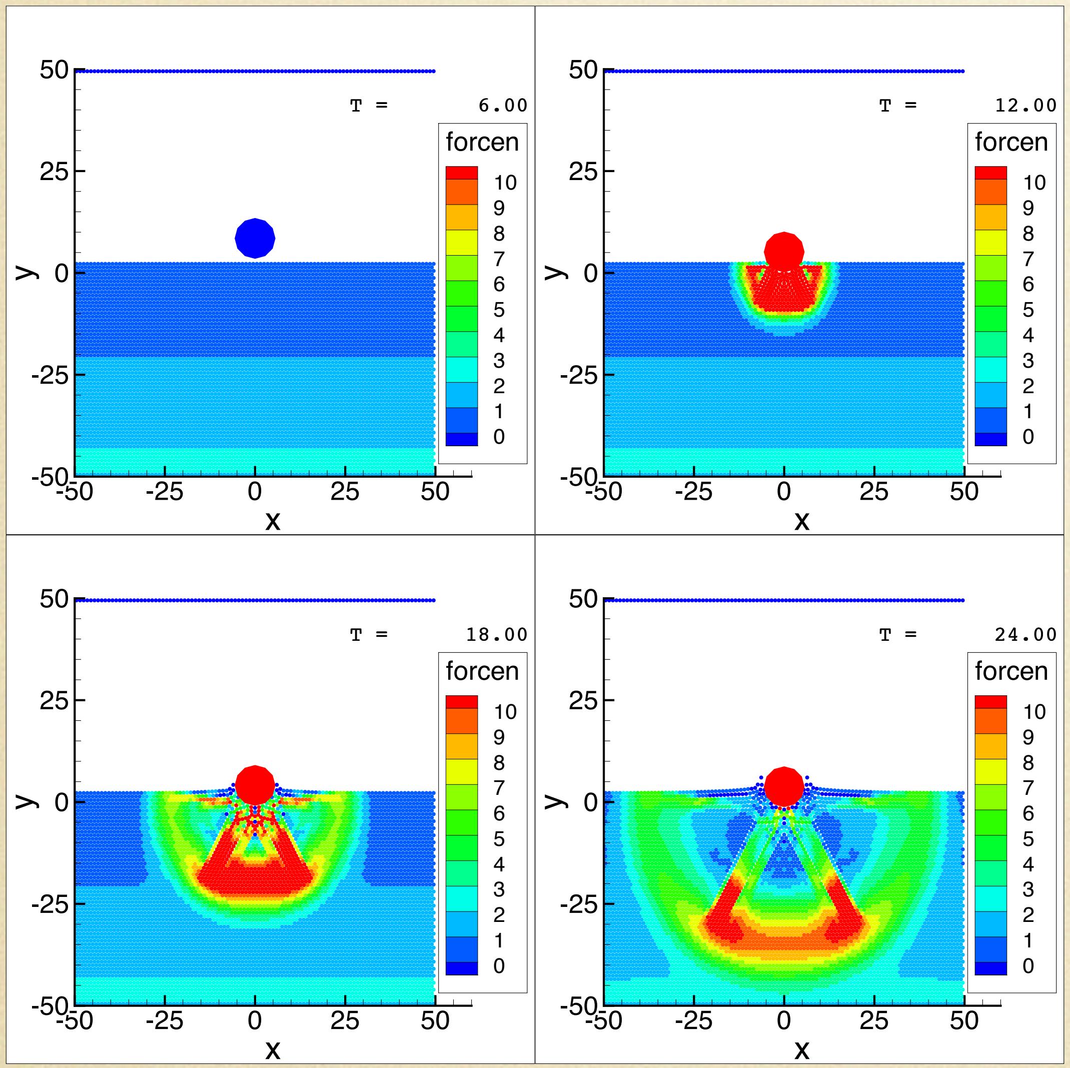
kinetic friction

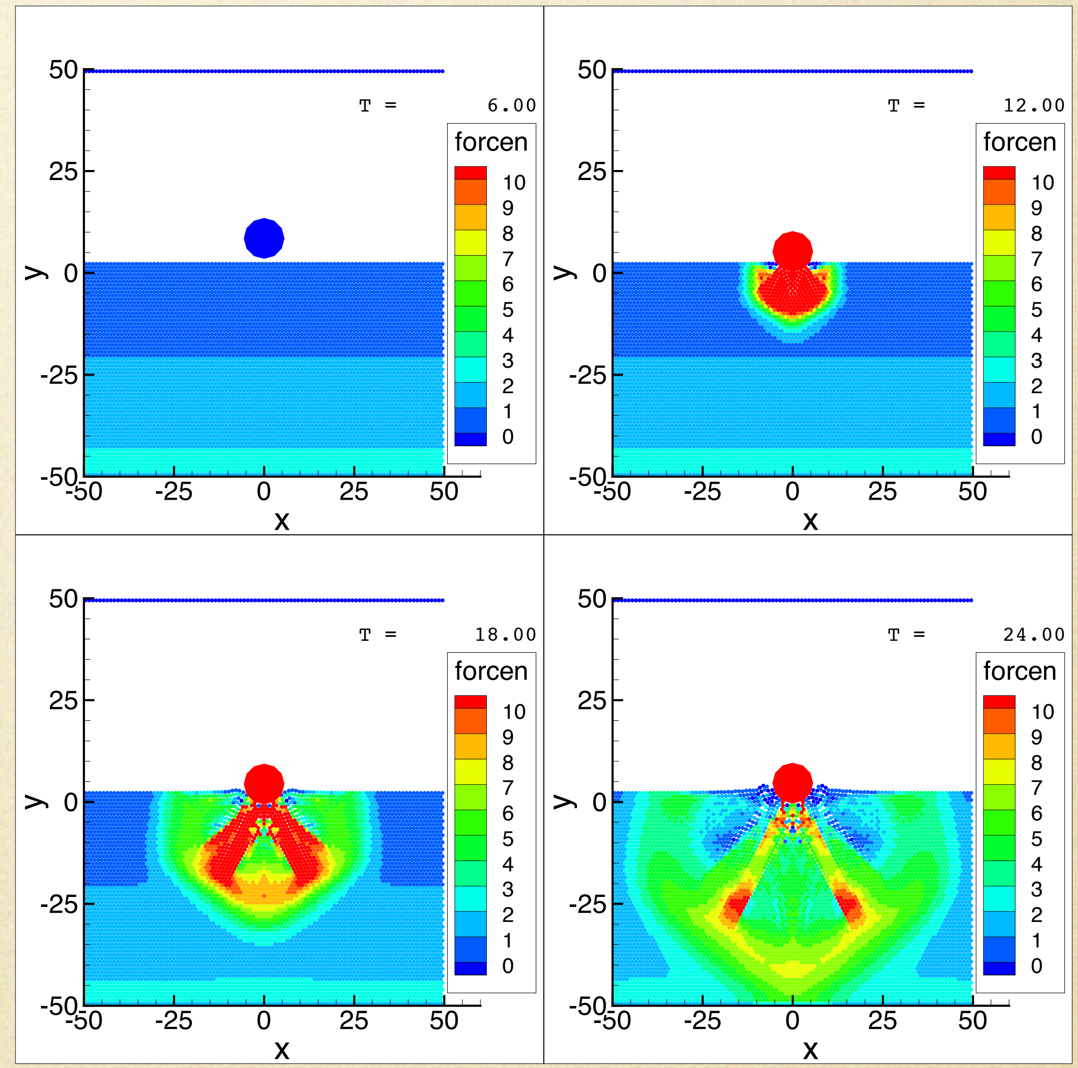
static friction

- Stronger tangential forces for static friction simulations: how to quantify the structure?

