"Vorticity deposition and evolution in accelerated inhomogeneous flows: Analysis, Computation, Experiment & Models"

Norman J. Zabusky

Weizmann Institute of Science, Dept of Complex Physics
Rutgers University, Dept MAE & CAIP Center
CSCAMM, INC ‘06

Copyright by N.J. Zabusky, 2006
“AIFS” Accelerated Inhomogeneous Flows

- **Domains:** Supernovae Astrophysics; GFD and Breaking Waves; Laser (IC) Fusion; Supersonic Combustion
- **Objectives:** Understand & model vortex physics & mixing
- **Approach:** Construct Reduced Models via Theory, Simulation, and Visiometrics with Experimental Juxtaposition
- **Specific Configurations:** Shock & Forced Acceleration Interactions with various geometries: Perturbed planar & Shock Cylinder
Topics

- Well-posedness and *finite initial transition layer*
- RM *a-dot* -> *constant* at intermediate times
- Circulation generation (*vortex bilayers*) & gradient Intensification
- Vortex Projectiles & Bounding box elongation
- Baroclinic Turbulence & Forcing
1. Rayleigh-Taylor, $g = \text{constant}$

2. Richtmyer-Meshkov, $g = G \delta(t)$, (impulse)

1. Rayleigh-Taylor, $g = \text{constant}$

Taylor’s Amplitude Formula

$$\omega = \pm \left[ g k_z A + k_z^3 \frac{T}{(\rho_1 + \rho_2)} \right]^{1/2},$$

$$\frac{d^2 a(t)}{dt^2} = k g A a(t)$$

where $A = \frac{(\rho_1 - \rho_2)}{(\rho_1 + \rho_2)}$ and $g$ is directed from 2 to 1.
Common geometries in studying AIFS flows

M = Mach No. , A = Atwood No. = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}
Dropped Tank, Incompressible
Jeff Jacobs, Pioneering Experiments

\[ M = \text{Mach No.}, \ A = \text{Atwood No.} = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \]
Sequence of images from an experiment with 1 & 1/2 waves and $ka_0=0.23$.

Times relative to the midpoint of spring impact are: (a) -14 ms, (b) 102 ms, (c) 186 ms, (d) 269 ms, (e) 353 ms, (f) 436 ms, (g) 520 ms, (h) 603 ms, (i) 686 ms, (j) 770 ms, (k) 853 ms, (l) 903 ms.

Thick 51mm

254mm
Classical RM Geometry

\[ M = \text{Mach No.} \quad A = \text{Atwood No.} = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \]
J. JACOBS & Colleagues, Laboratory Experiments

“Spike”

“Bubble”

2a
R-M a-dot comparisons

M=1.5, A=0.5, a₀/λ=0.05
Gas Dynamics Euler Equations for 2D Compressible RM Simulations

\[
\begin{bmatrix}
\rho \\
\rho u \\
\rho v \\
E
\end{bmatrix}_t + \begin{bmatrix}
\rho u \\
\rho u^2 + p \\
\rho uv \\
u(E + p)
\end{bmatrix}_x + \begin{bmatrix}
\rho v \\
\rho uv \\
\rho v^2 + p \\
v(E + p)
\end{bmatrix}_y = 0
\]

where, total energy \( E = e + (u^2 + v^2) / 2 \)

Equations of State (EOS): \( p = (\gamma - 1) \rho e \) (for closure)
Vorticity Evolution Equation

\[
\frac{\partial \omega}{\partial t} = - \mathbf{u} \cdot \nabla \omega + \mathbf{\omega} \cdot \nabla \mathbf{u} - \mathbf{\omega} (\nabla \cdot \mathbf{u}) + \frac{1}{\rho^2} (\nabla \rho \times \nabla p)
\]
Schematic of regular refraction, for three shocks at a fast-slow interface. Incident, $i$, reflected, $r$, and transmitted, $t$, shocks intersecting at a node on the interface, $m$.

Local, Shock-Polar Analysis Yields Circulation pu Length

\[
\frac{d\Gamma}{ds'} \equiv u_1 - u_2, \quad \Gamma' \equiv \frac{d\Gamma}{ds} = (u_1 - u_2) \frac{\cos \alpha}{\cos(\alpha - \delta_b)}
\]

\[
\Gamma'_4 = \frac{2\gamma^{\frac{1}{2}}}{\gamma + 1} (1 - \eta^{-\frac{1}{2}}) \sin \alpha (1 + M^{-1} + 2M^{-2})(M - 1).
\]

See Samtaney & Zabusky (1994)
Inclined Planar

Use symmetry

$M = \text{Mach No.}$, $A = \text{Atwood No.} = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$
Vortex Paradigm, Hawley & Zabusky

