"Vorticity deposition and evolution in accelerated inhomogeneous flows: Analysis, Computation, Experiment & Models"

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CSCAMM, INC '06

"AIFS" **Accelerated Inhomogeneous Flows**

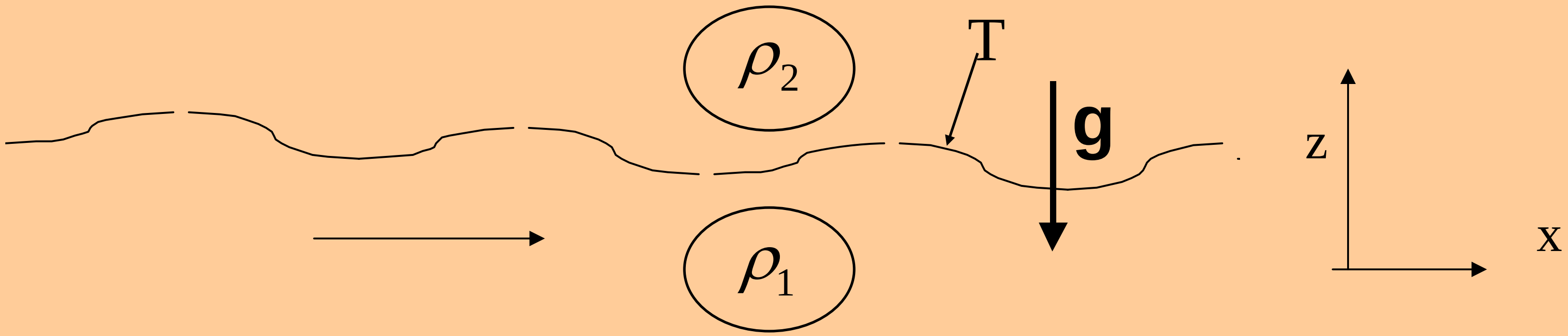
- **Domains:** **Supernovae Astrophysics; GFD and Breaking Waves; Laser (IC) Fusion; Supersonic Combustion**
- **Objectives: Understand & model *vortex physics* & *mixing***
- **Approach: Construct Reduced Models** via **Theory, Simulation, and *Visiometrics* with Experimental Juxtaposition**
- **Specific Configurations: Shock & Forced Acceleration Interactions with various geometries : Perturbed planar & Shock Cylinder**

OVERVIEW: "*AIFS*"   2D Richtmeyer-Meshkov

- **Topics**
  - **Well-posedness and *finite initial transition layer***
  - **RM *a-dot ->constant* at intermediate times**
  - **Circulation generation (*vortex bilayers)* & gradient Intensification**
  - **Vortex Projectiles & Bounding box elongation**
  - **Baroclinic Turbulence & Forcing**

Rayleigh-Taylor & Richtmyer-Meshkov Incompressible

[For RT, see *Hydrodynamic & Hydromagnetic Stability* by S. Chandrasekhar, p.483]



1. **Rayleigh-Taylor, g = constant**
2. **Richtmyer-Meshkov, g=G δ(t), (impulse)**

1. Rayleigh-Taylor, g = constant

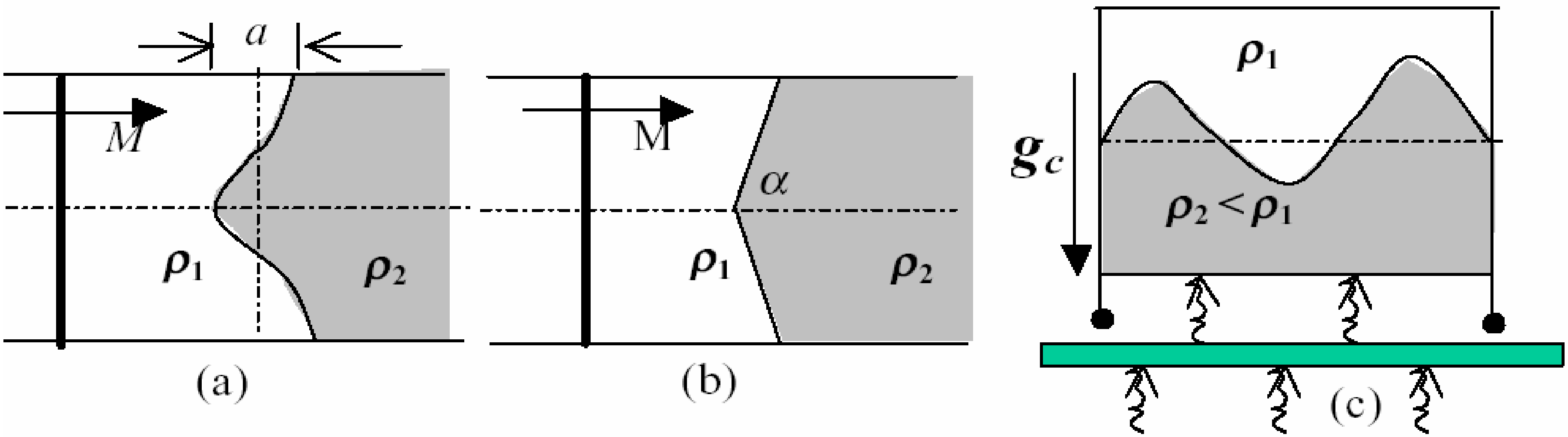
Taylor's Amplitude Formula

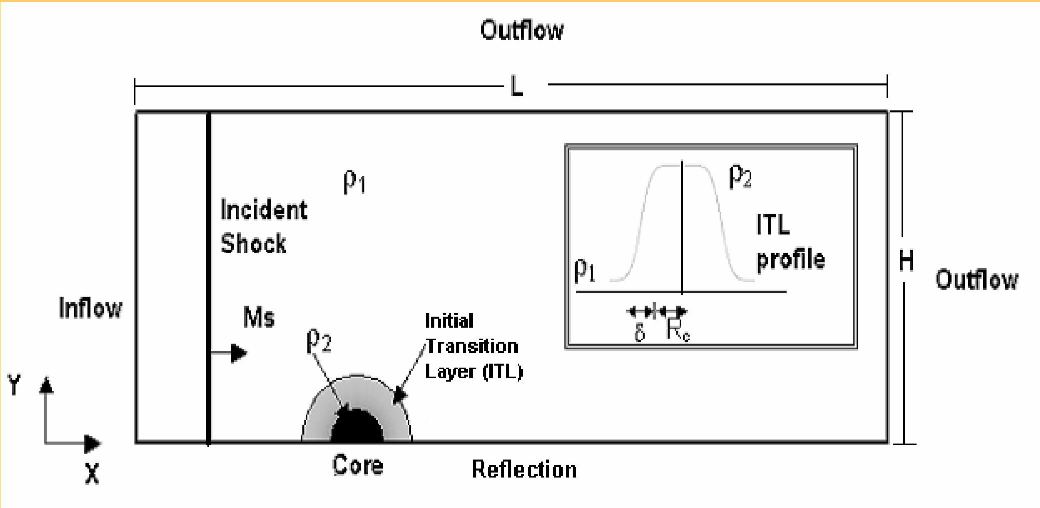
$$\omega = \pm[\, g k_z A + k_z{}^3 \frac{T}{(\rho_1 + \rho_2)}]^{1/2},$$

$$\frac{d^2 a(t)}{dt^2} = k g A\, a(t)$$

$$where \;\; A = \frac{(\rho_1 - \rho_2)}{(\rho_1 + \rho_2)} \;\; and \;\; g \; is \; directed \;\; from \; 2 \; to \; 1.$$

Geometries



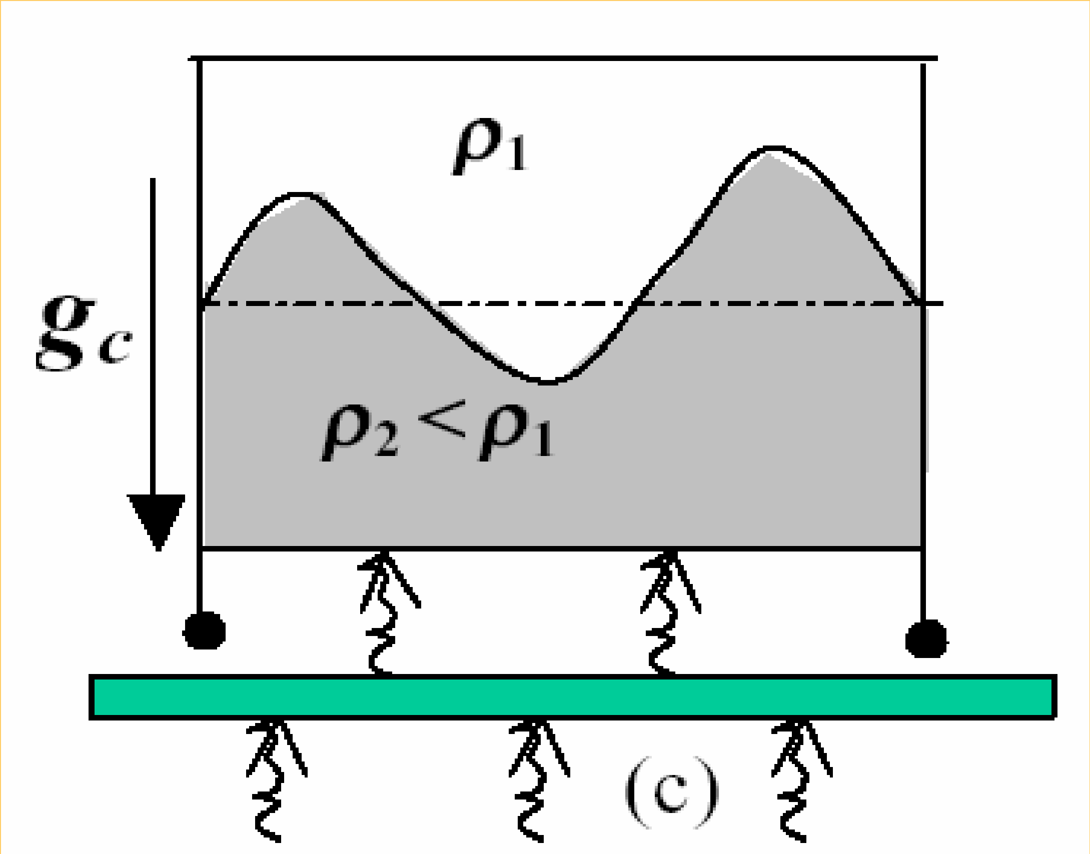


Common geometries in studying *aifs* flows

M=Mach No. , A = Atwood No. = $\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

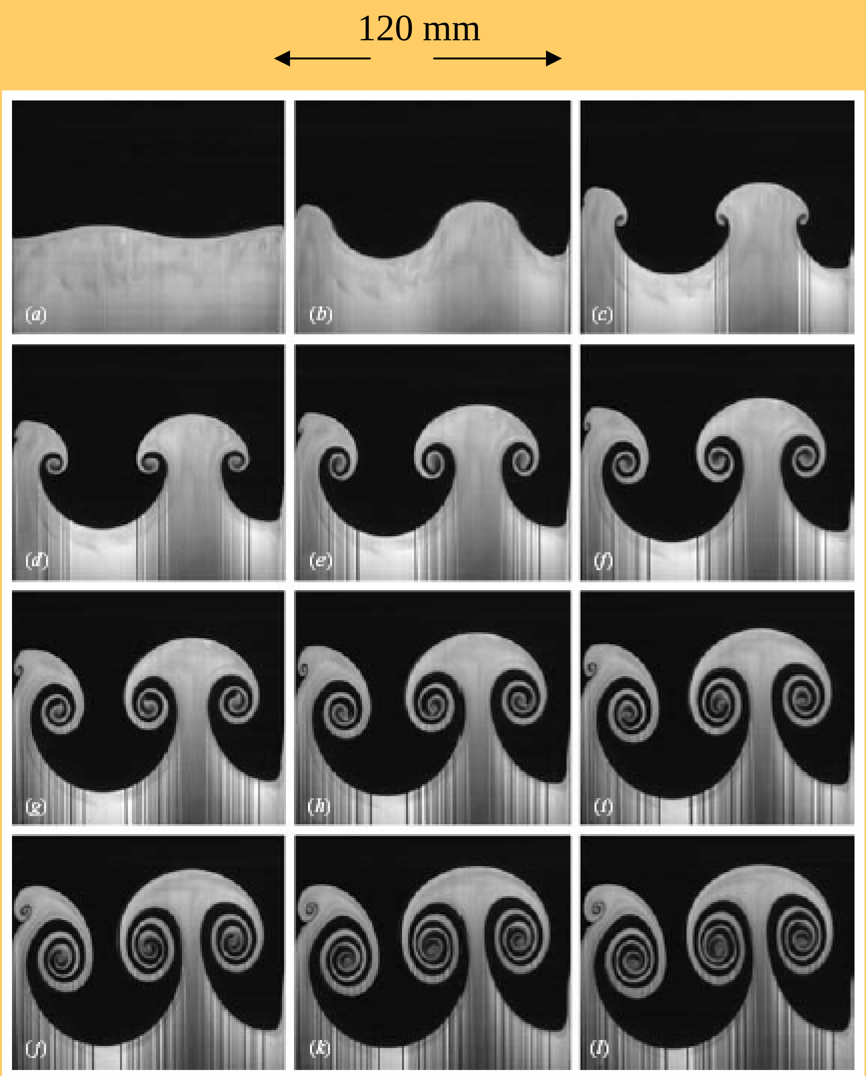
Dropped Tank, Incompresible

Jeff Jacobs, Pioneering Experiments



M=Mach No. , A = Atwood No. = $\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

Jacobs & Niederhaus , Experiment, 2003

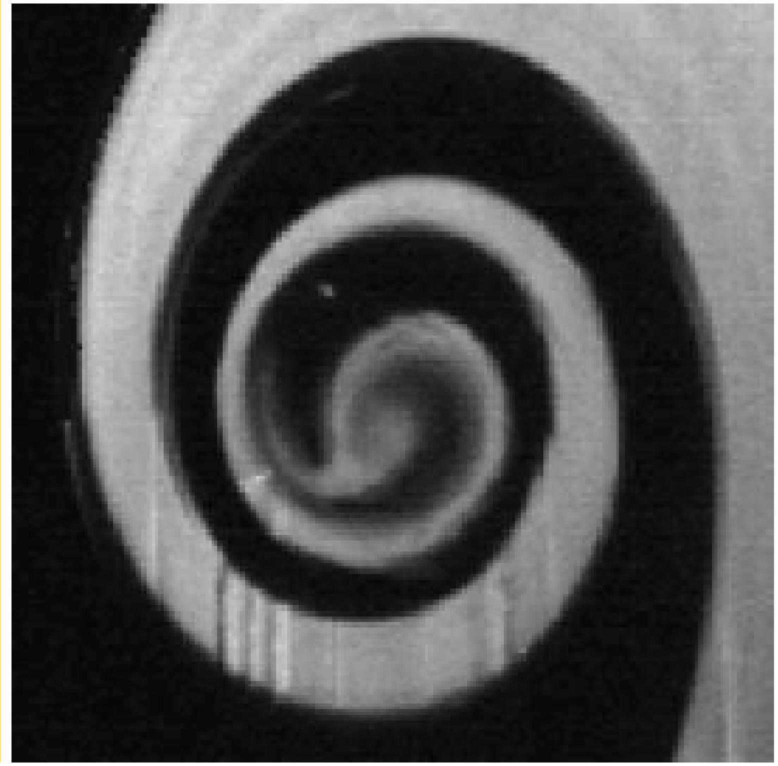


Sequence of images from an experiment with 1 &1/2 waves and ka_0=0.23.