Force networks for different friction models: Normal force

monodisperse (ordered) system





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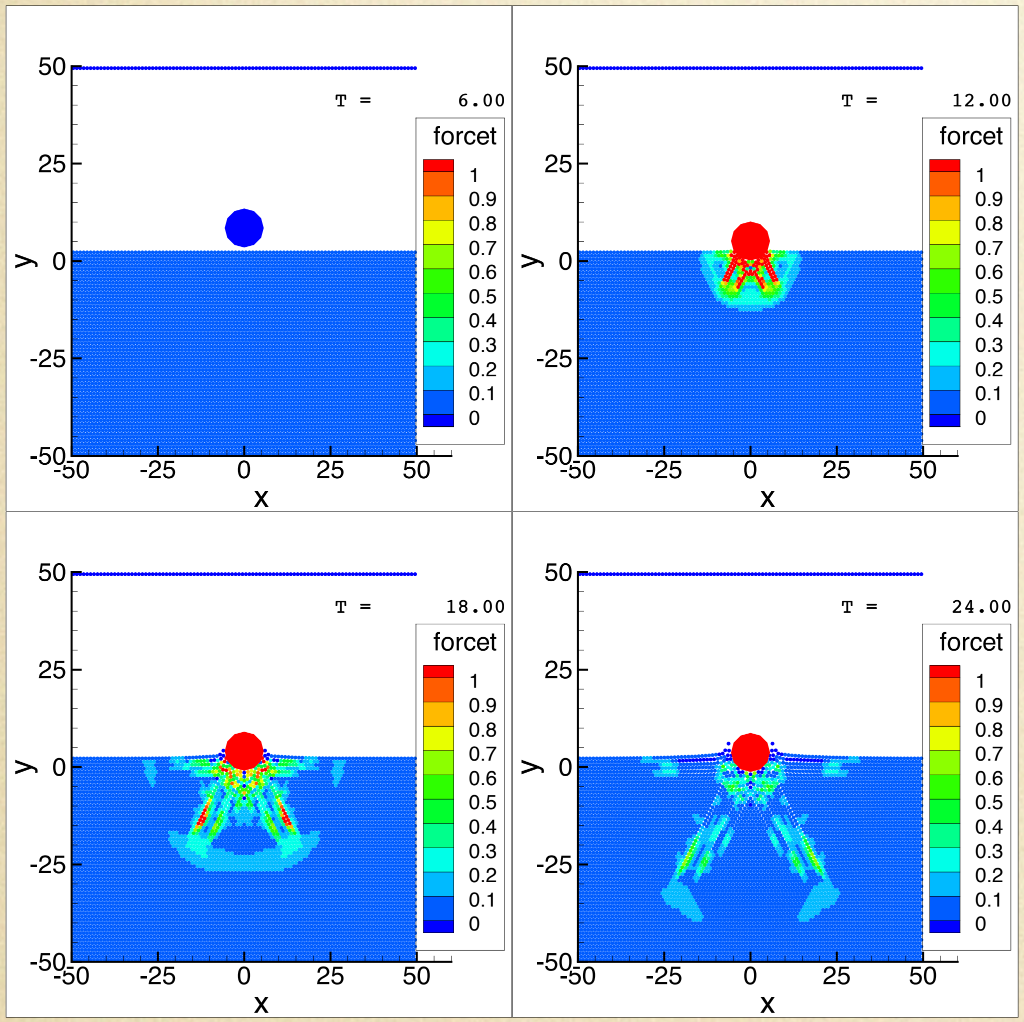
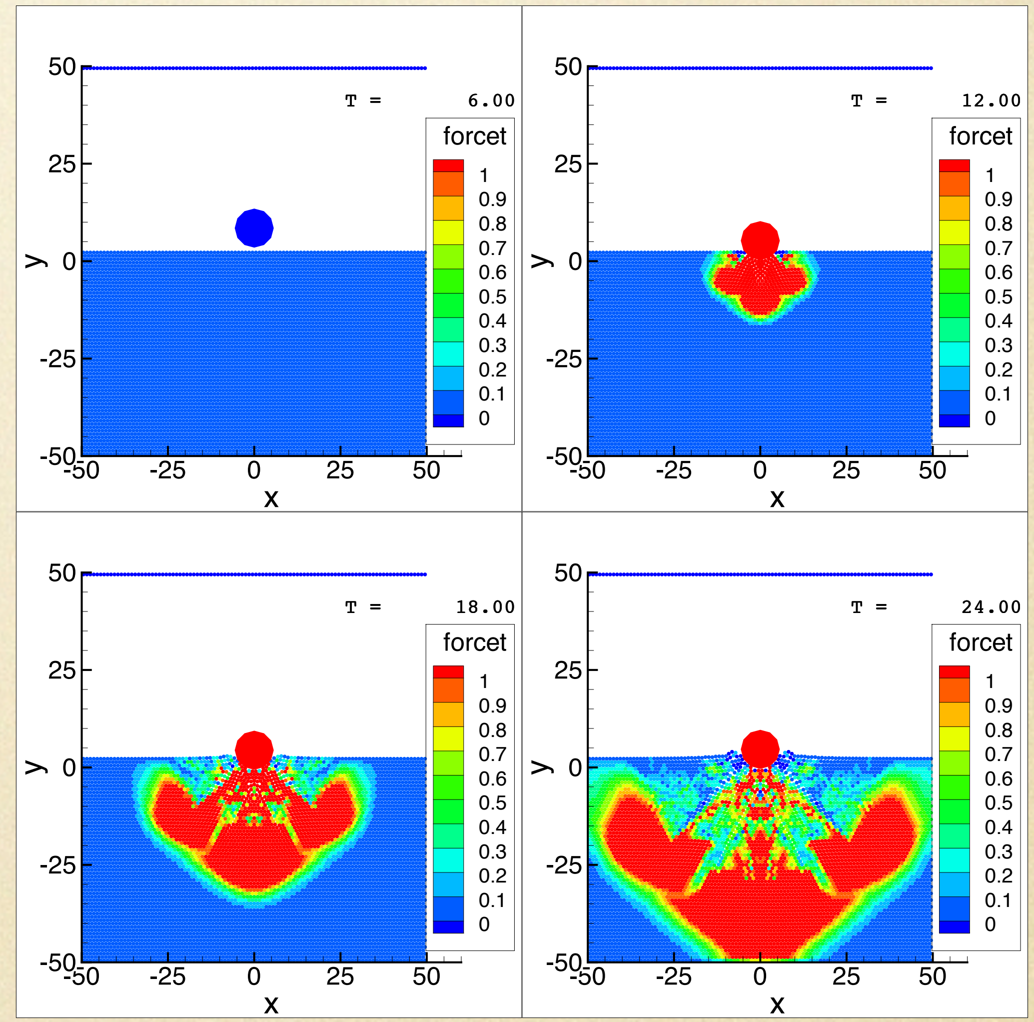


kinetic friction



static friction

Current work

- impact of heavy, non-circular intruders (collaborative with experiments at Duke)
- analysis of force structure properties (connectivies, clusters, percolation)
- 3D simulations

Current work

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  (Duke)
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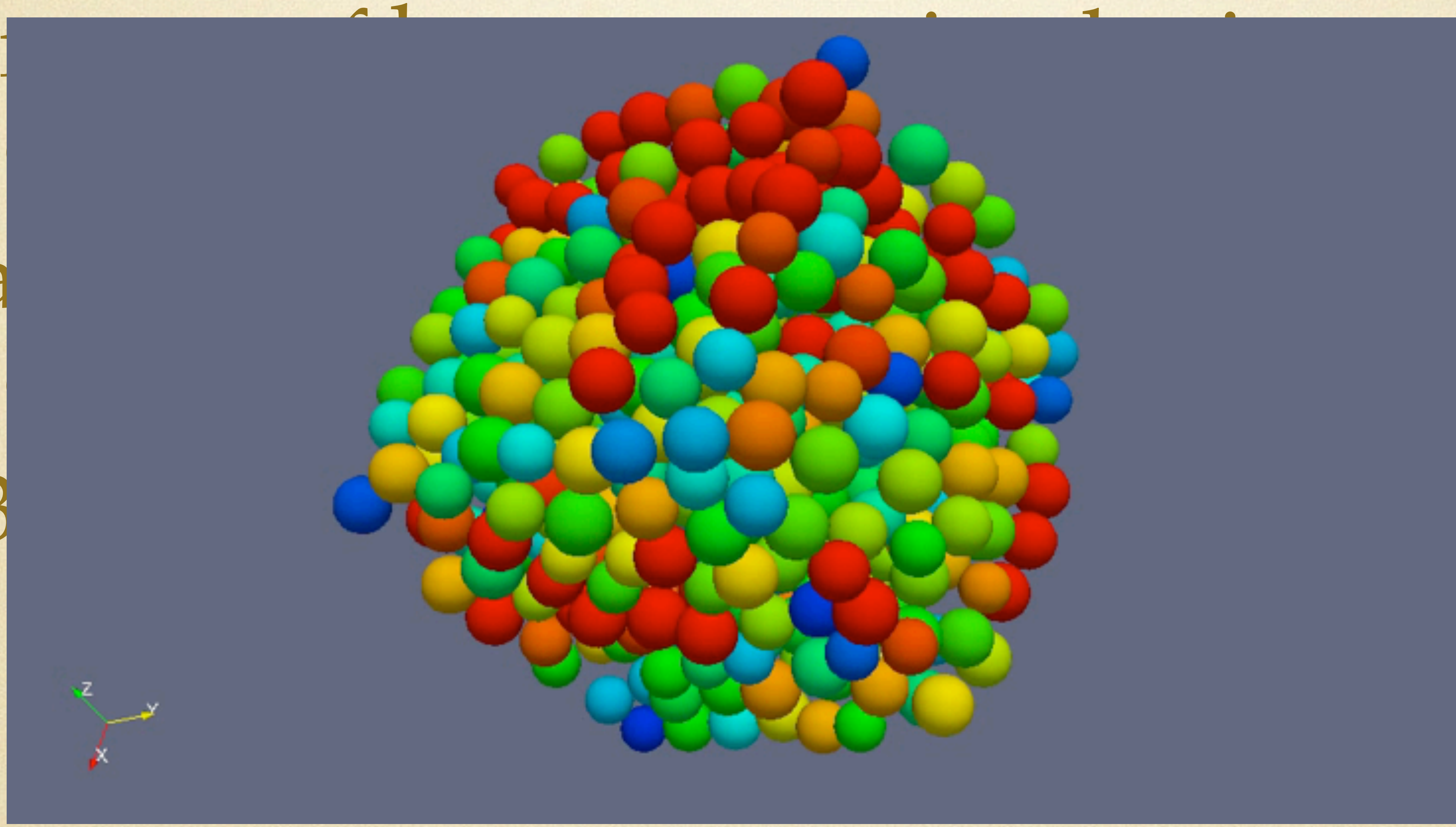

Force field structure:

- Rigidity phase transition, Aharonov & Sparks PRE '99
- Force distributions, Radjai etal, Chaos '99
- Force networks due to localized perturbation, Goldenberg & Goldhirsch, PRL '02
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Is there a simple but precise way to describe force chains?