PRL, 1989
OVERVIEW: RM

A Vortex Approach

- Topics
  - Well-posedness and
    - finite initial transition layer (ITL)
RT & RM Finite-Time singularity
(ill-posed nature of vortex sheet evolution)

Kelvin-Helmholtz of vortex sheets

- *Finite-time Moore curvature singularity*
  - Similar to finite time singularity in first derivative of
  - non-controllable *numerical rollups* due to grid perturbations
- *Regularization* - Finite Interfacial Transition Layer (ITL)

\[ u_t + uu_x = 0 \]

**Density**: Shock inclined interface (Samtaney '96)
Richtmyer-Meshkov Planar inclined Interface
Density (left) Vorticity (right)

Vortex Paradigm

(Hawley & Zal PRL, 1989)

(R. Samtaney & NJ Zabusky JFM)
Integrated vorticity space-time diagram, $M=2; A=0.5$
y-integrated vorticity for a shock interacting with a planar layer: 
(M = 1.5, A=0.5; [Air/R22], at an angle of 60 deg)
Classical RM Geometry

\[ M = \text{Mach No.}, \quad A = \text{Atwood No.} = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \]
RT & RM  Finite-Time singularity
(ill-posed nature of vortex sheet evolution)

Kelvin-Helmholtz of vortex sheets

– *Finite-time Moore curvature singularity* for **single sine wave**

– *Regularization* via finite ITL

\[ u_t + uu_x = 0 \]

Vorticity

Sinusoidal interface

\[ \nabla^2 \rho \]
Vizlab Simulations (PPM) of G. Peng and S. Zhang & Jacob & Krivets’ Experiment (PLIF)
(M = 1.3, A = 0.635) Juxtaposition: Columns of Vorticity, Density, Experimental PLIF

Extracted interfaces (in high gradient regions)

- (a) 0.045
- (d) 3.69
- (b) 1.27
- (e) 4.17
- (c) 2.42
- (f) 5.26
- (g) 6.17
OVERVIEW: “AIFS” Accelerated Inhomogeneous Flows

A Vortex Approach

Topics

- Well-posedness and *finite initial transition layer*
- RM *$a-dot \rightarrow constant$* at intermediate times
Amplitude growth $a(t)$: 

simulation and experiment

$k(a-a_0^*)$ vs $t/t_M$
a & a-dot for $A = 0.2, 0.635 & 0.9$

a

a-dot

a-dot $\to$ cnst at intermediate times
A-dot comparisons:
Simulations & proposed
*adjusting-one vortex model*

\[
\dot{a}_{\text{vortex}} = -\frac{k \Gamma \sin kx_c}{4 \pi} \left( \frac{1}{\cosh(kd_s) - \cos kx_c} \right) + \frac{1}{\cosh(kd_b) + \cos kx_c},
\]
OVERVIEW: RM

Topics

- Well-posedness and initial transition layer
- RM \textit{a-dot} \textit{-> constant} at intermediate times
- Circulation generation (vortex bilayers) & gradient Intensification
Global quantifications: Circulation & Enstrophy

Circulation \[ \Gamma_+, \Gamma_-, \Gamma = \Gamma_+ + \Gamma_- \]

\[
\int_D [\omega, \omega_\pm](x, y, t) \, dx \, dy
\]

Enstrophy

\[
\int_D \omega^2(x, y, t) \, dx \, dy
\]
Circulations vs time, RM with $A=0.635$, $M=1.3$

- Curved shock in volume & noise
- Shock-related
- Vortex-accelerated deposition
- Total
RM
Baroclinic vorticity

0.5t_m

1.0t_m

1.5t_m
Vortex-accelerated “secondary” baroclinic vorticity deposition

\[
\frac{D \omega}{Dt} = \frac{\nabla \rho \times \nabla p}{\rho^2} + \omega \cdot \nabla \mathbf{u} - \omega \nabla \cdot \mathbf{u}
\]
FIG. 13. Density gradient magnitude (normalized by the preshock initial maximum density gradient magnitude) distribution for $A^* = 0.635$ at times $t/t_M = 1.0$, 5.12, 7.2, and 12.
Definitions:
\[ u = u e_x + v e_y \quad \omega = \omega e_z = (-\partial_y u + \partial_x v) e_z, \]
\[ \Gamma_D = \iint_D \omega \, dx \, dy. \]

Circulation Rate of Change
\[ \partial_t \Gamma = -\frac{1}{2} \oint \nabla (u \cdot u) \cdot ds + \oint \nabla (u \times \omega) \cdot ds + \oint \rho^{-1} \nabla p \cdot ds \]
\[ = 0 + 0 + \oint \rho^{-1} dp \]
\[ = -\int_{e_1 e_2} p \rho^{-2} dp - \int_{f_1 f_2} p \rho^{-2} dp \]
\[ \partial_t \Gamma = -(p_b - p_t) \left( \frac{1}{\rho_1} - \frac{1}{\rho_2} \right) = -(p_b - p_t) \frac{\rho_2 - \rho_1}{\rho_2 \rho_1} \]