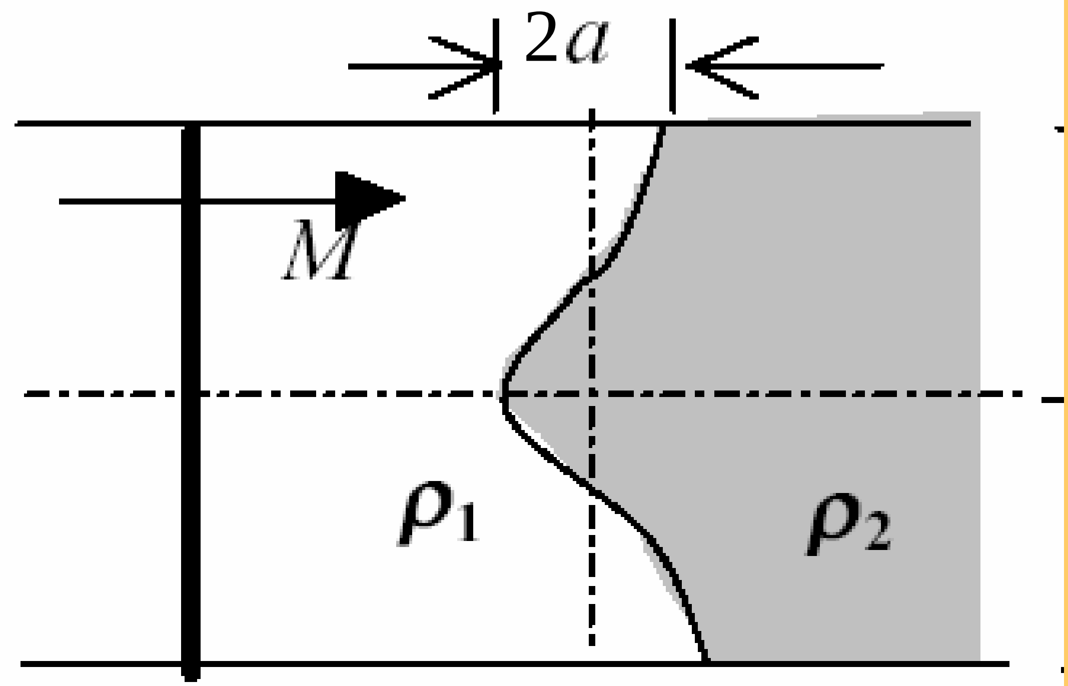
Times relative to the midpoint of spring impact are: (a) -14 ms, (b) 102 ms, (c) 186 ms, (d) 269 ms, (e) 353 ms, (f) 436 ms, (g) 520 ms, (h) 603 ms, (i) 686 ms, (j) 770 ms, (k) 853 ms, (l) 903 ms.

Thick 51mm

254mm

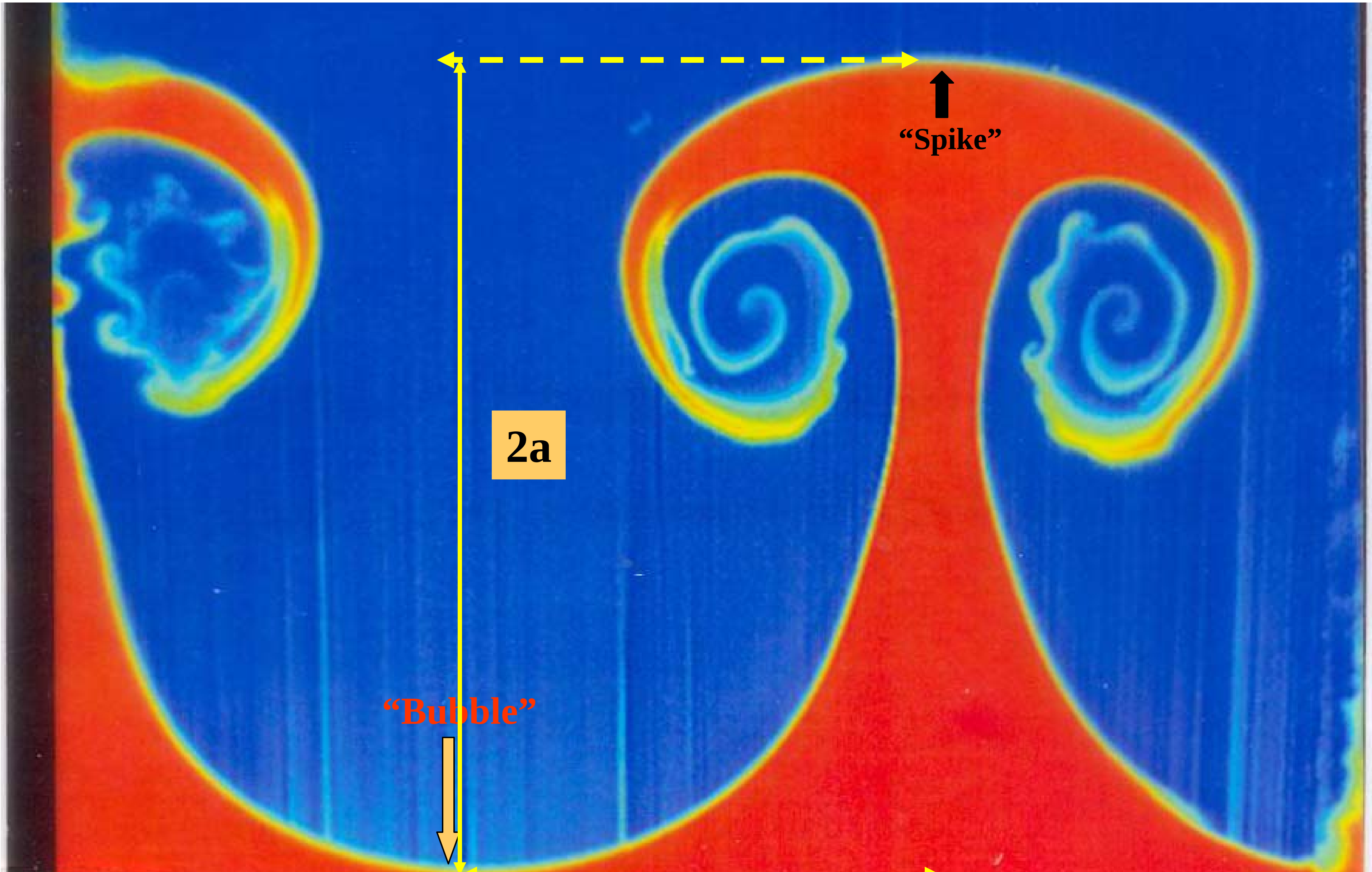


Classical RM Geometry



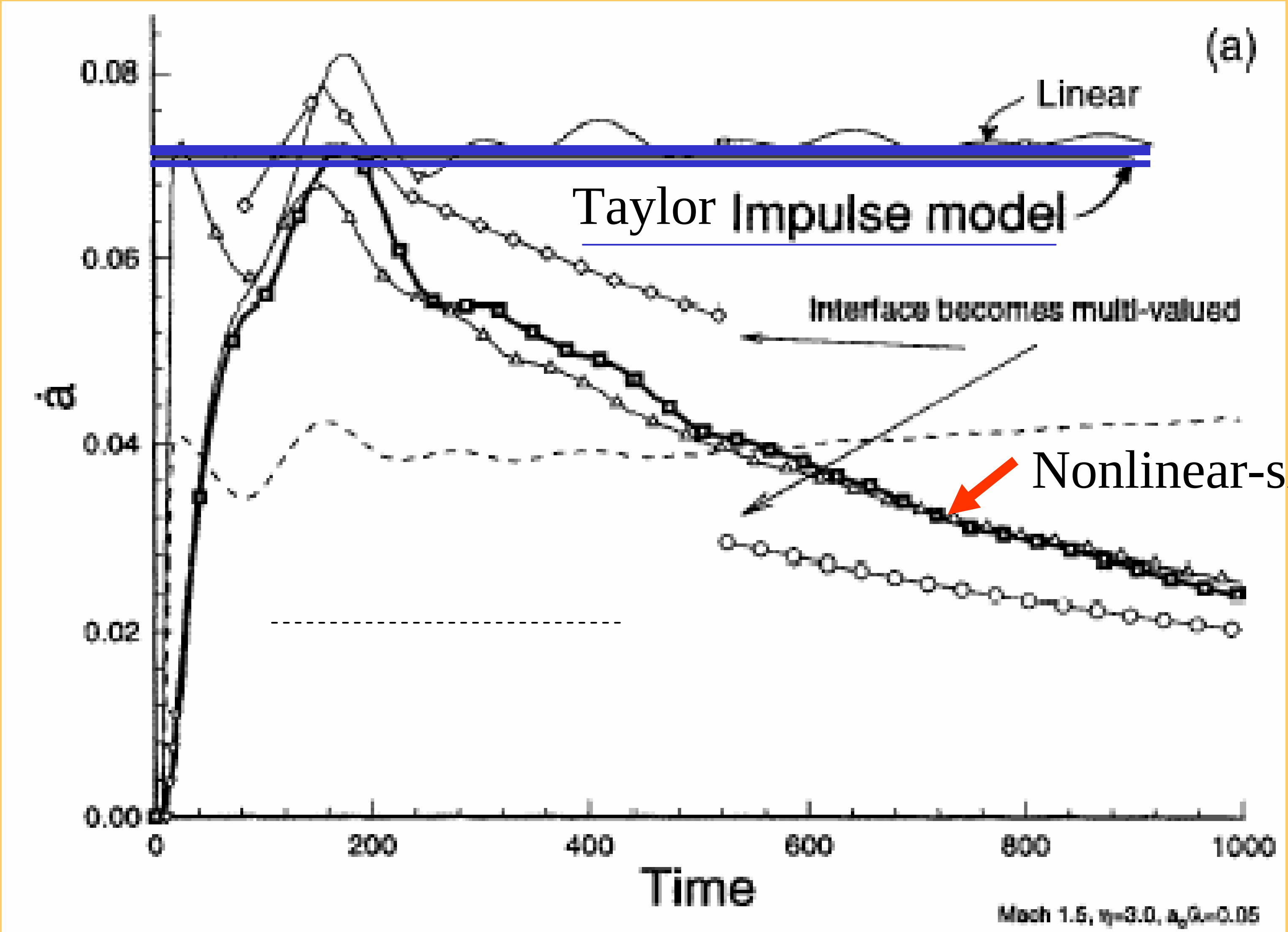
(a)

M=Mach No. , A = Atwood No. = $\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$



J. JACOBS & Colleagues, Laboratory Experiments

R-M a-dot comparisons



M=1.5, A=0.5, a_0/λ=0.05

Gas Dynamics Euler Equations for 2D Compressible *RM* Simulations

$$\begin{bmatrix} \rho \\ \rho u \\ \rho \mathrm{v} \\ E \end{bmatrix}_t + \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u \mathrm{v} \\ u(E+p) \end{bmatrix}_x + \begin{bmatrix} \rho \mathrm{v} \\ \rho u \mathrm{v} \\ \rho \mathrm{v}^2 + p \\ \mathrm{v}(E+p) \end{bmatrix}_y = 0$$

where, total energy $E = e + (u^2 + \mathrm{v}^2)/2$

Equations of State (EOS): $p = (\gamma - 1)\rho e$ (for closure)

Vorticity Evolution Equation

$$\frac{\partial \omega}{\partial t} = -\mathbf{u} \cdot \nabla \omega + \omega \cdot \nabla \mathbf{u} - \omega(\nabla \cdot \mathbf{u}) + \frac{1}{\rho^2}(\nabla \rho \times \nabla p)$$

Schematic of regular refraction, for *three* shocks at a fast-slow interface.

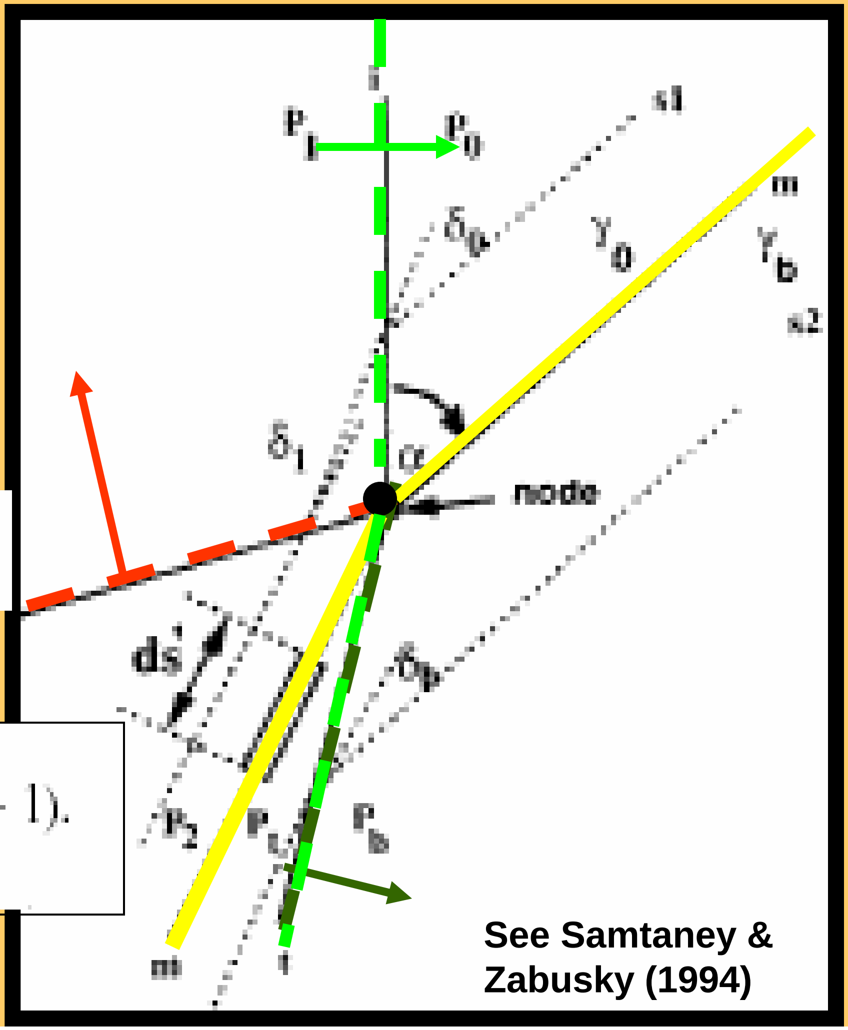
incident, *i* , reflected, *r* , and transmitted, *t*, shocks intersecting at a node on the interface, *m*.