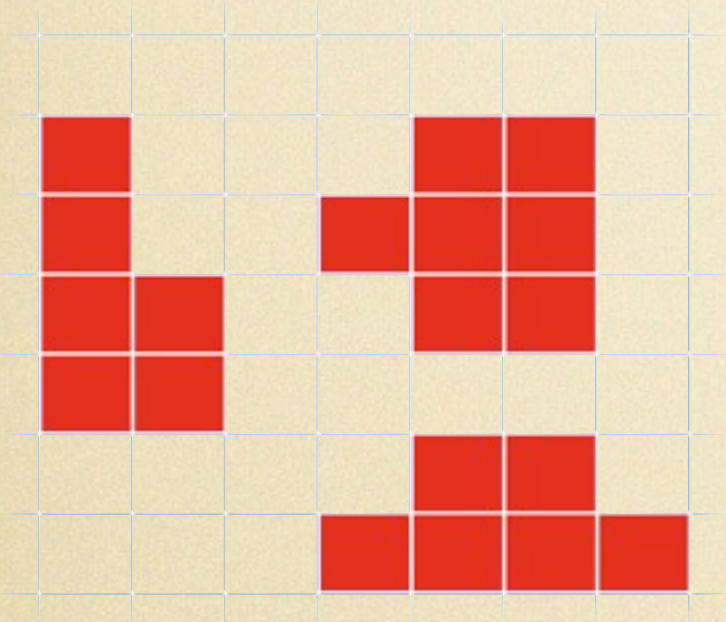
Topological measures

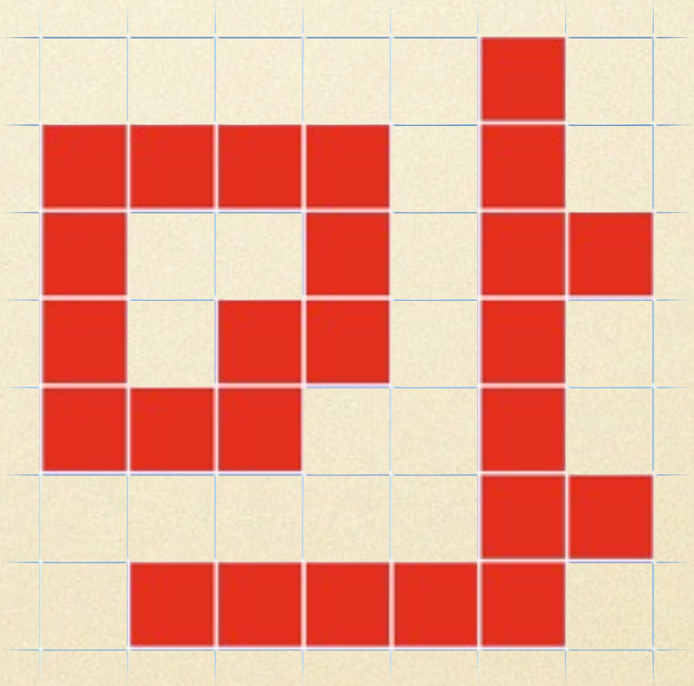
- Consider topological measures of the force field
- Here: concentrate on the simplest measures specifying connectivity and holes that form as a function of force magnitude: *Betti numbers*
- Consider a very simple system: isotropically compressed system of circular disks in 2D
- Discuss how the topological measures, quantified by *Betti numbers,* change as volume fraction is modified

Exploring force field structure

- looking for an objective measure of global features of the force field
- compute connectivity of the granular particles as a function of total force (normal or tangential) experienced: quantify in terms of Betti numbers measuring number of components (clusters) and number of holes (`loops')

http://chomp.rutgers.edu/





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- here concentrate on B0

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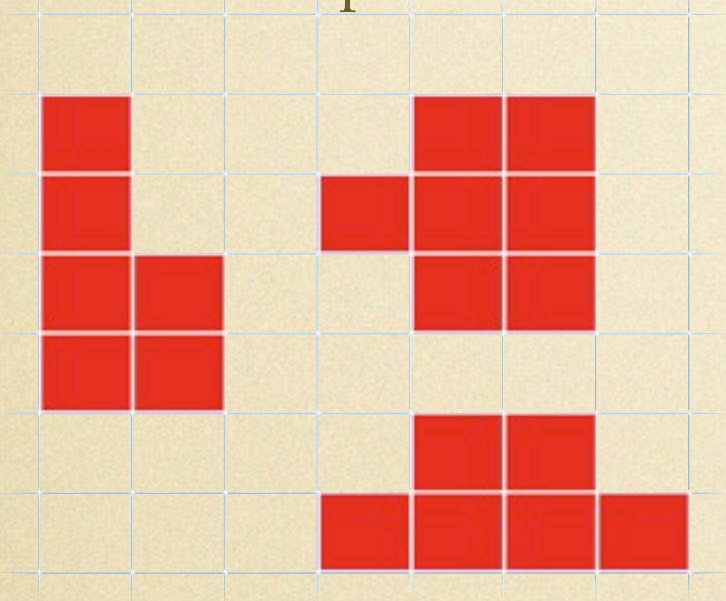
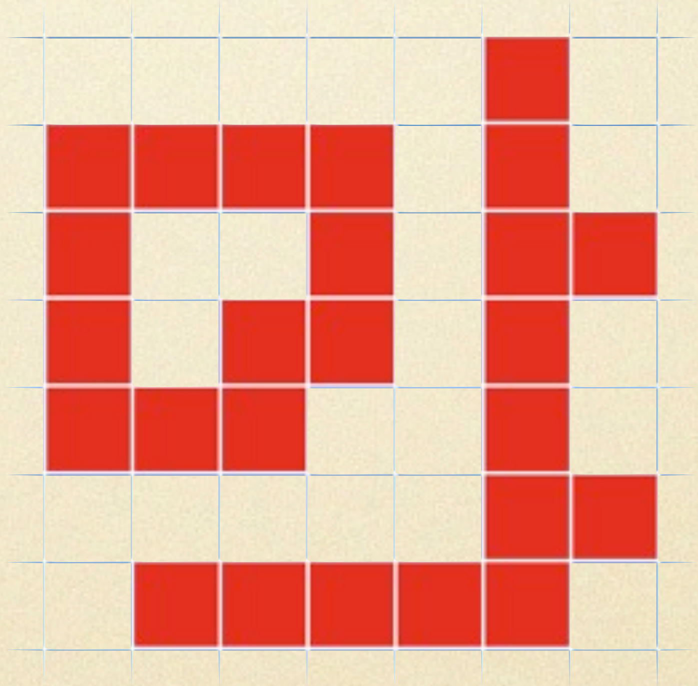
Example 1



$B_0 = 3$

$B_1 = 0$



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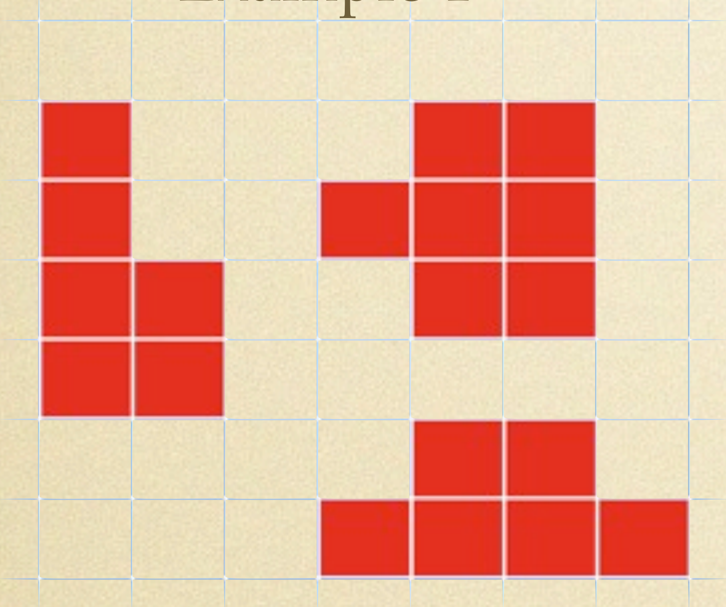

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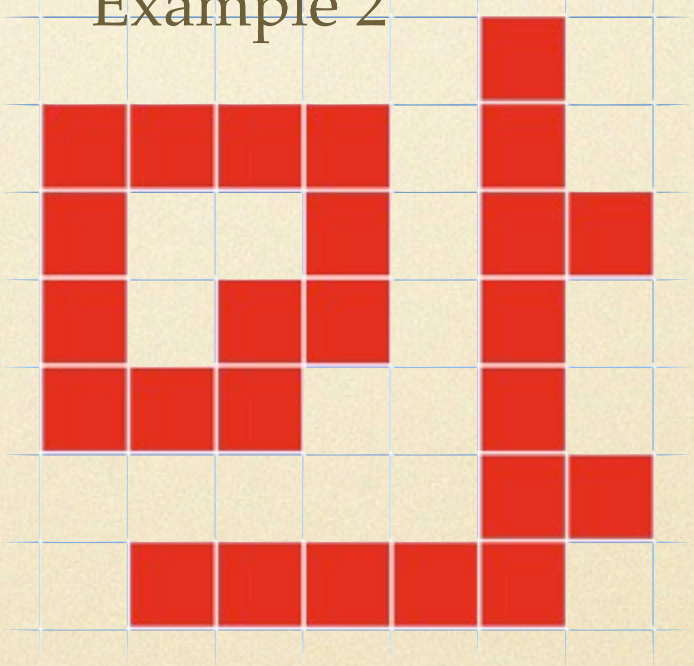
Example 2



$B_0 = 2$

$B_1 = 1$

- compute Betti numbers and observe the features of force network for different systems
- here concentrate on B0