Choose domain to select positive or negative, etc
\[ \dot{\Gamma} \approx - \int_{a_1}^{a_2} p(d\rho / \rho^2) - \int_{b_1}^{b_2} p(d\rho / \rho^2), \]  
\[ f_{ij}(t_n) = \sum_{t_m = t_M/6}^{t_m = t_M/6} f_{ij}(t_m) \Phi(t_n; t_m) \]

where \( \Phi(t_n; t_m) = T^{-1} [1 + \cos (\pi (t_n - t_m) / T)], -T/2 \leq t_m \leq T/2, \) \& \( T = t_M/3. \)

\[ \dot{\Gamma}^*(t_n) = \frac{\tilde{p}_a(t_n) - \tilde{p}_b(t_n)}{(\rho_2 - \rho_1) / \rho_2 \rho_1} \]  
\[ \dot{\Gamma}^#(t_n) = \sum_{D}^2 \left[ \frac{\tilde{\omega}_{i,j}(t_{n+1}) - \tilde{\omega}_{i,j}(t_{n-1})}{2\delta t} \right]. \]
Summary: 2D Vortex paradigm for the evolution of RM interfaces through intermediate times

- Intermediate-time dominance of vortex-accelerated vorticity deposition (VAVD) process; [Peng et al (2003)]

- Quantification procedures & formulas for net circulation rate of change, \( \dot{\Gamma}_D \), comprise a vortex paradigm for the evolution of RM & RT interfaces through intermediate times. [Lee, Peng & Zabusky (Sept. 2006)].

- Special features observed [Lee, Peng & Zabusky (2006)] are signatures of physically important phenomena and include:
  - Gradient intensification of interfacial transition layer
  - \( N \) for \( t/t_M > 1 \) that increases with \( \Lambda^* \) and the
  - \( \dot{\Gamma}_D \) scaling at intermediate times: near-constant negative values that increase with \( \Lambda^* \), for \( 0.5 < \Lambda^* < 0.75 \).

- Generalization to other accelerated inhomogeneous flow configurations (axisymmetry, shock-cylinder [S. Zhang, N. J. Zabusky, G. Peng, and S. Gupta, PoF, 2004], etc)
OVERVIEW: “AIFS” - RM

Topics

- Well-posedness and initial transition layer
- RM $a\text{-}dot \to \text{constant}$ at intermediate times
- Circulation generation (vortex bilayers) gradient Intensification

Vortex Projectiles
RM New Results

• **Secondary Baroclinic Circulation** is much greater than Primary (Deposited by Shock). *PoF ‘03: G. Peng, S. Zhang & N. Zabusky*,
  – Due to *vortex acceleration & gradient intensification of* transition layer (TL)

• **Vortex Projectiles**: Dipolar/Ring-like objects active at all times in determining turbulence and mixing

• **New diagnostic**: Rate of change of circulation in bubble to spike domain. *PoF, ’06 : D.K. Lee, G. Peng, & NJZ*
Extraction Algorithm

- $|\text{GRAD } \rho| > 0.1 \{\text{max}|\text{GRAD } \rho|_0\}$
- $\text{LAPLACE (density)} = 0$
Initial domain

ρ₁ = 1.0x10⁻³ (Air)
ρ₂ = 5.0x10⁻³ (SF₆)
Initial domain

$\rho_1 = 1.0 \times 10^{-3}$ (Air)

$\rho_2 = 5.0 \times 10^{-3}$ (SF$_6$)
Visiometrics: uncertainty quantification & numerical validation

(a). Experimental images (LANL)

(c). Simulation with visiometrics (SZhang@Vizlab)
Compare Jacobs’ Experiment

Experiment: PLIF

(t = 0 ms)

Cyl 2

Shock

Cyl 3

Slice y=0

(t = 0.22 ms)

protuberance

PLIF Colormap

Density Colormap

Vorticity Colormap
Comparing with Jacobs’ Experiment cont.

Experiment: PLIF

Cyld 2

Cyld 3

$\tau = 0.37 \text{ ms}$

$\tau = 0.59 \text{ ms}$

PLIF Colormap

Density Colormap

Vorticity Colormap

VBL

VP
Comparing with Jacobs’ Experiment cont.
Variation of transition layer thickness: IC uncertainty

Sim I: Linear transition profile;
Sim II: Error function profile, with initial SF6 concentration 60%
Integrated vorticity space (x)- time (t) diagram (Hawley & Zabusky: PRL 1989)