SAMTANEY-ZABUSKY FORMULA

Local, Shock-Polar Analysis Yields Circulation pu Length

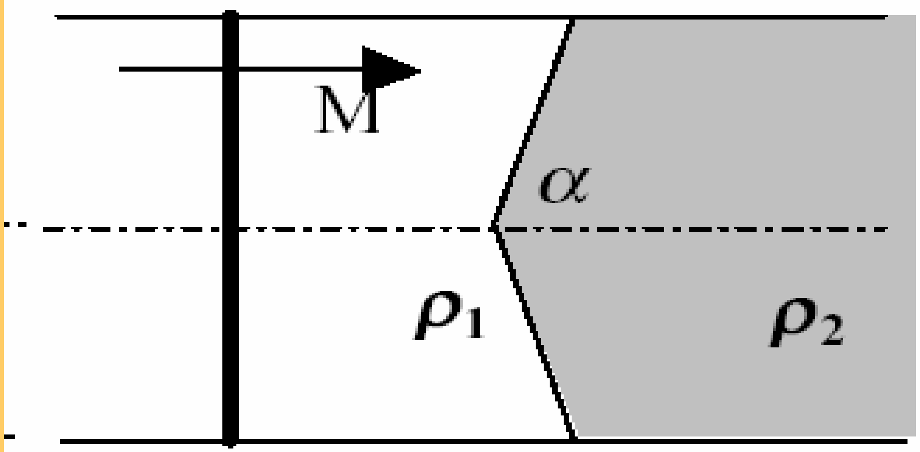
$$\frac{d\Gamma}{ds'} \equiv v_t - v_2, \quad \Gamma' \equiv \frac{d\Gamma}{ds} = (v_t - v_2)\frac{\cos\alpha}{\cos(\alpha - \delta_b)}$$

$$\Gamma'_{+} = \frac{2\gamma^{\frac{1}{2}}}{\gamma + 1}(1 - \eta^{-\frac{1}{2}})\sin\alpha(1 + M^{-1} + 2M^{-2})(M - 1).$$



See Samtaney & Zabusky (1994)

Inclined Planar



(b)

Use symmetry

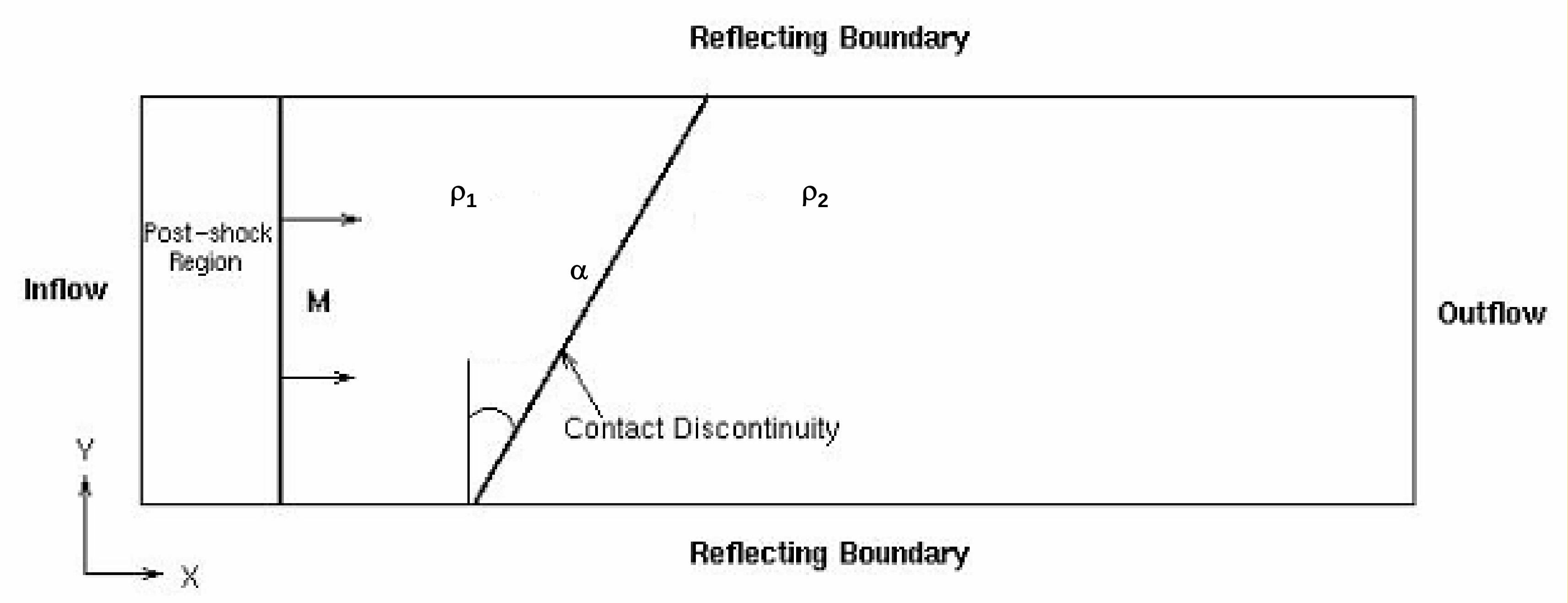
M=Mach No. , A = Atwood No. = $\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

Richtmyer-Meshkov Planar inclined Interface

B. Sturtevant, pioneering Experiments

Vortex Paradigm, Hawley & Zabusky

PRL,1989



OVERVIEW: *RM*

A Vortex Approach

- Topics
  - Well-posedness and
    - *finite initial transition layer (ITL)*

RT & RM Finite-Time singularity

(ill-posed nature of vortex *sheet* evolution)

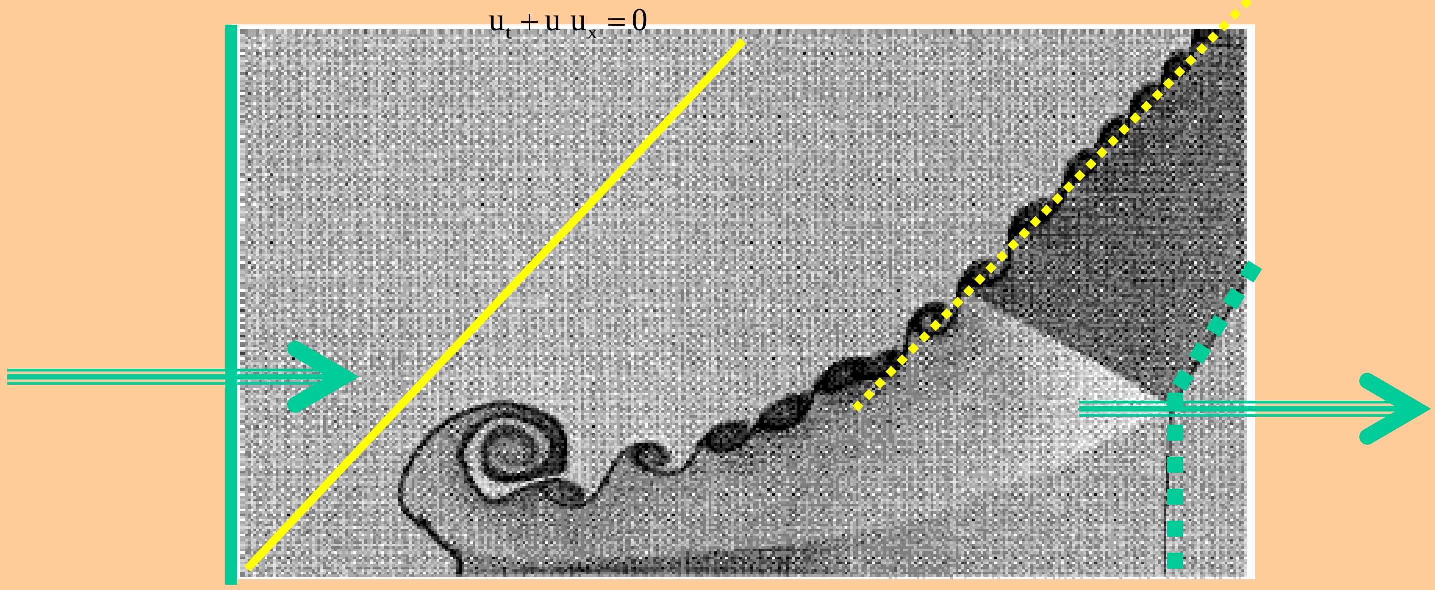
Kelvin-Helmholtz of vortex sheets

– *Finite-time Moore curvature singularity*

- Similar to finite time singularity in first derivative of $u_t + uu_x = 0$

– non-controllable *numerical rollups* due to grid perturbations

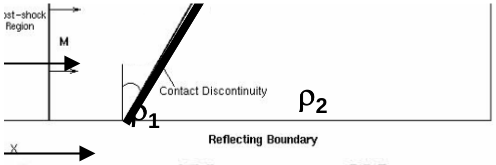
– *Regularization* - Finite Interfacial Transition Layer (ITL)

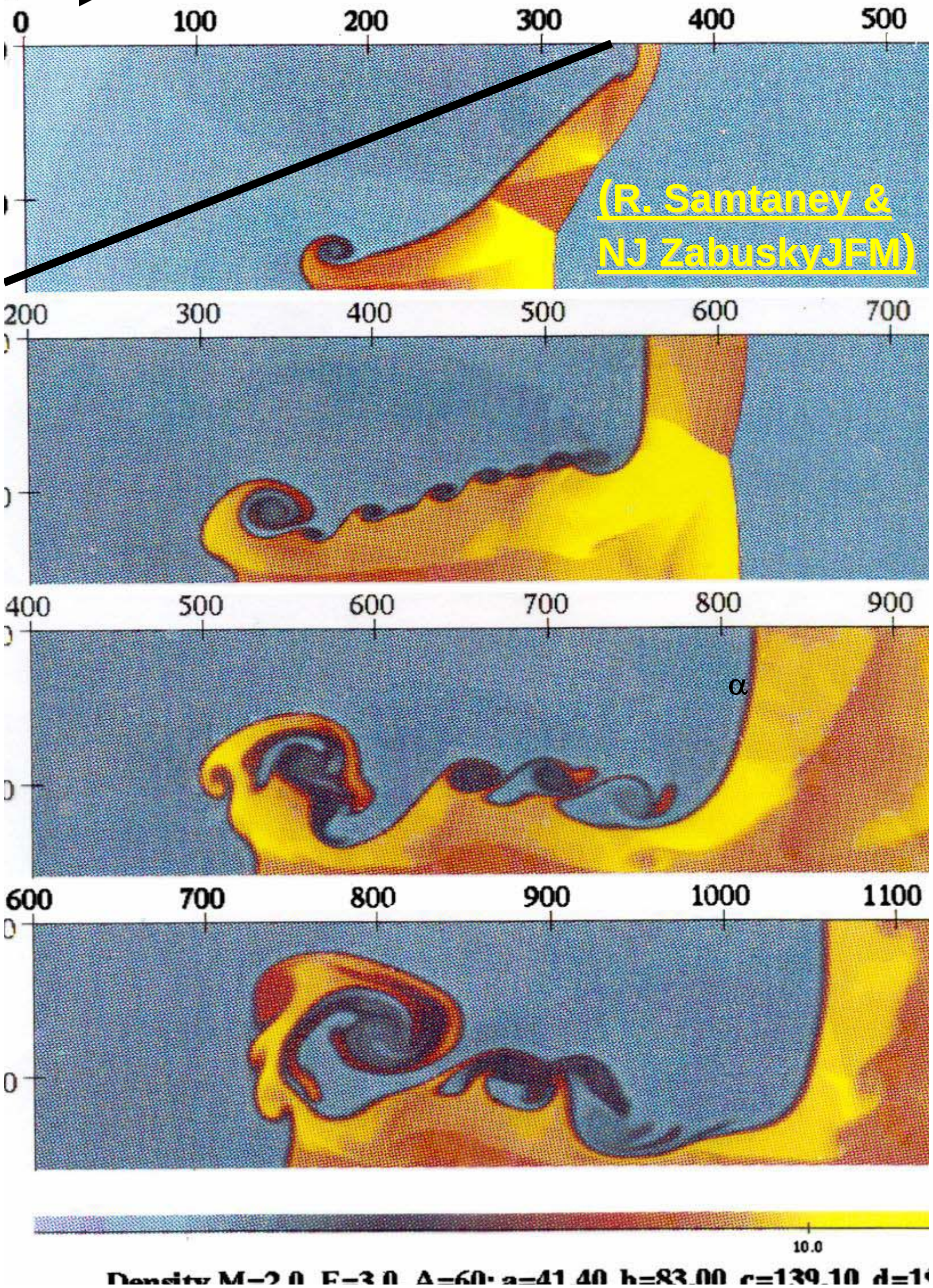


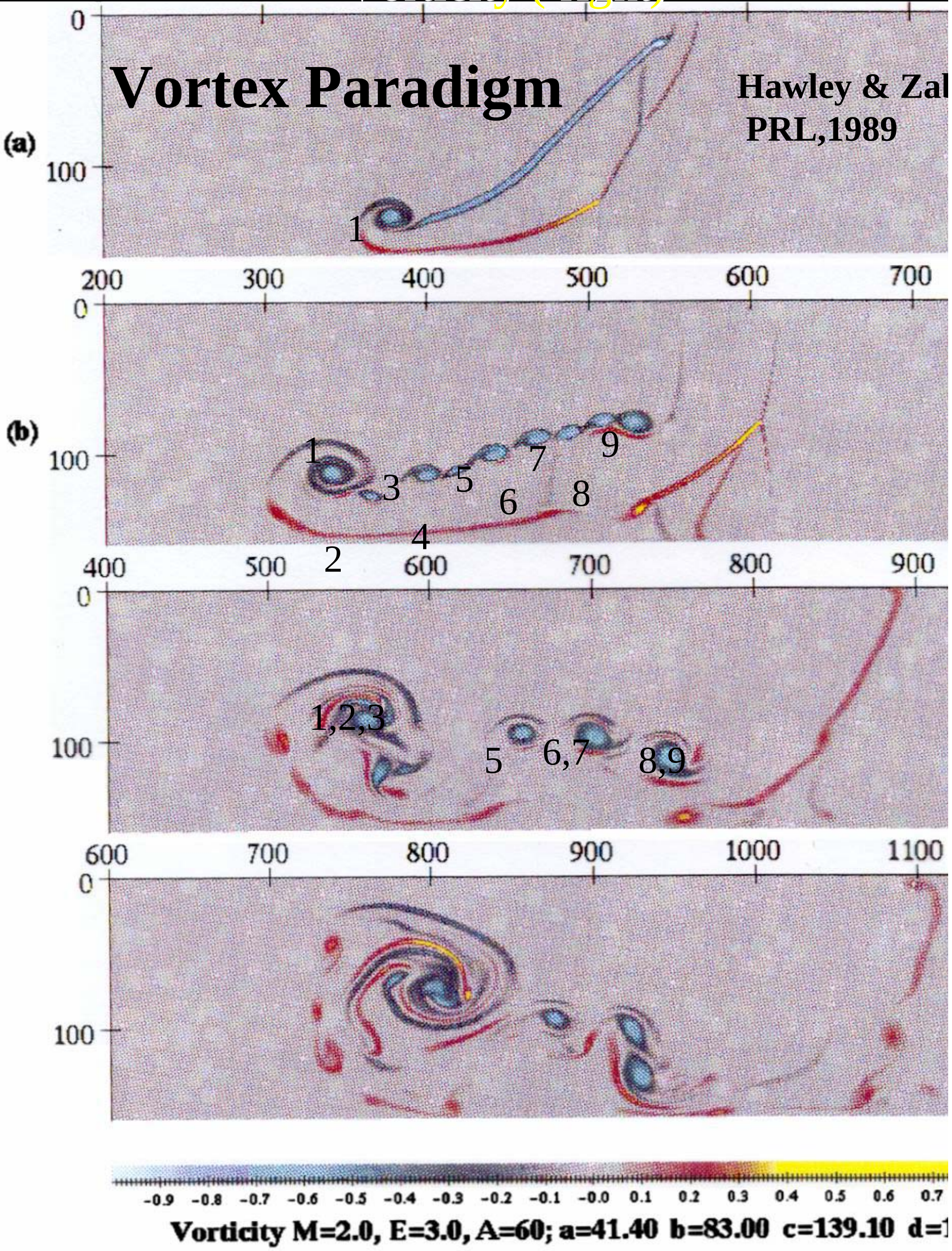
<u>Density</u>: Shock inclined interface (Samtaney '96)