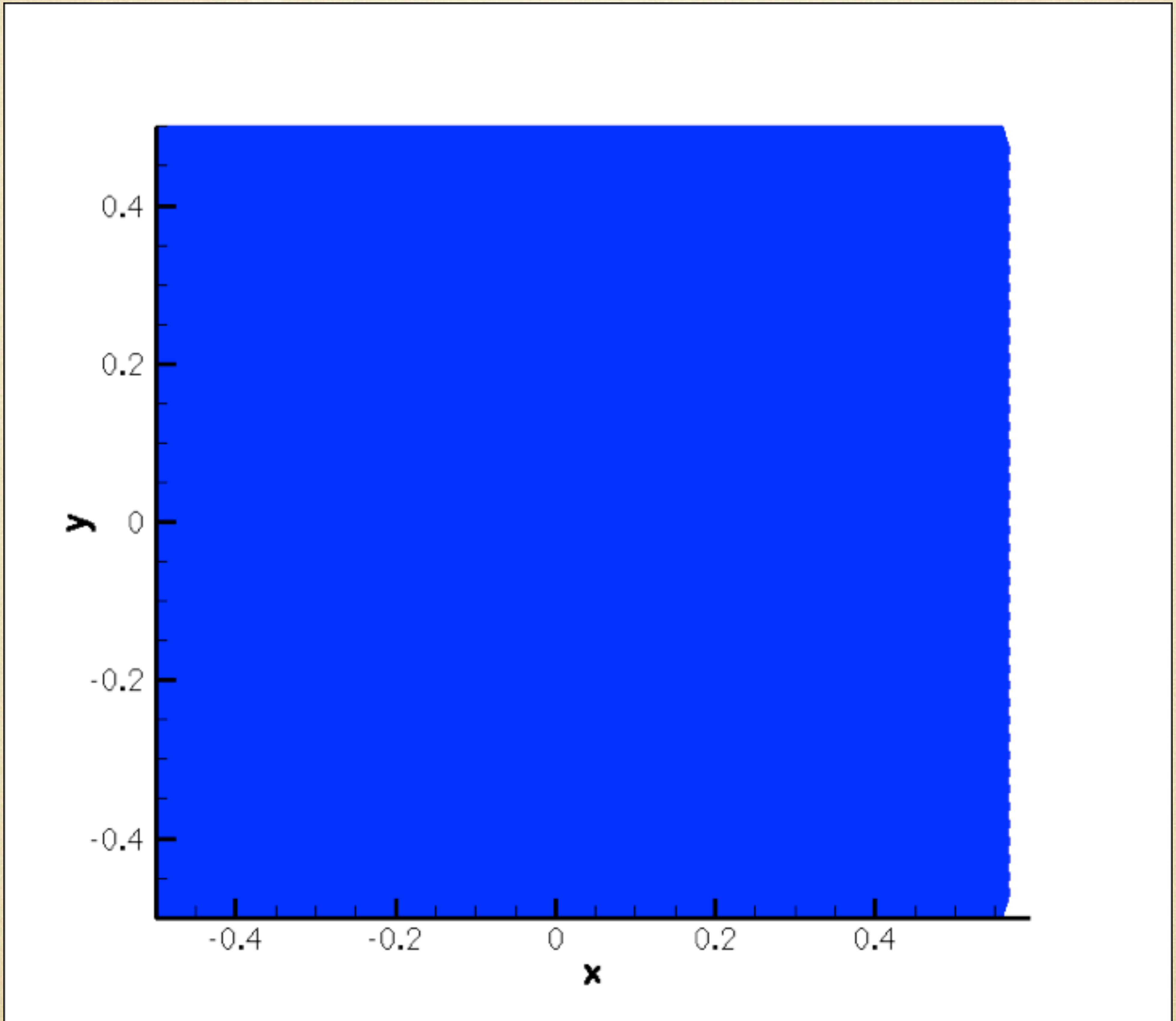
Topology Tools: CHomP

- CHomP: computational package by Mischaikow and collaborators used to compute Betti numbers (and many other things) of various spatial and temporal structures

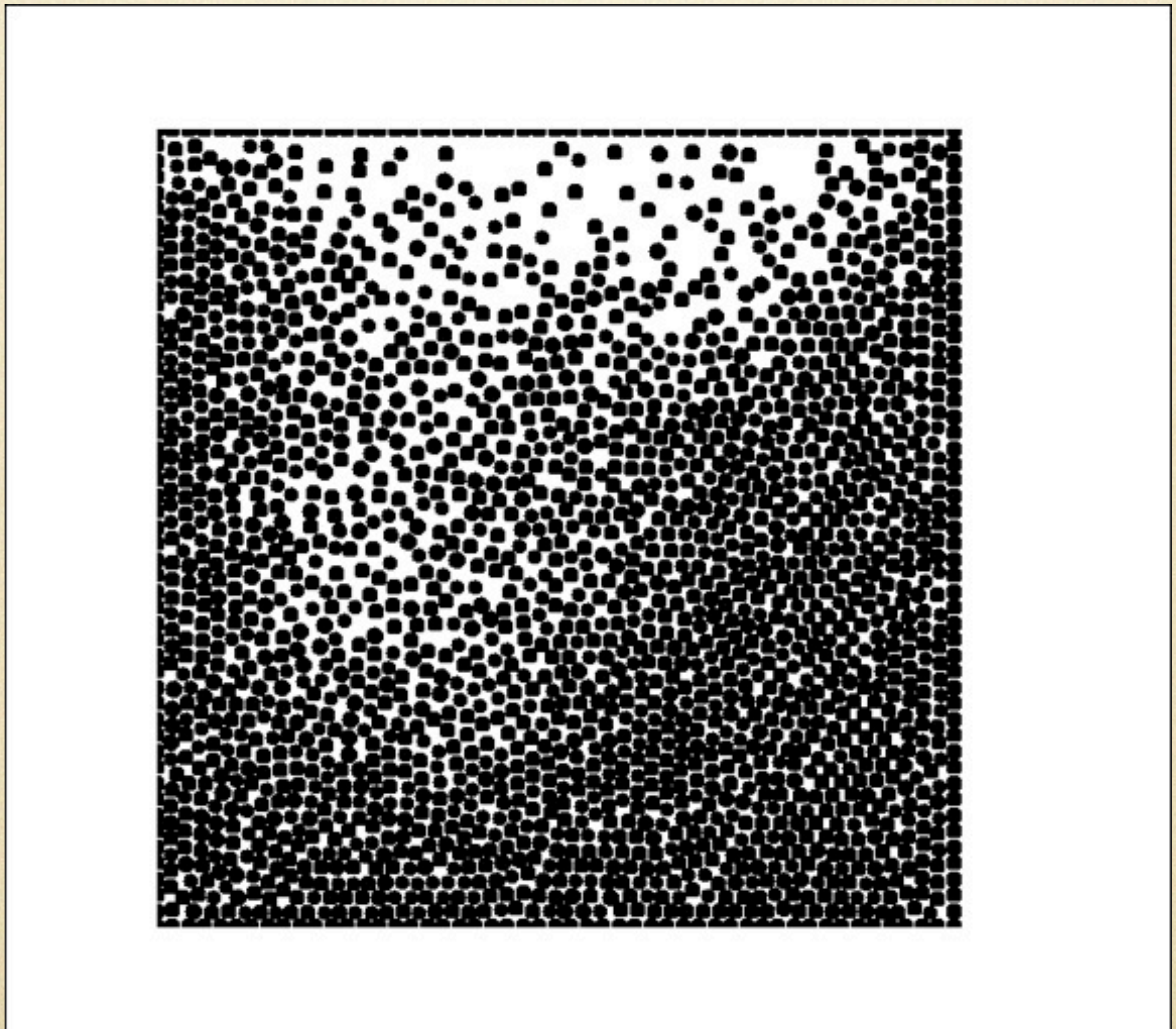
  http://chomp.rutgers.edu/

- Here: concentrate on
  - B0: number of connected components
  - B1: number of holes
- Structure: force field in a compressed granular system

Example of simulations



Forces and structures:thresholds (for a given time)



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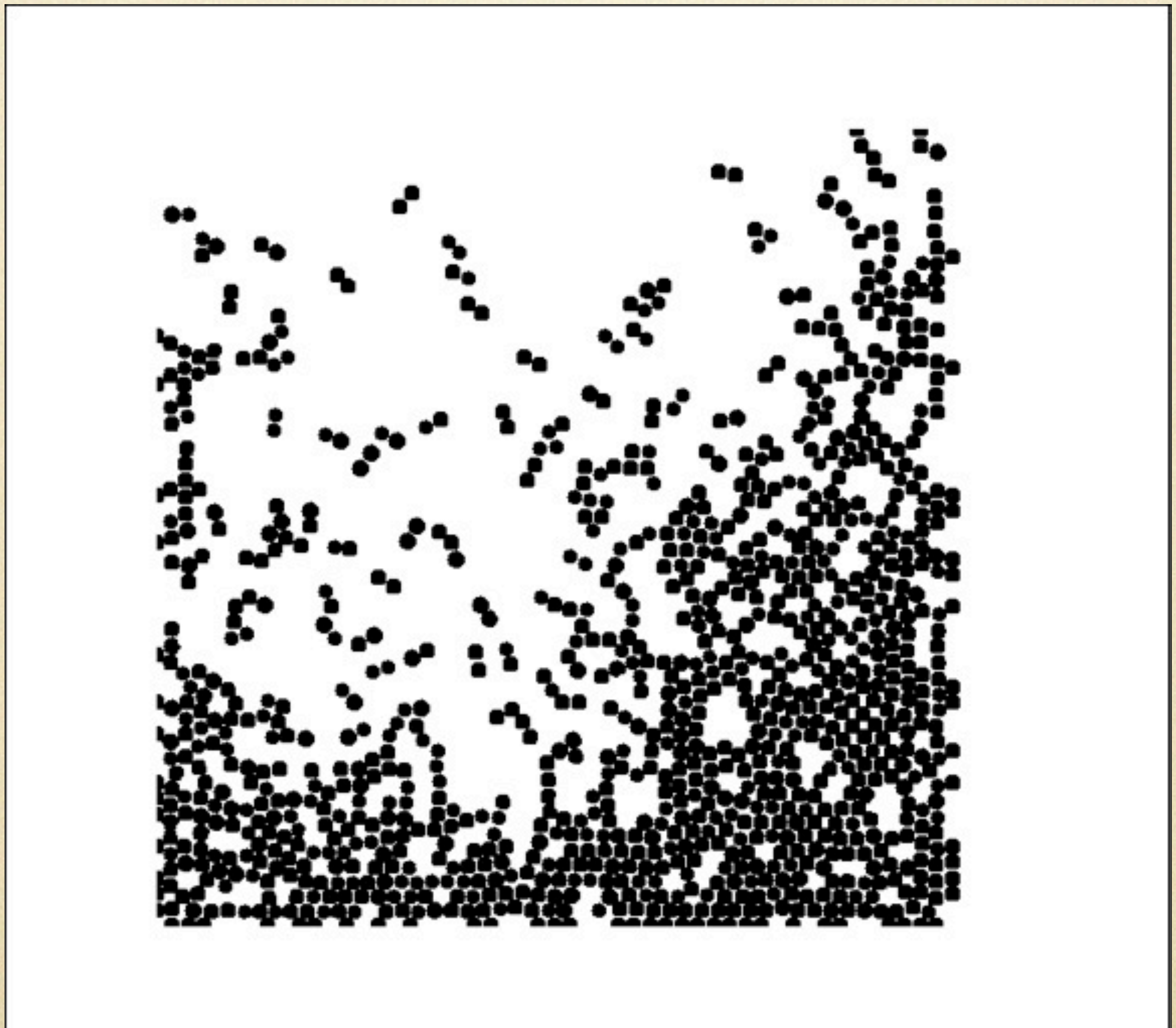

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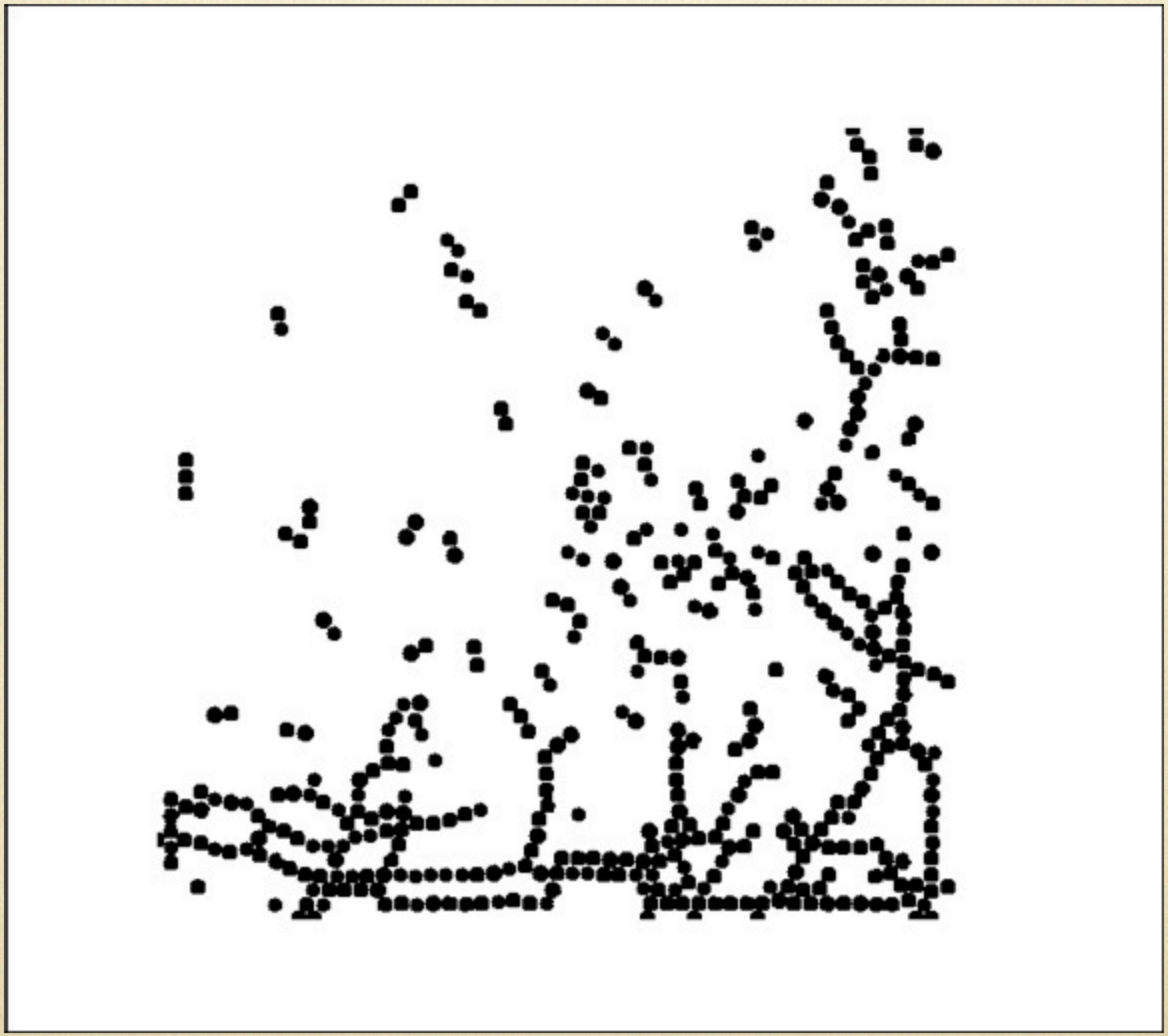

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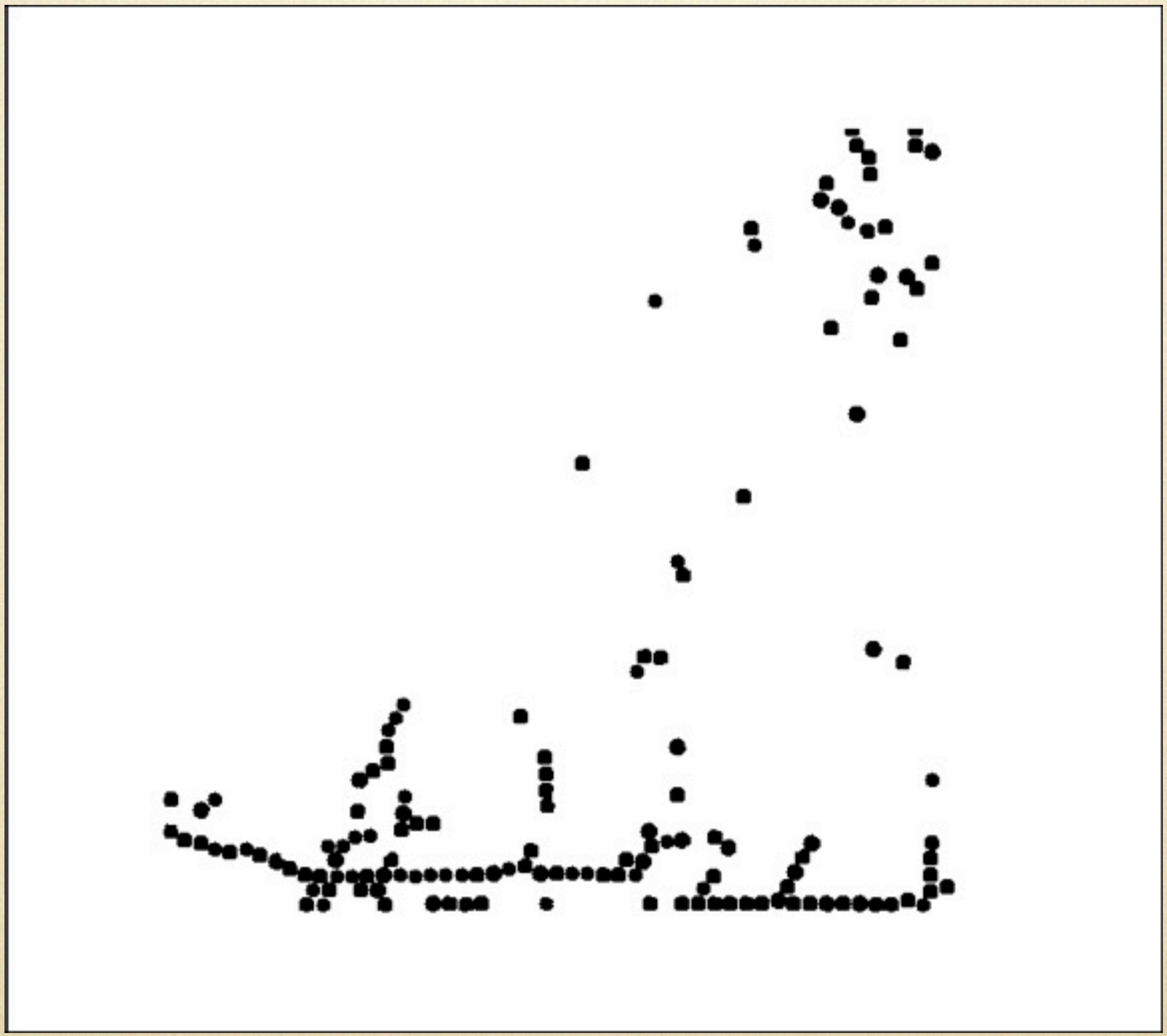