Richtmyer-Meshkov Planar inclined Interface

Density (left) Vorticity (right)

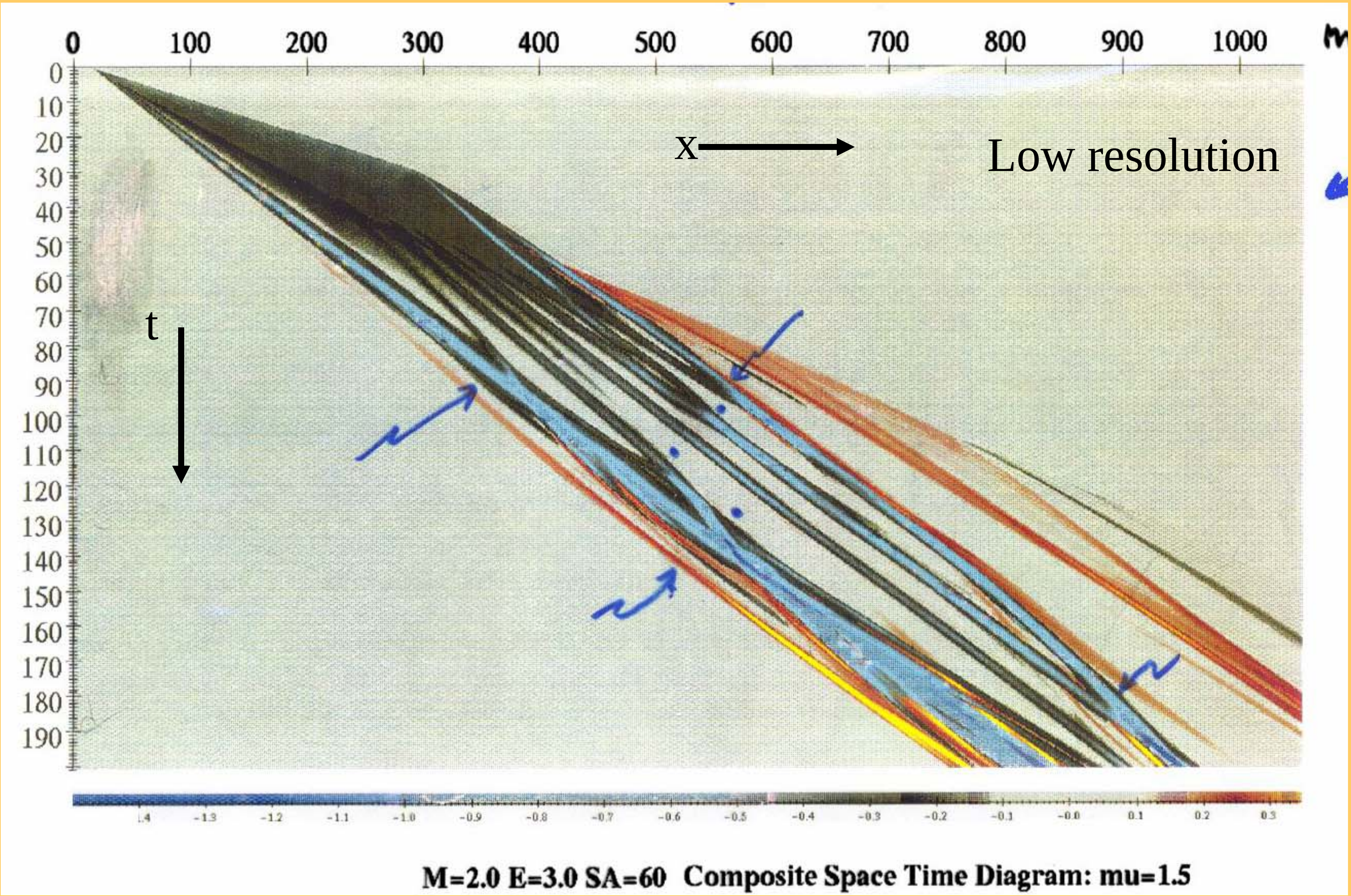






Integrated vorticity

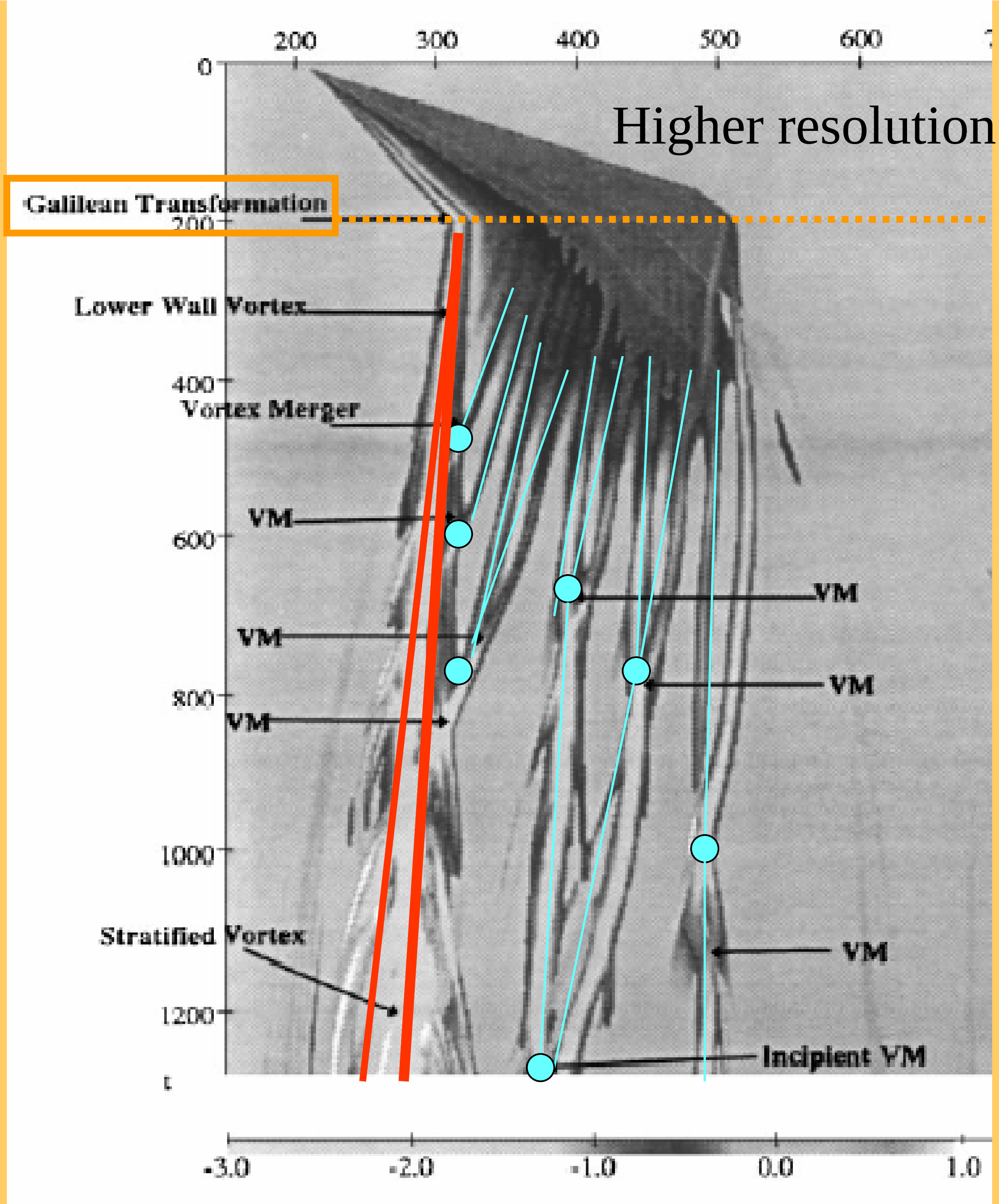
space- time diagram, M=2; A=0.5



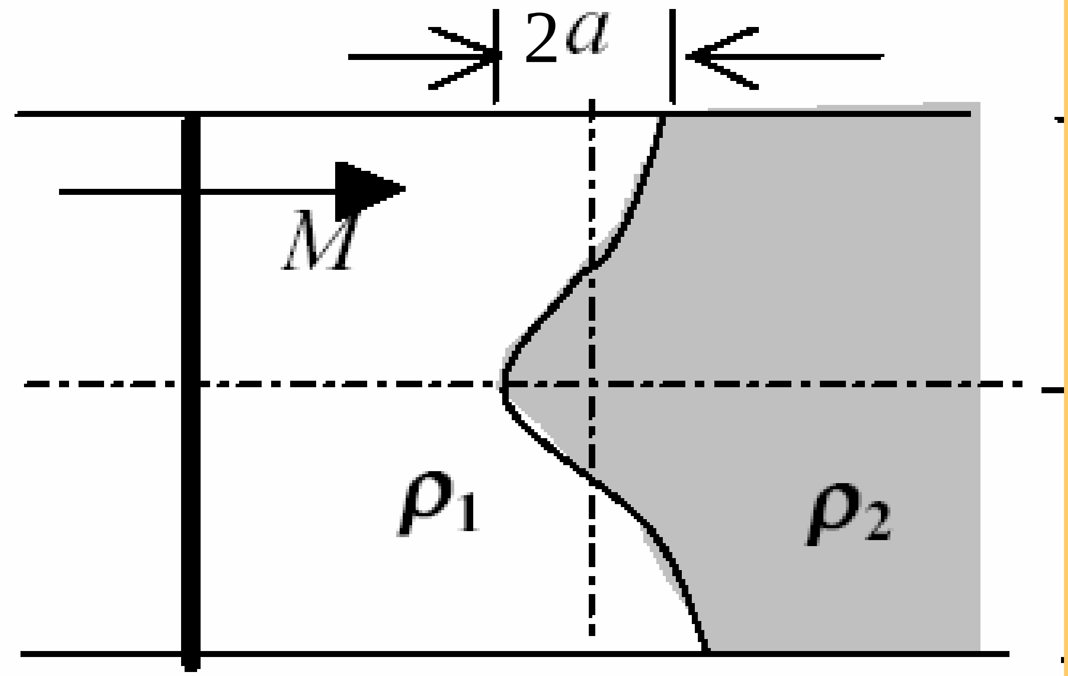
y-integrated vorticity

for a shock interacting with a planar layer:

(M = 1.5, A=0.5;
[Air/R22], at an angle of 60 deg)



Classical RM Geometry



(a)

M=Mach No. , A = Atwood No. = $\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

RT & RM Finite-Time singularity

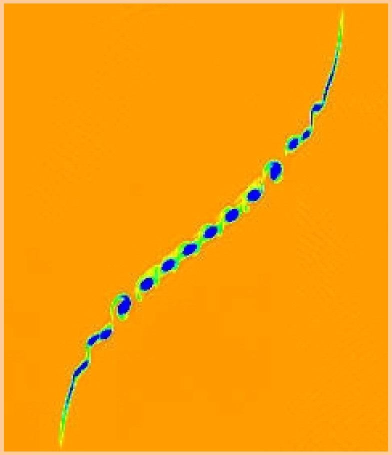
(ill-posed nature of vortex sheet evolution)

Kelvin-Helmholtz of vortex sheets

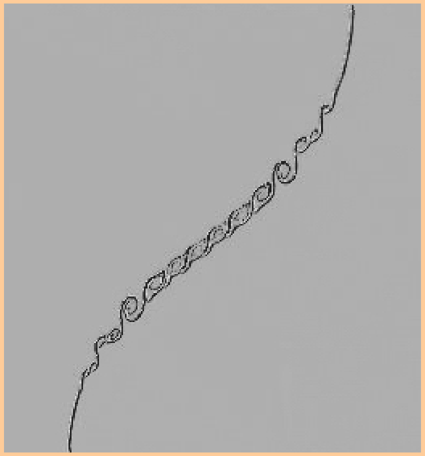
– *Finite-time Moore curvature singularity* for **single sine wave**