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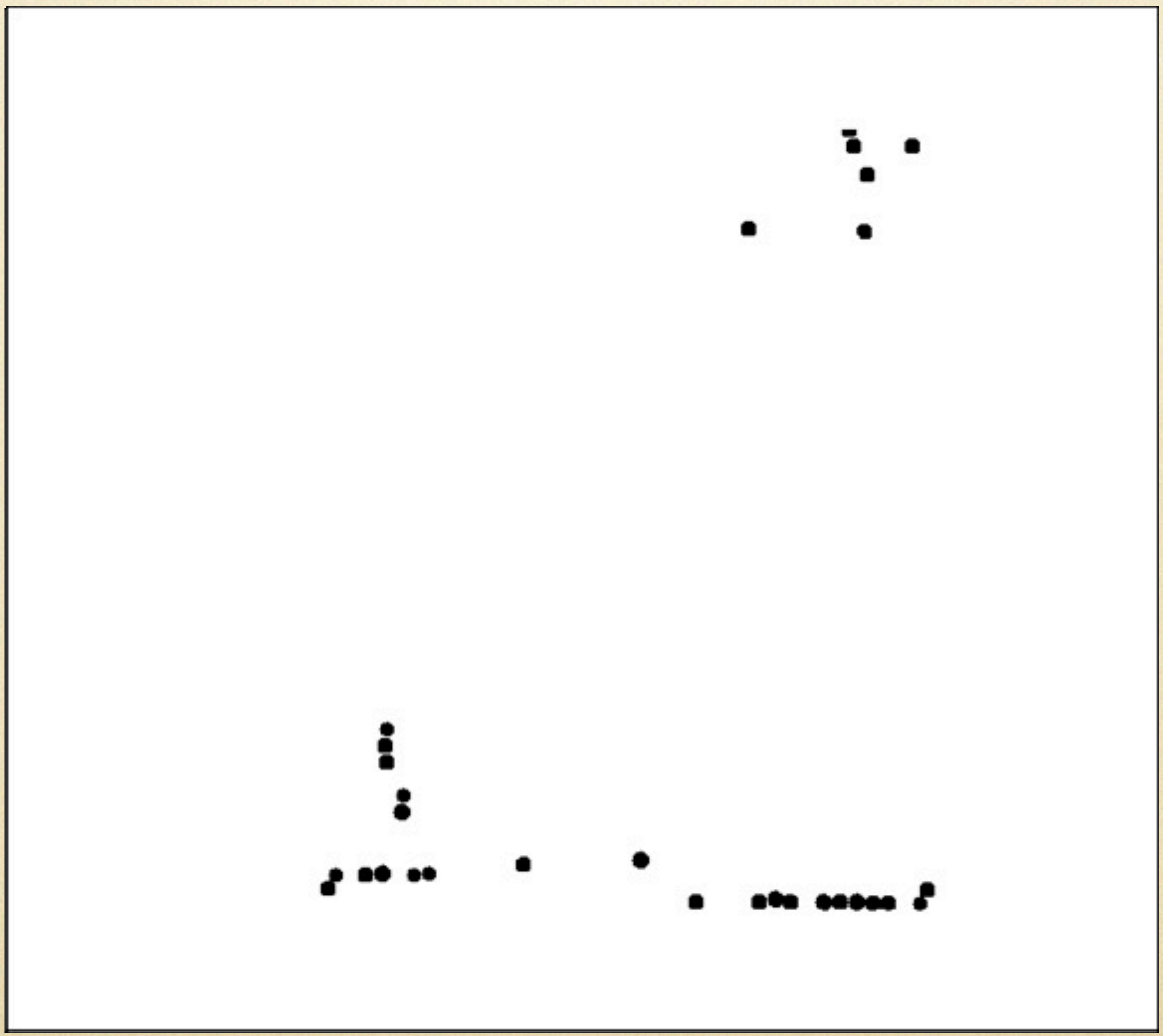
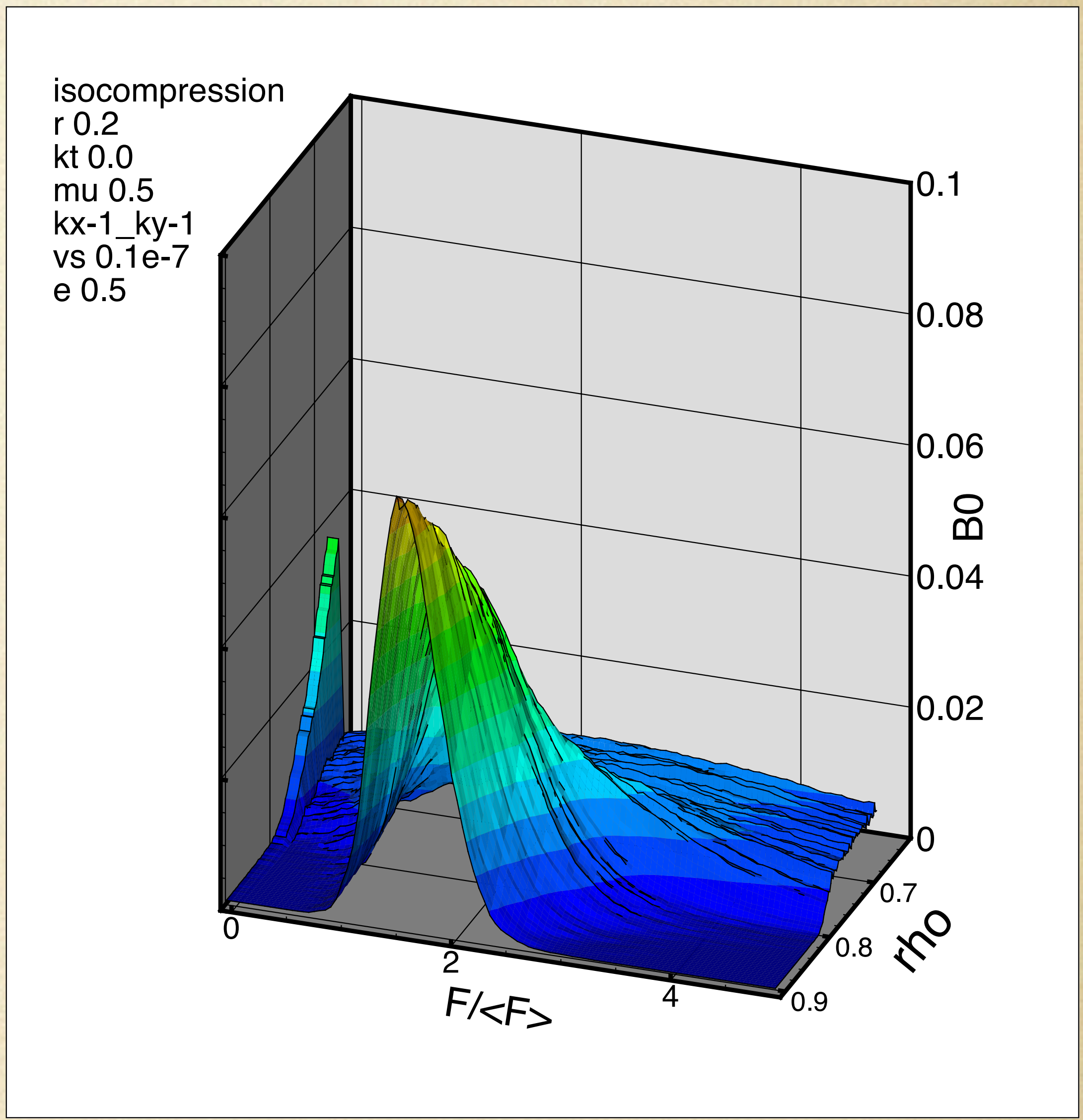


Betti numbers: B0

- polydisperse system
- dynamic friction
- average 20 realizations
- normalized by number of particles
- system-size independent
- main features of the results are rate independent



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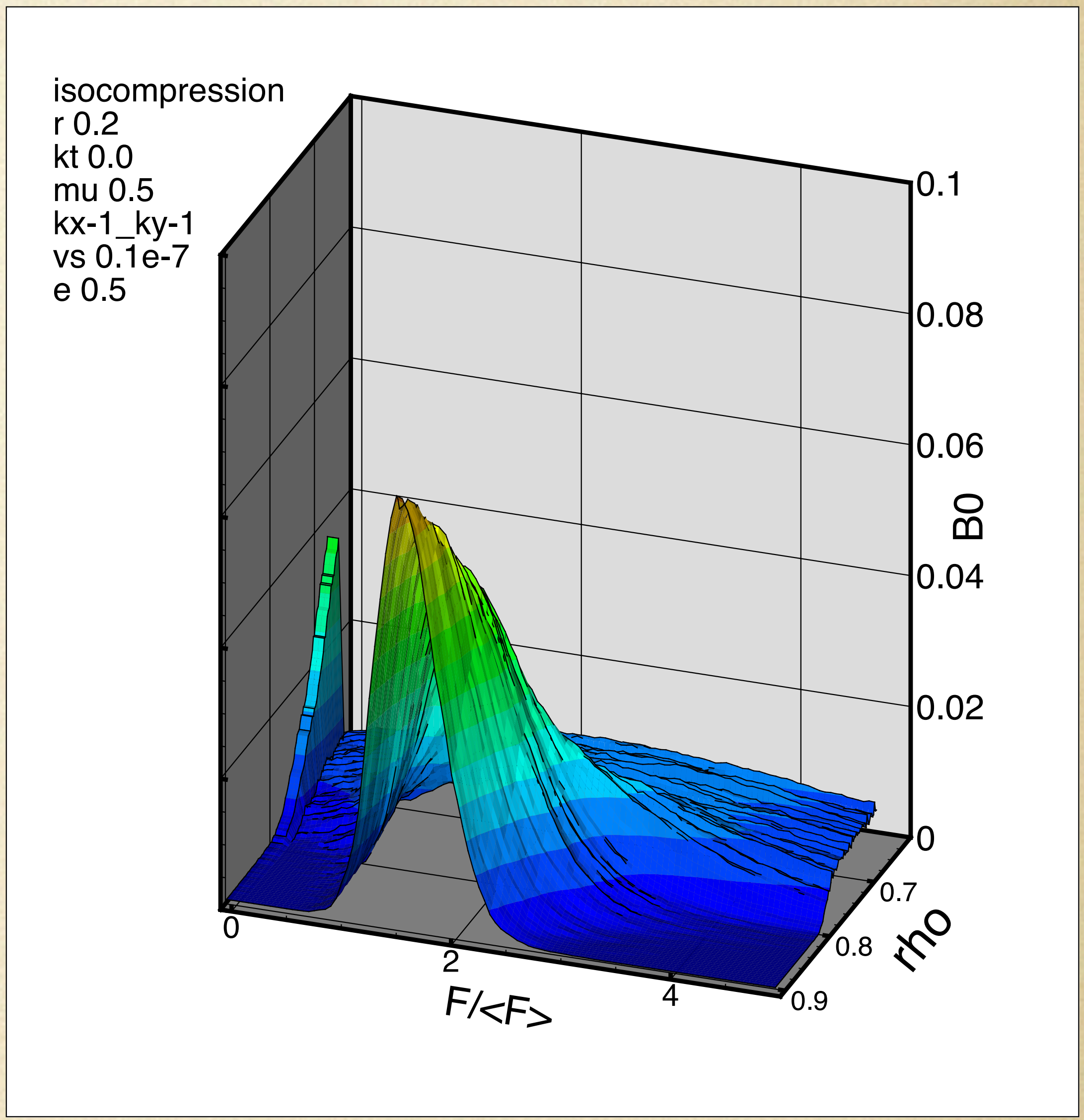
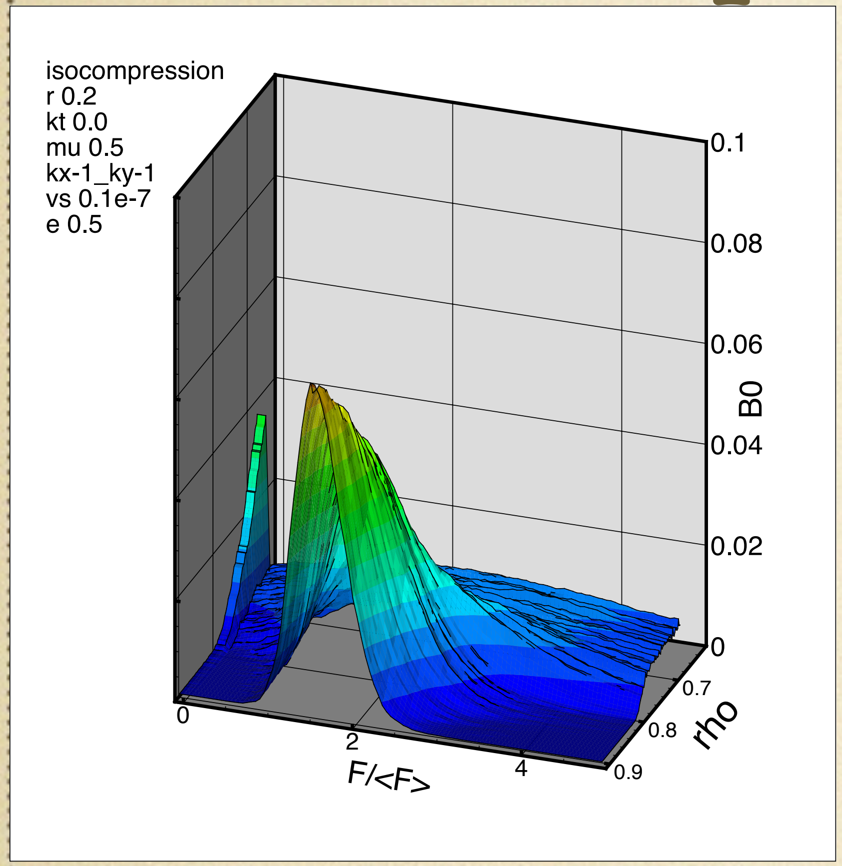


- zero threshold: showing fabric of the material
- large volume fraction: huge increase of the number of components

Is this intrinsic property of dense jammed granular systems?

Which of material properties influence topology of force chains?



polydisperse, dynamic friction (static friction produces similar results)

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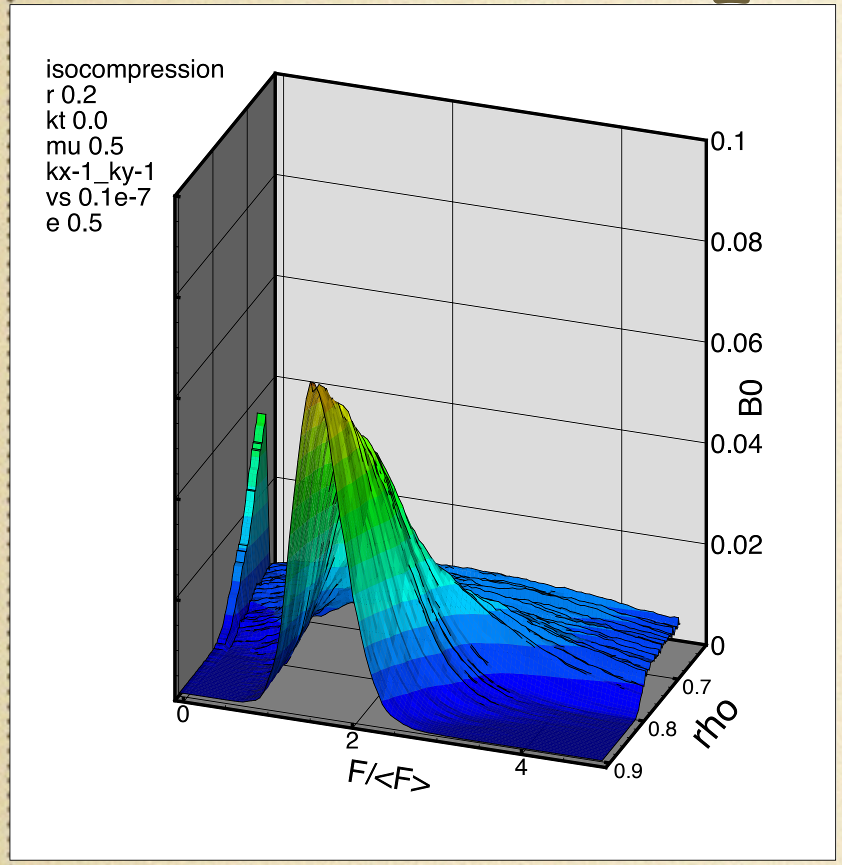

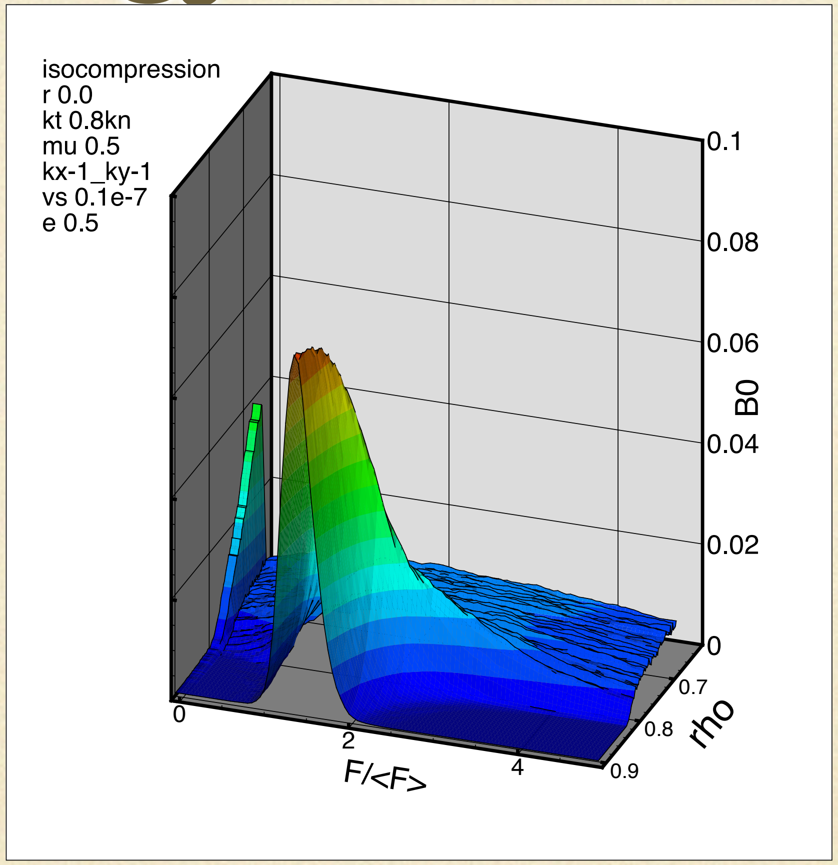

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monodisperse, static friction

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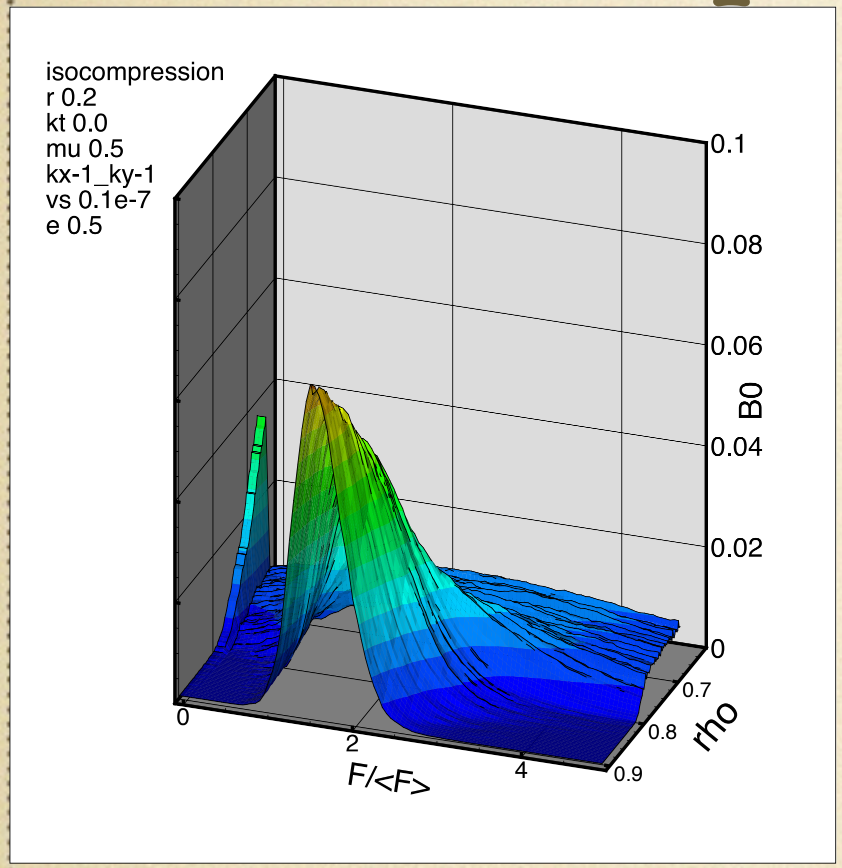