– *Regularization* via finite ITL

$$u_t + u u_x = 0$$



Vorticity

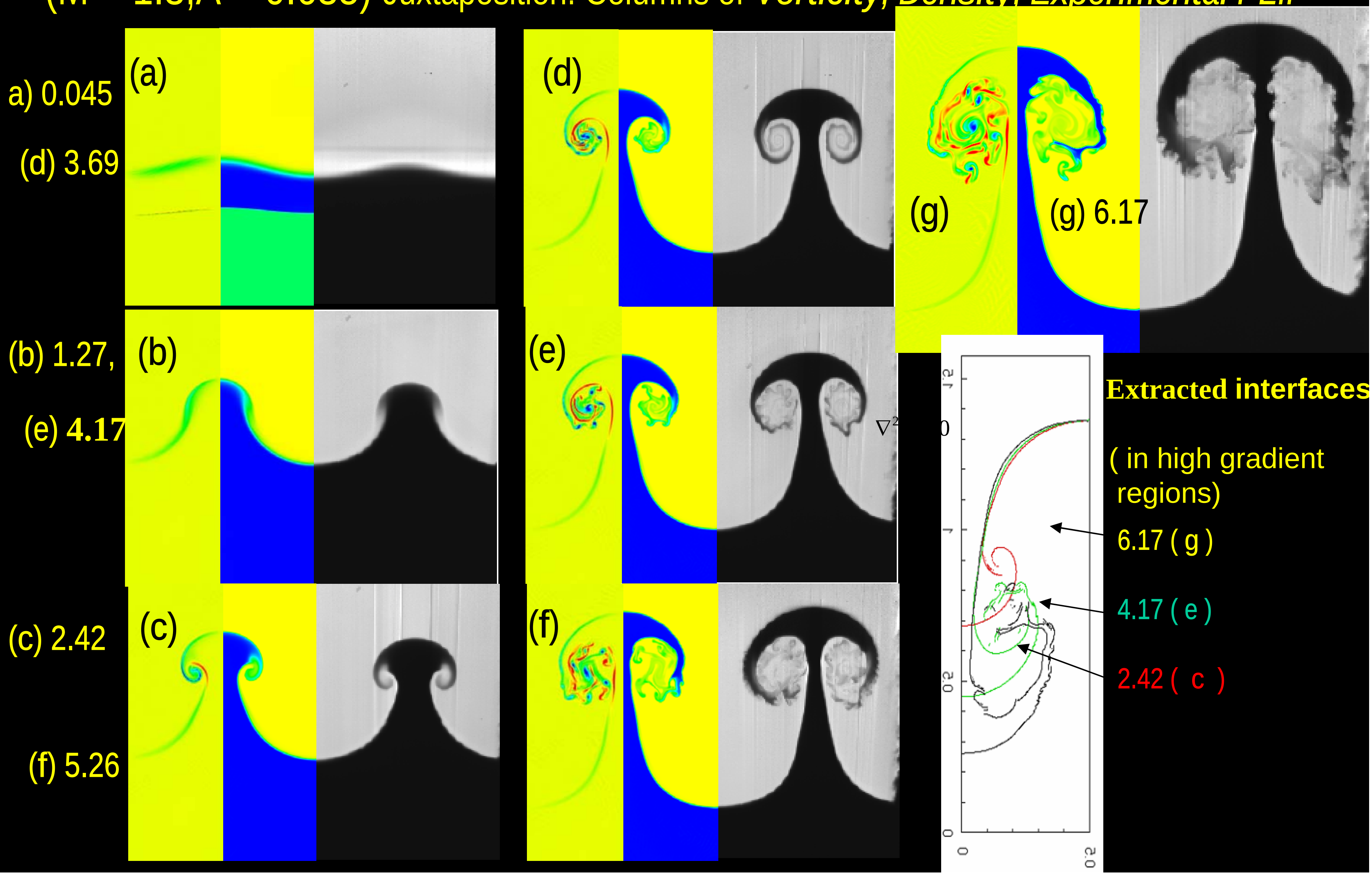


$\nabla^2 \rho$

Sinusoidal interface

Vizlab Simulations (PPM) of G. Peng and S. Zhang & Jacob & Krivets' Experiment (PLIF)

(M = 1.3,A = 0.635) Juxtaposition: Columns of *Vorticity, Density, Experimental PLIF*



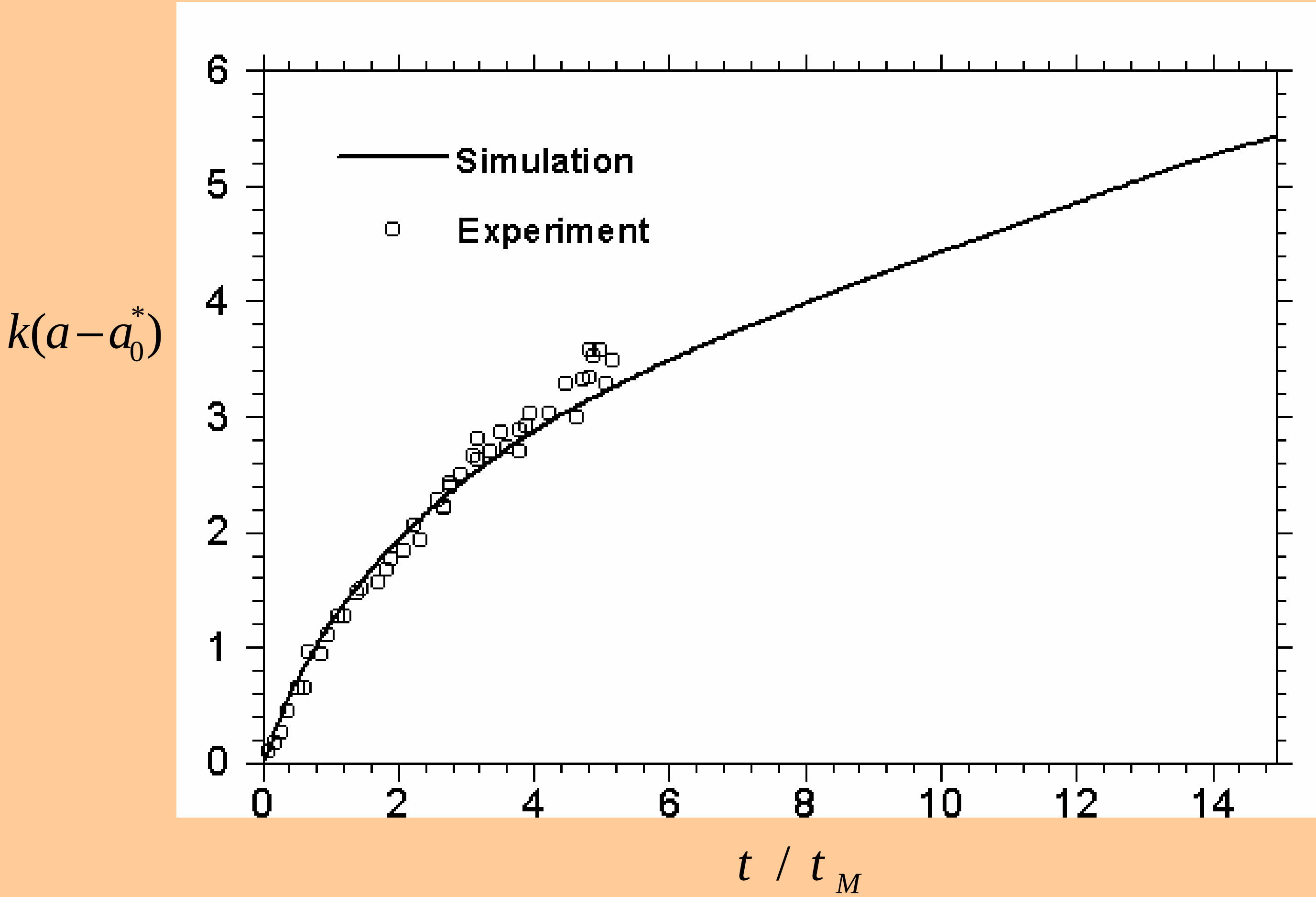
OVERVIEW: "*AIFS*" Accelerated Inhomogeneous Flows

A Vortex Approach

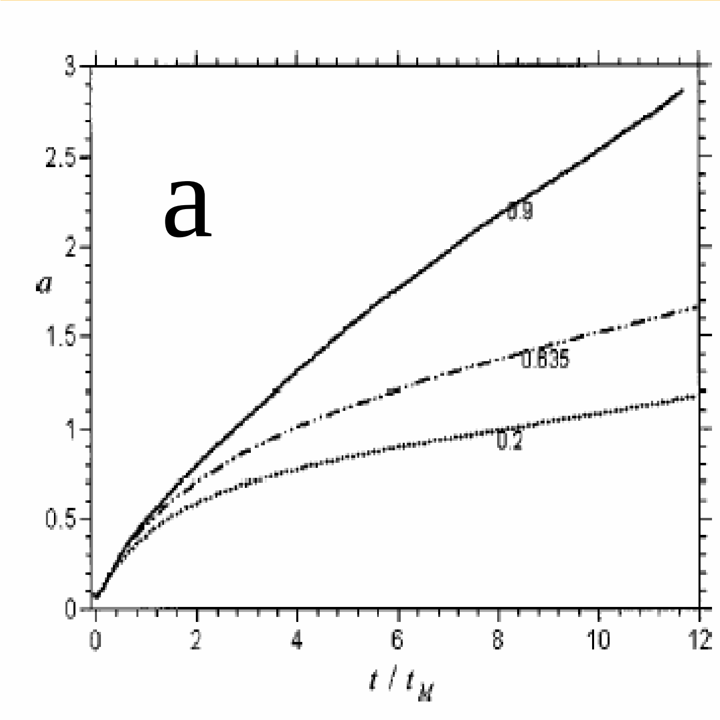
- **Topics**
  - **Well-posedness and *finite initial transition layer***
  - **RM *a-dot ->constant* at intermediate times**
  -

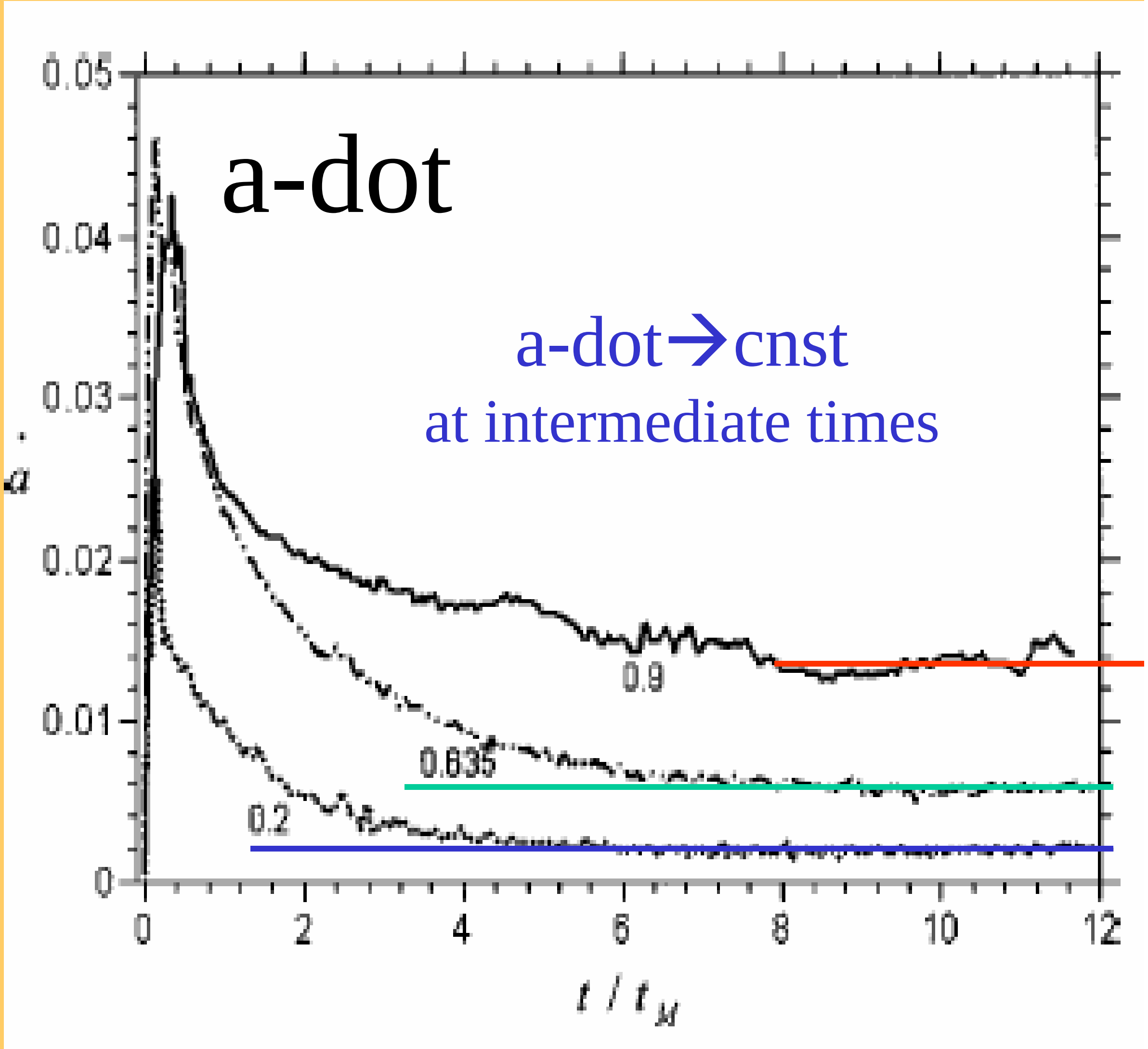
Amplitude growth a(t):

simulation and experiment



a & a-dot for **A**= **0.2, 0.635 & 0.9**



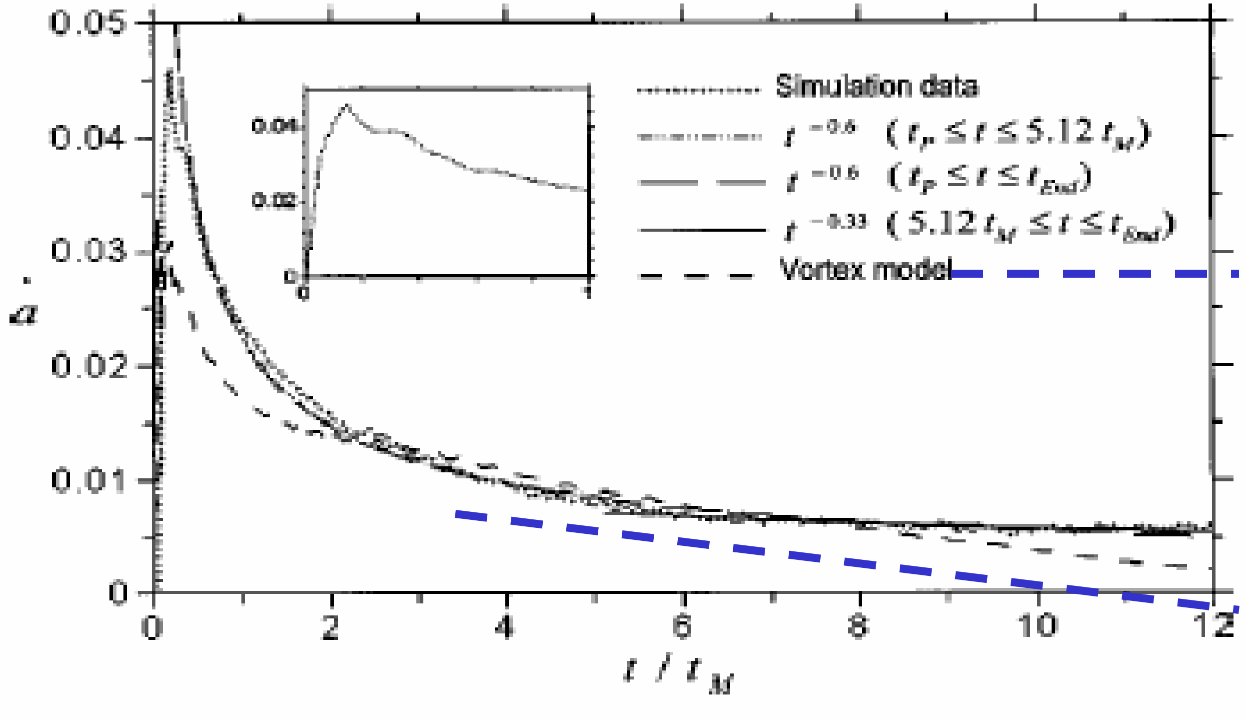


A-dot comparisons:

Simulations & proposed

adjusting-one vortex model

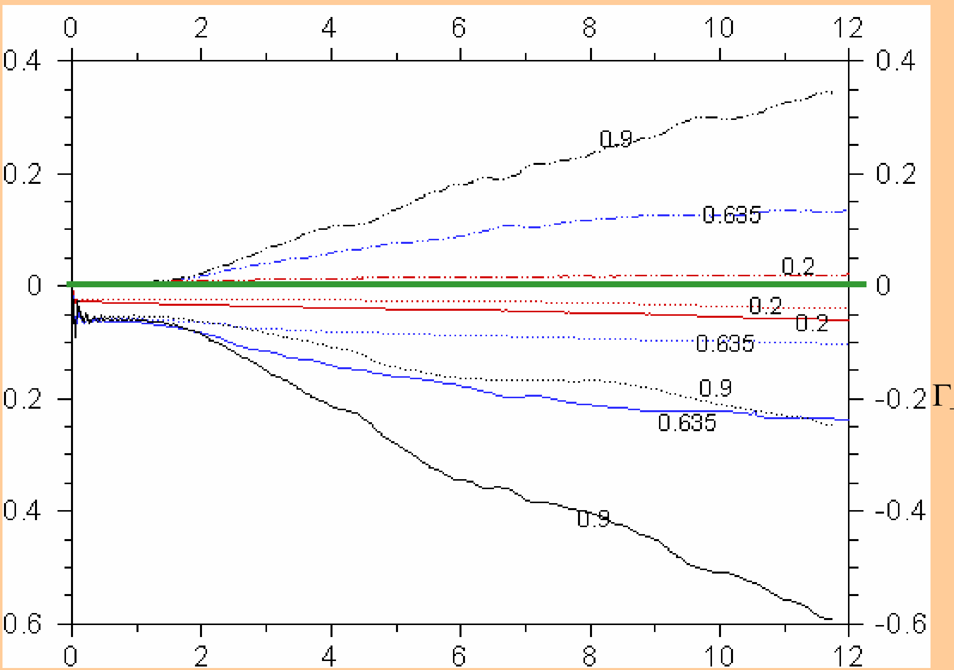
$$\dot{a}_{\text{vortex}} = -\frac{k\Gamma \sin kx_c}{4\pi}\left(\frac{1}{\cosh(kd_s) - \cos kx_c} + \frac{1}{\cosh(kd_b) + \cos kx_c}\right),$$



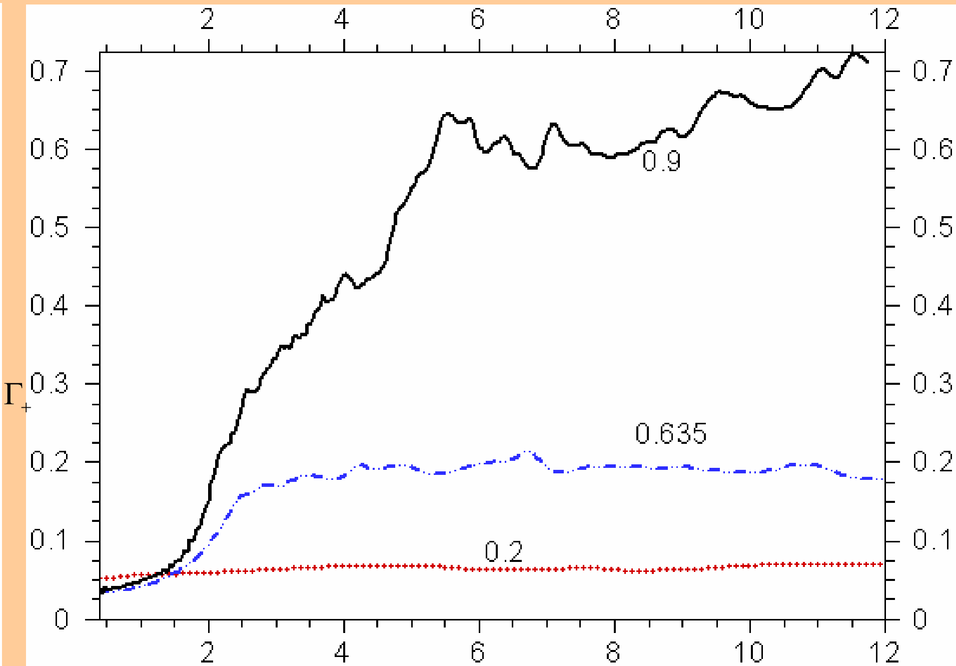
OVERVIEW:RM

- **Topics**
  - **Well-posedness and initial transition layer**
  - **RM *a-dot ->constant* at intermediate times**
  - **Circulation generation (*vortex bilayers)* & gradient Intensification**

Global quantifications: Circulation & Enstrophy



$t\,/\,t_M$



$t\,/\,t_M$

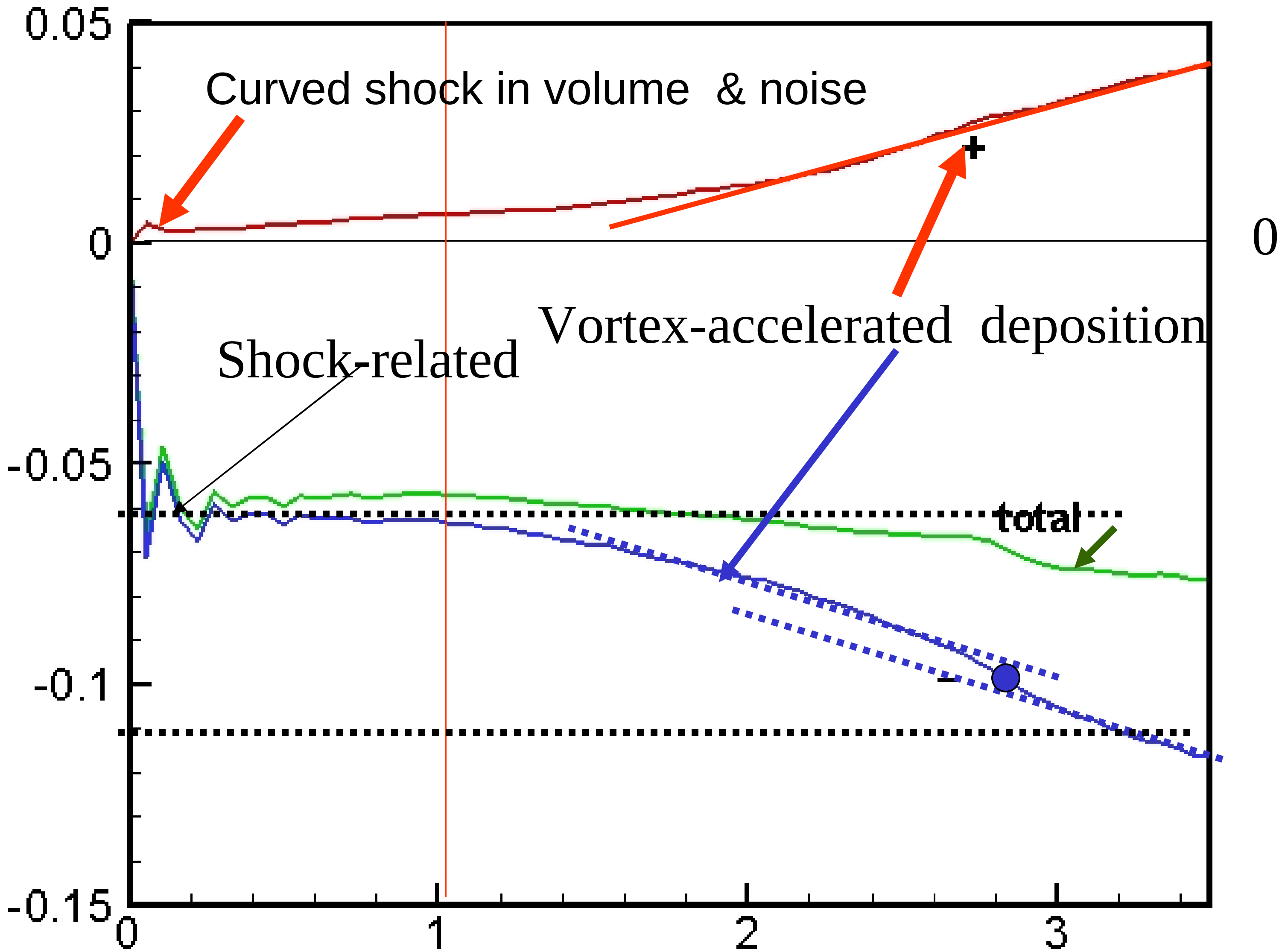
Circulation $\Gamma_+, \Gamma_-\,,\;\; \Gamma = \Gamma_+ + \Gamma_-,$

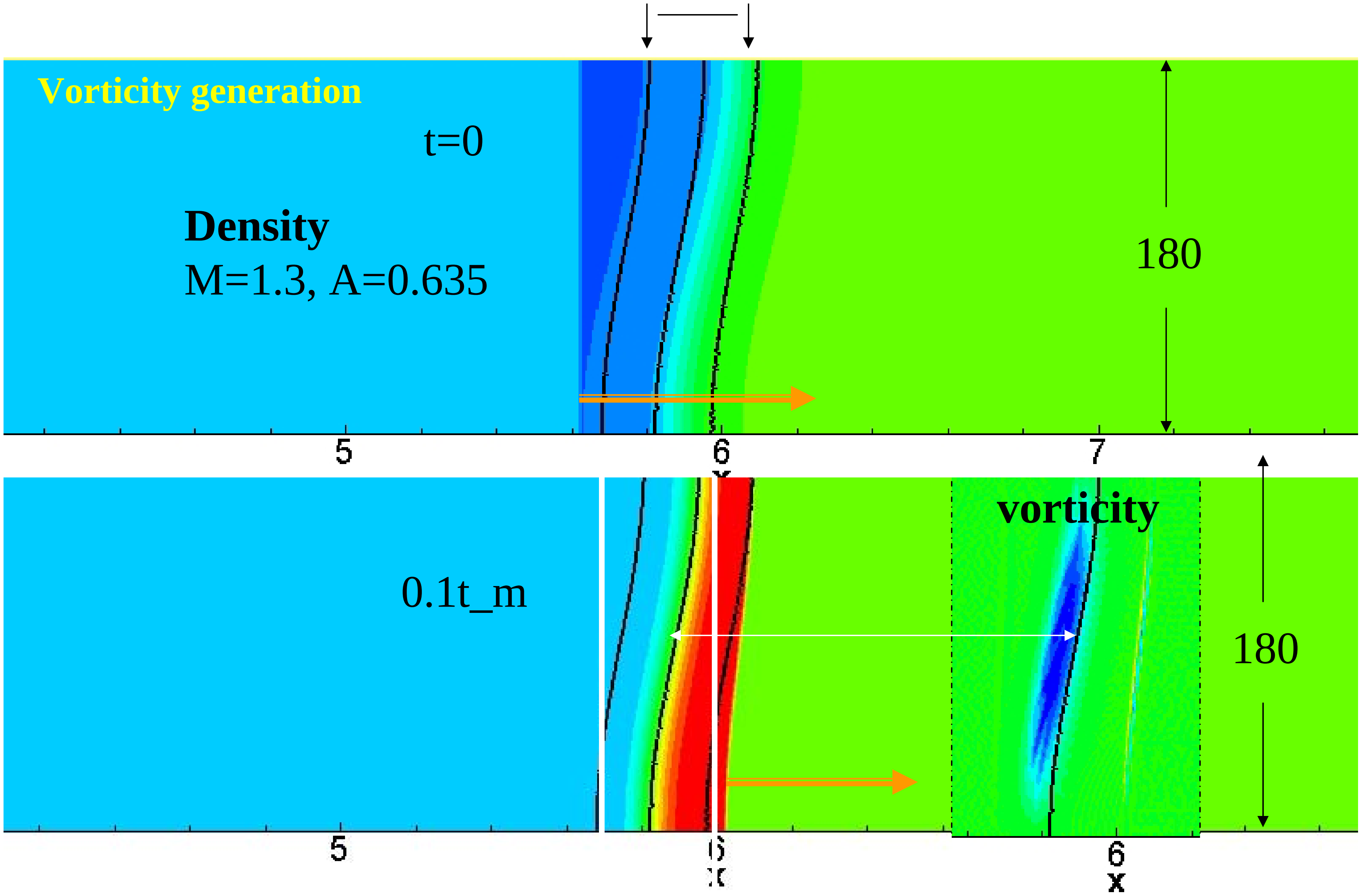
$$\int_D [\omega, \omega_\pm](x, y, t)\,dx\,dy$$

Enstrophy

$$\int_D \omega^2 (x, y, t)\,dx\,dy$$

Circulations vs time, RM with A=0.635, M= 1.3

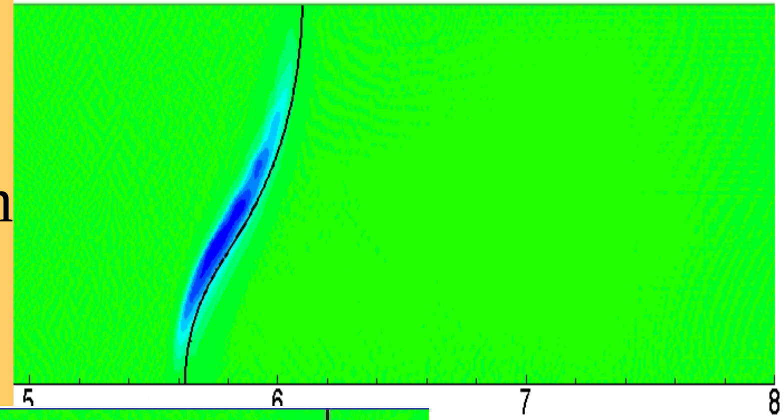




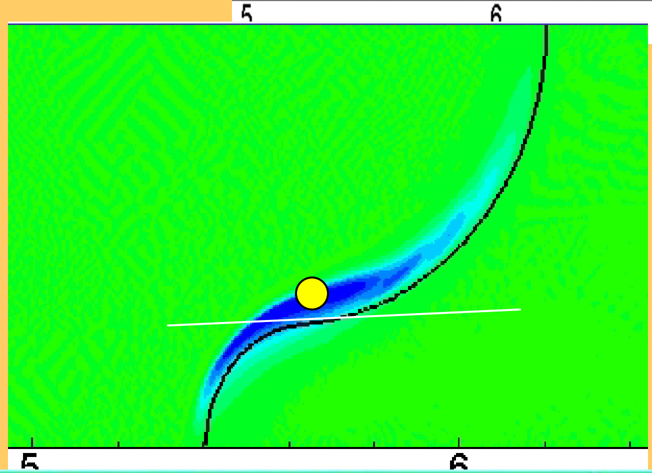
Vorticity generation
t=0
Density
M=1.3, A=0.635
180
5
6
7
0.1t_m
vorticity
180
5
6
6
x