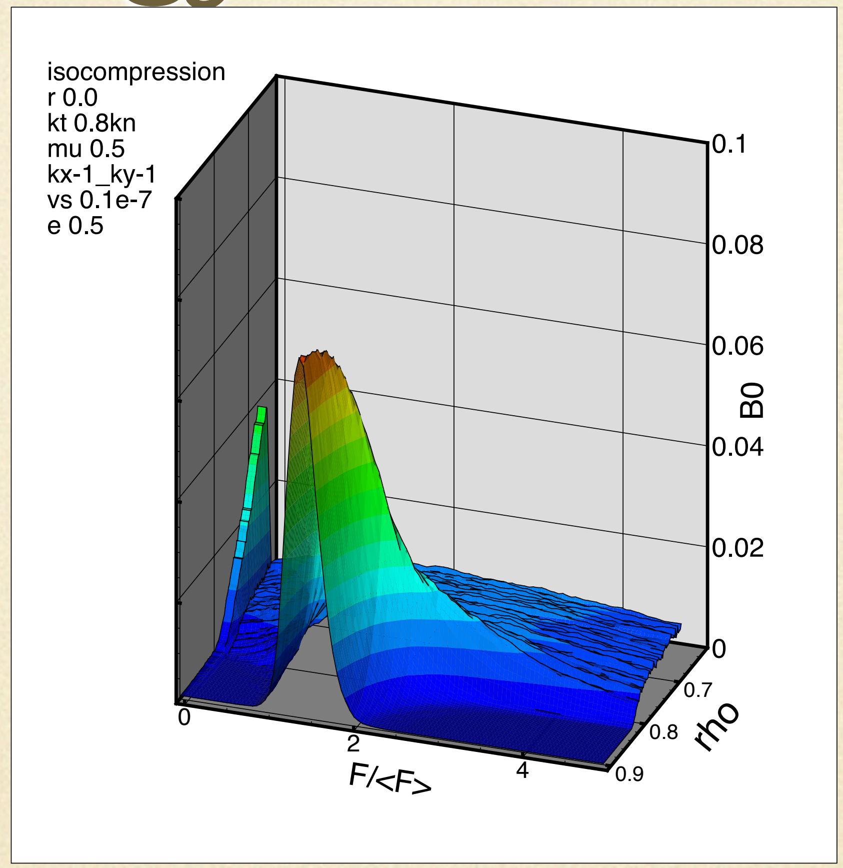

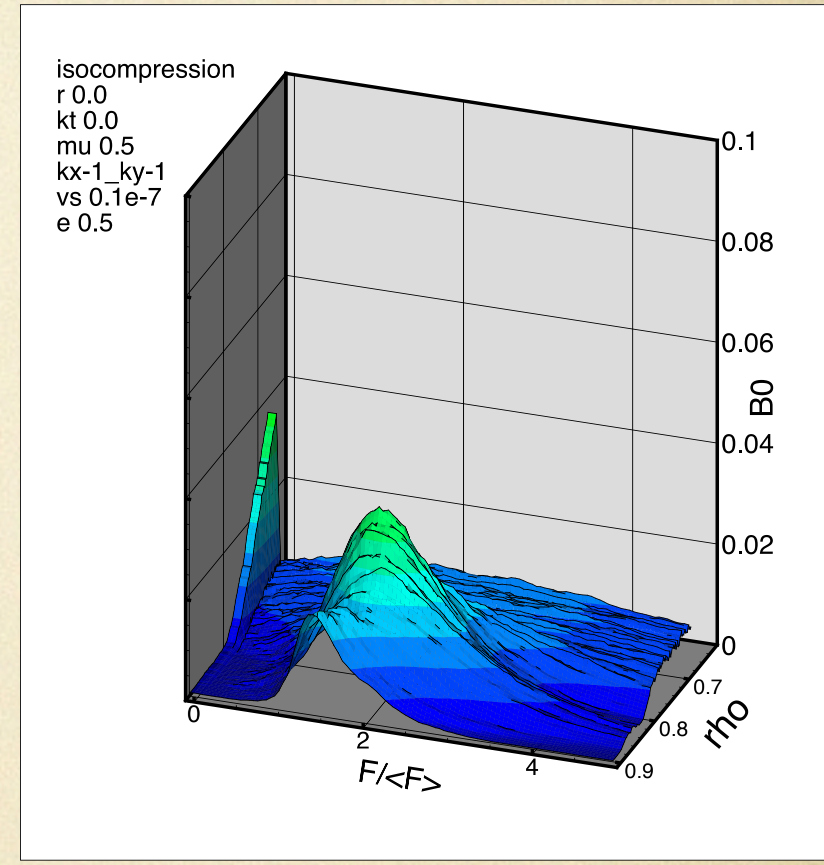

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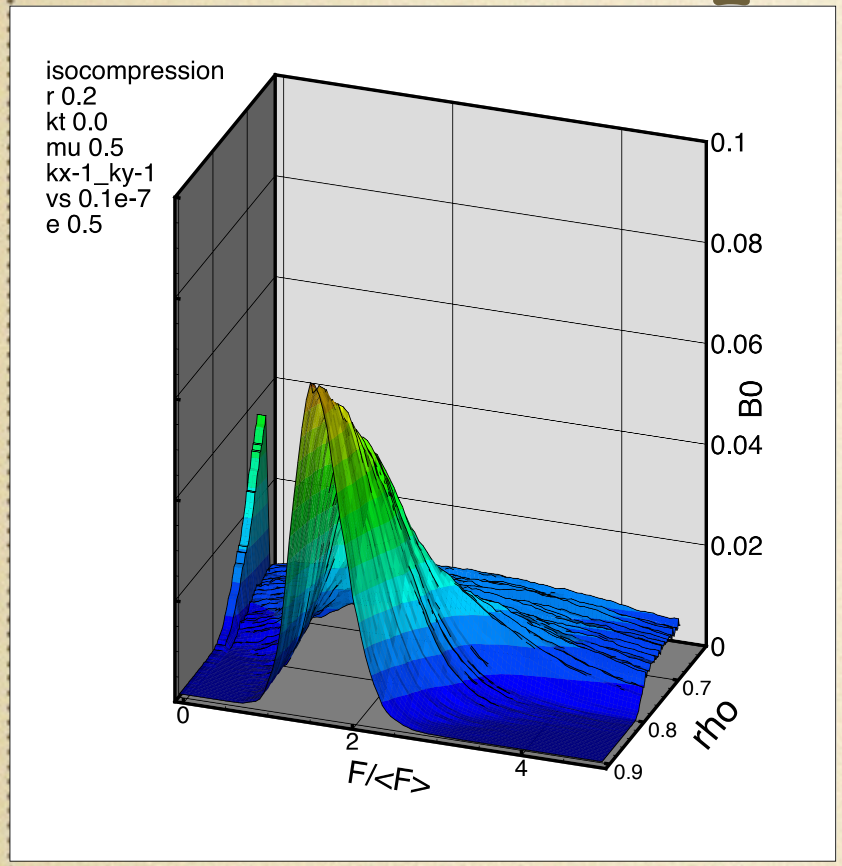

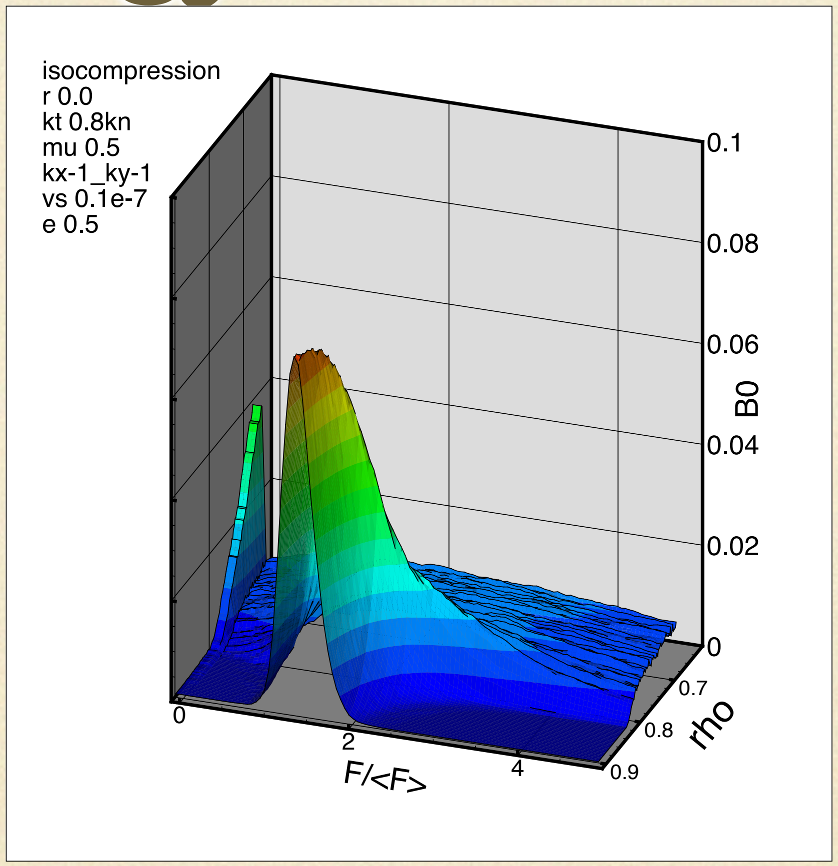

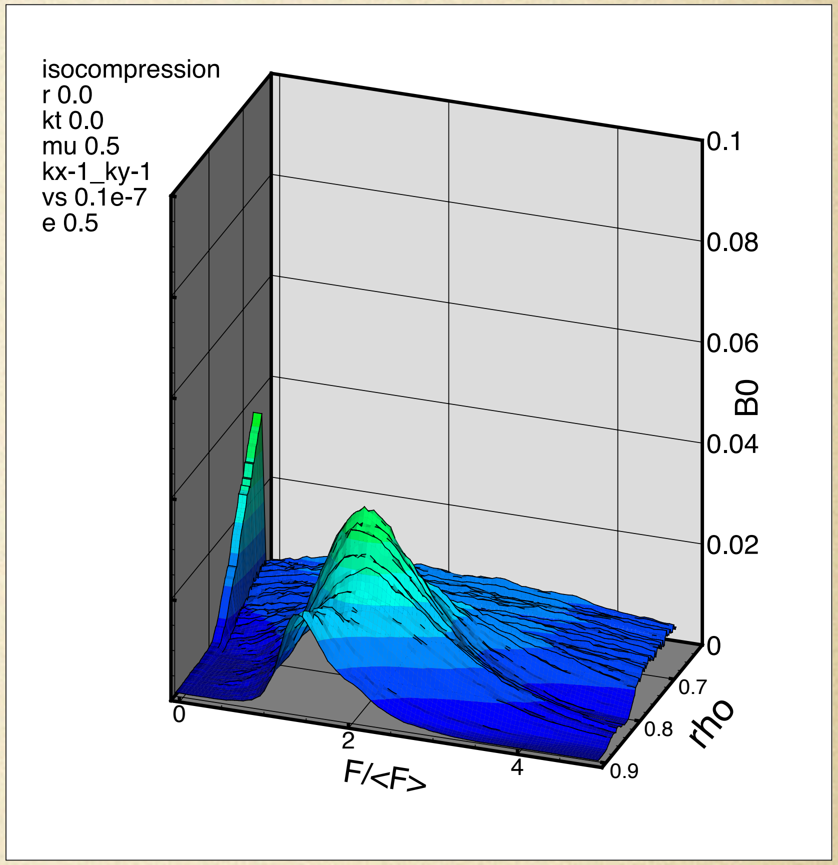

polydisperse, dynamic friction (static friction produces similar results)

monodisperse, static friction

monodisperse, dynamic friction

B0 captures the influence of friction on the structure of force network

Summary

- discrete element simulations are very useful in analyzing granular systems
- significant information can be extracted and connected to continuum models
- computing hardware and software allows for considering problems and questions which could not be approached just few years ago
- there is a large number of relevant and important problems which should be addressed: 3D systems; cohesive particles; particles of irregular shapes, and many others