RM Baroclinic vorticity

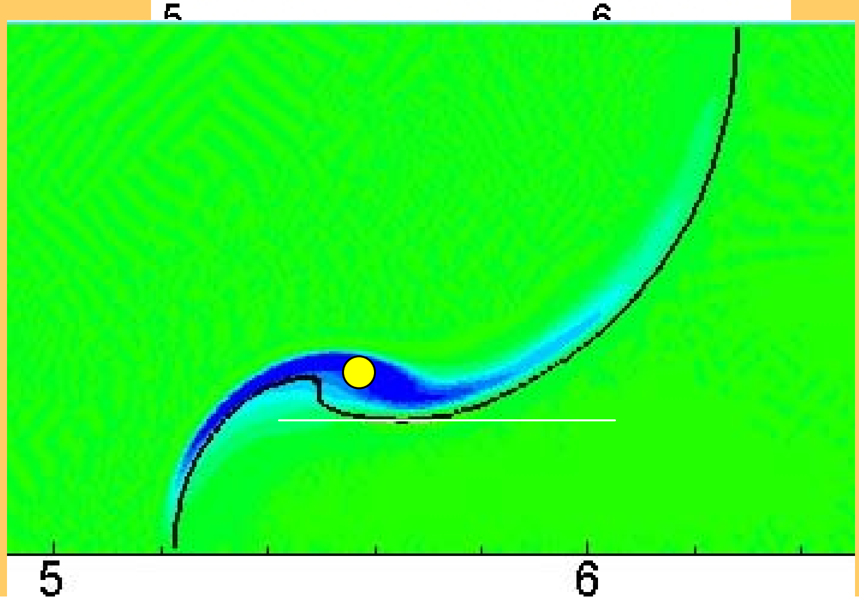
0.5t_m



1.0t_m



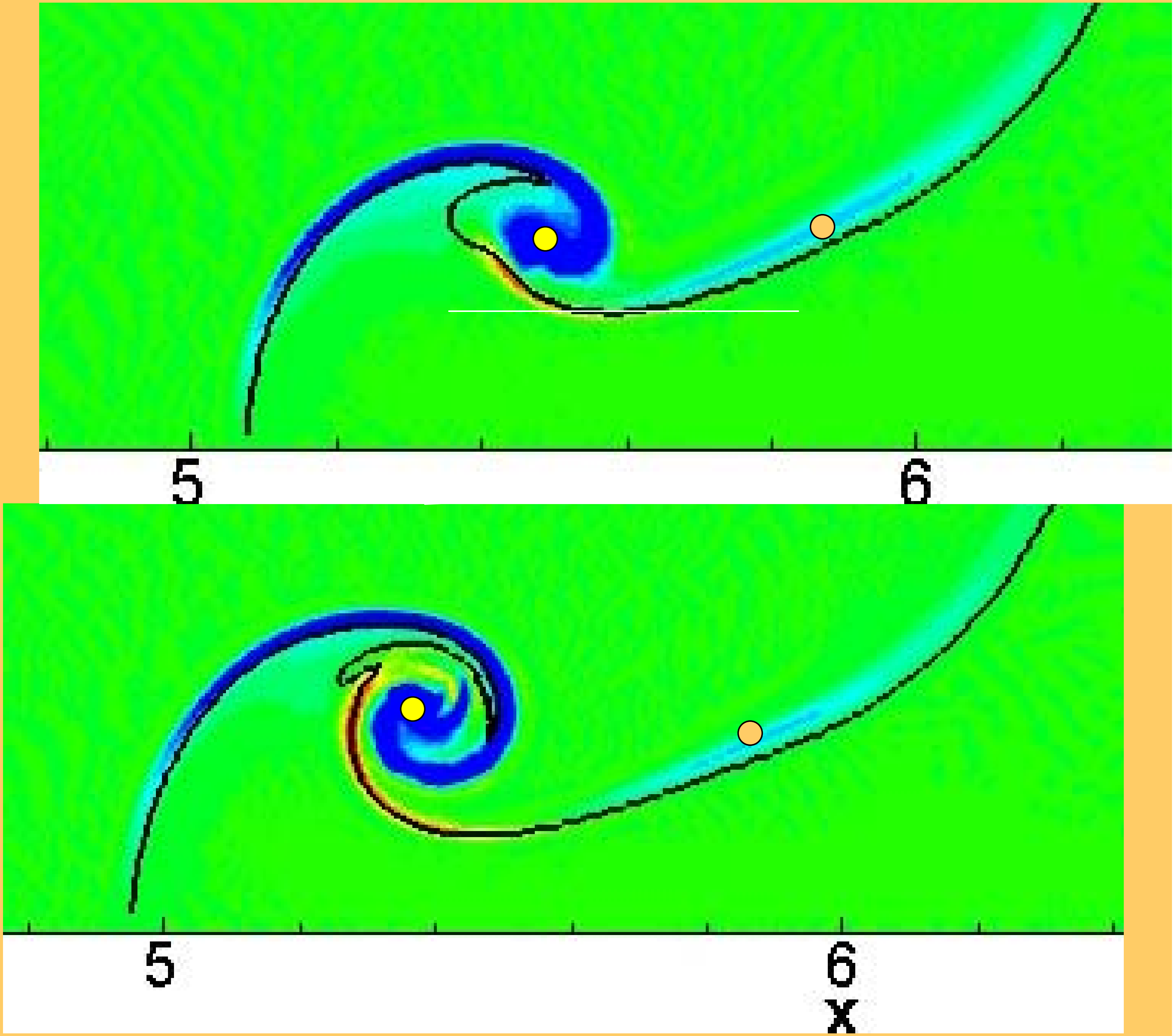
1.5t_m



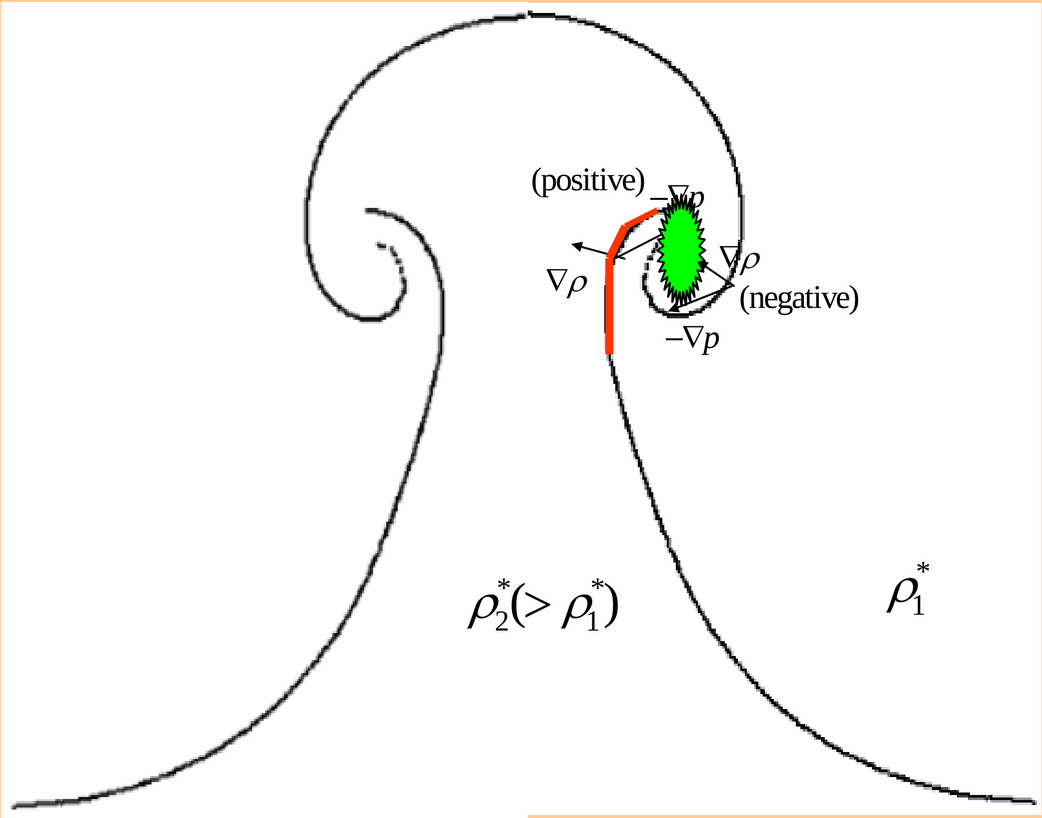
2.0t_m

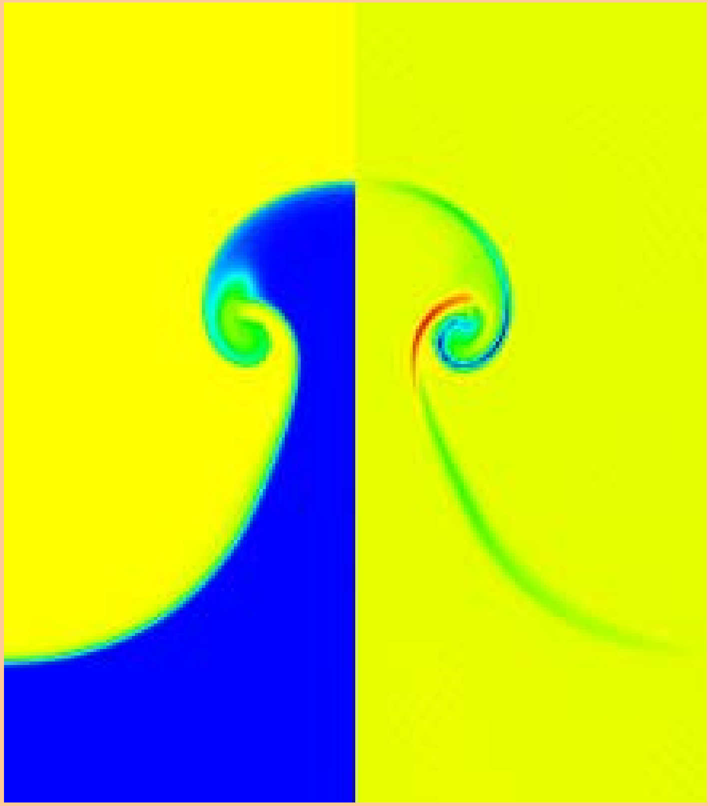
RM Baroclinic vorticity

2.5t_m



Vortex-accelerated "secondary" baroclinic vorticity deposition





$$\frac{D\omega}{Dt} = \frac{\nabla\rho\times\nabla p}{\rho^2} + \omega\cdot\nabla\mathbf{u} - \omega\nabla\cdot\mathbf{u}$$

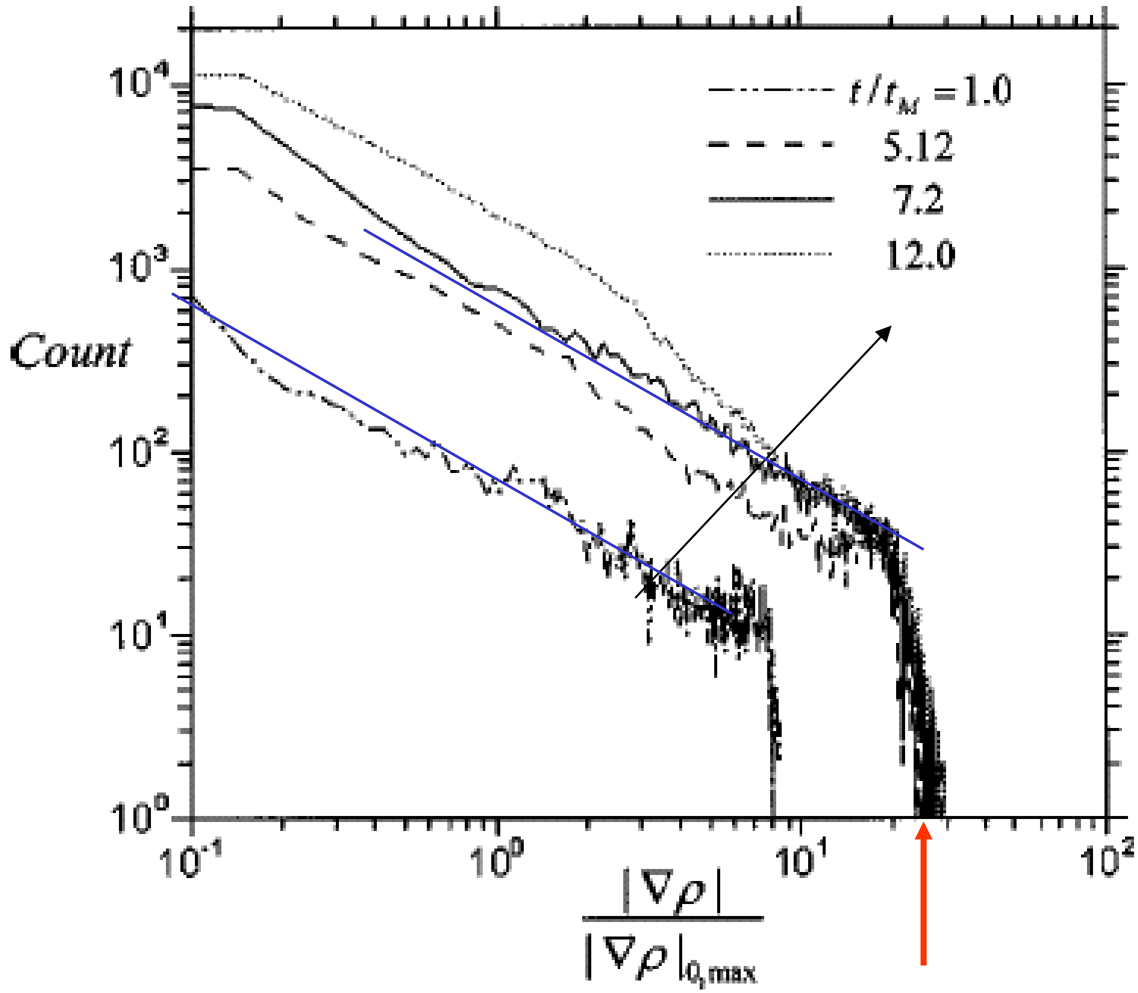
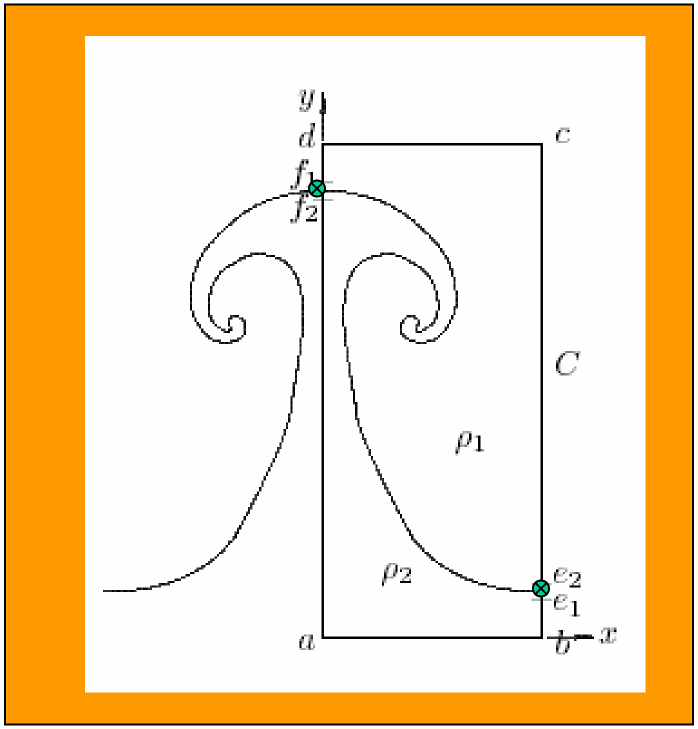


FIG. 13. Density gradient magnitude (normalized by the preshock initial maximum density gradient magnitude) distribution for $A^*=0.635$ at times $t/t_M=1.0$, 5.12, 7.2, and 12.



Definitions:

$$\mathbf{u} = u\,\mathrm{e}_x + v\,\mathrm{e}_y \quad \omega = \omega\,\mathrm{e}_z = (-\partial_y u + \partial_x v)\,\mathrm{e}_z\,,$$

$$\Gamma_D = \iint_D \omega\, dxdy\,.$$

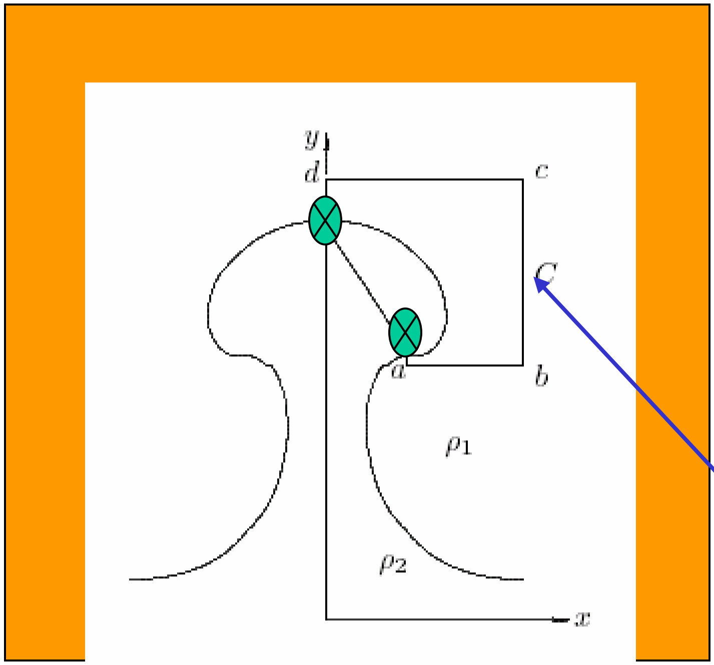
Circulation Rate of Change

$$\partial_t \Gamma = -\frac{1}{2}\oint \nabla(u \bullet u)\bullet ds + \oint \nabla(u \times \omega)\bullet ds + \oint \rho^{-1}\nabla p \bullet ds$$

$$= 0 + 0 + \oint \rho^{-1} dp$$

$$= -\int_{e1e2} p\rho^{-2} d\rho - \int_{f1f2} p\rho^{-2} d\rho$$

$$\partial_t \Gamma = -(p_b - p_t)\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) = -(p_b - p_t)\frac{(\rho_2 - \rho_1)}{(\rho_2 \rho_1)}$$



Choose domain to select positive or negative, etc

Circulation Data : Computing & Filtering

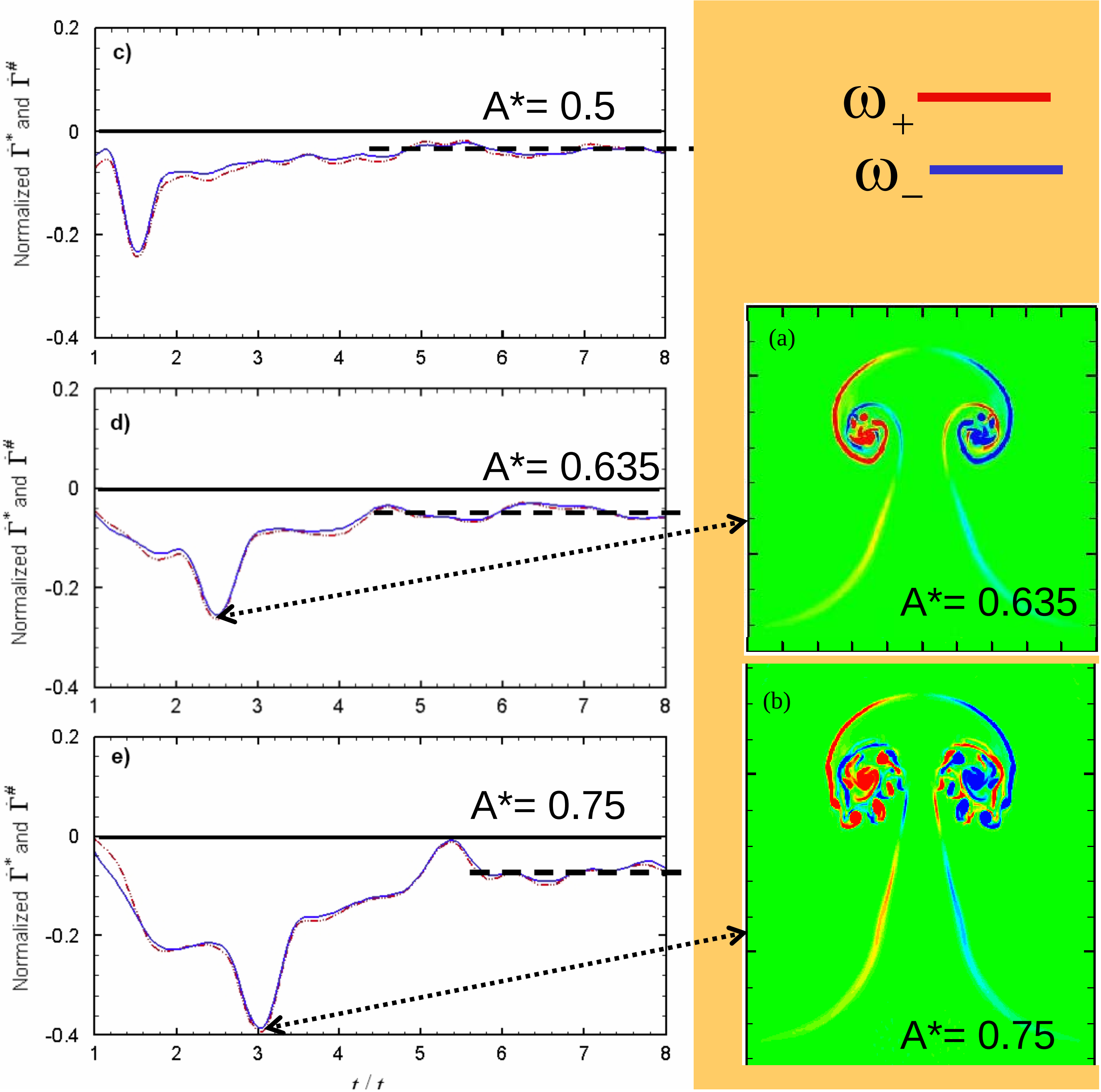
$$\dot{\Gamma} \approx -\int_{a1}^{a2} p(d\rho/\rho^2) - \int_{b1}^{b2} p(d\rho/\rho^2)\,, \qquad (3)$$

$$\tilde{f}_{i,j}(t_n) = \sum_{t_m=-t_M/6}^{t_m=+t_M/6} f_{i,j}(t_m)\Phi(t_n;t_m)$$

where $\Phi(t_n;t_m) = T^{-1}[1+\cos(2\pi(t_n - t_m)/T)],\ -(T/2) \le t_m \le (T/2),\quad \&\ T = t_M/3.$

$$\dot{\Gamma}^*(t_n) \equiv \frac{\tilde{p}_a(t_n) - \tilde{p}_b(t_n)}{(\rho_2 - \rho_1)/\rho_2\rho_1} \qquad (4)$$

$$\dot{\Gamma}^\#(t_n) \equiv \sum_D^2 h\left[\frac{(\tilde{\omega}_{i,j}(t_{n+1}) - \tilde{\omega}_{i,j}(t_{n-1}))}{2\delta t}\right]. \qquad (5)$$



c)
A*= 0.5
d)
A*= 0.635
e)
A*= 0.75
Normalized $\dot{\Gamma}^*$ and $\dot{\Gamma}^\#$
t / t
ω_+
ω_-
(a)
A*= 0.635
(b)
A*= 0.75

Summary: 2D Vortex paradigm for the evolution of RM interfaces through intermediate times

- Intermediate-time dominance of *vortex-accelerated vorticity deposition* (VAVD) process; [Peng *et al* (2003)]

- Quantification procedures & formulas for *net circulation rate of change*, $\dot{\Gamma}_D$, comprise a vortex paradigm for the evolution of RM &RT interfaces through intermediate times. [Lee, Peng & Zabusky (**Sept.** 2006)].

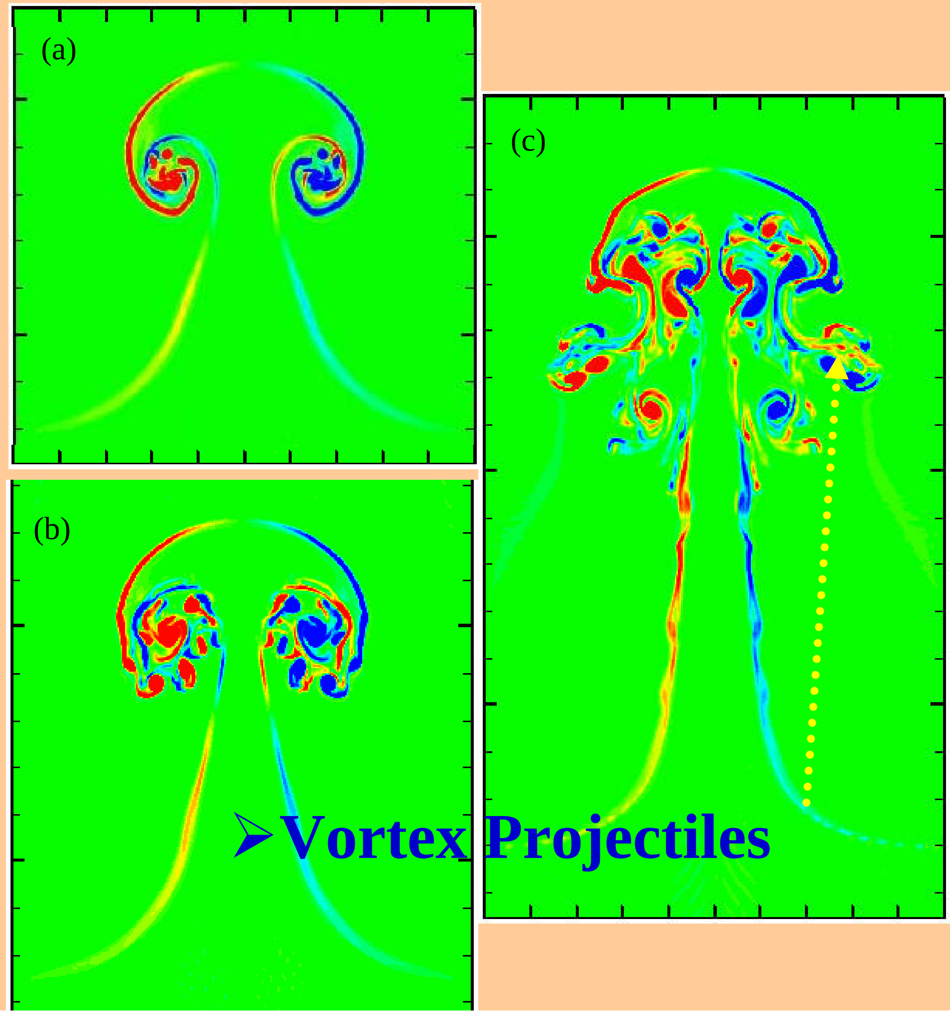
- Special features observed [Lee, Peng & Zabusky (2006)] are signatures of physically important phenomena and include:
    - o *Gradient intensification* of interfacial transition layer
    - o *NdpÓ* for $t/t_M > 1$ that increases with A* and the
    - o $\dot{\Gamma}_D$ *scaling at intermediate times*: near-constant negative values that increase with A*, for $0.5 < A^* < 0.75$.

- Generalization to other accelerated inhomogeneous flow configurations (axisymmetry, shock-cylinder [S. Zhang, N. J. Zabusky, G. Peng, and S. Gupta, PoF ,2004], etc)

<u>OVERVIEW: “*AIFS*” - *RM*</u>

- **Topics**
  - **Well-posedness and initial transition layer**
  - **RM *a-dot ->*<u>*constant*</u> at intermediate times**
  - **Circulation generation (*vortex bilayers)* gradient Intensification**

- **Vortex Projectiles**



RM New Results

- **Secondary Baroclinic Circulation is much greater than Primary (Deposited by Shock).** ***PoF '03: G. Peng , S. Zhang & N. Zabusky,***
  - Due to ***vortex acceleration*** & ***gradient intensification of*** transition layer (TL)

- **Vortex Projectiles: Dipolar/Ring-like objects active at all times in determining turbulence and mixing**

- **New diagnostic: R**ate of change of circulation in bubble to spike domain. ***PoF, '06 : D.K. Lee, G. Peng, & NJZ***

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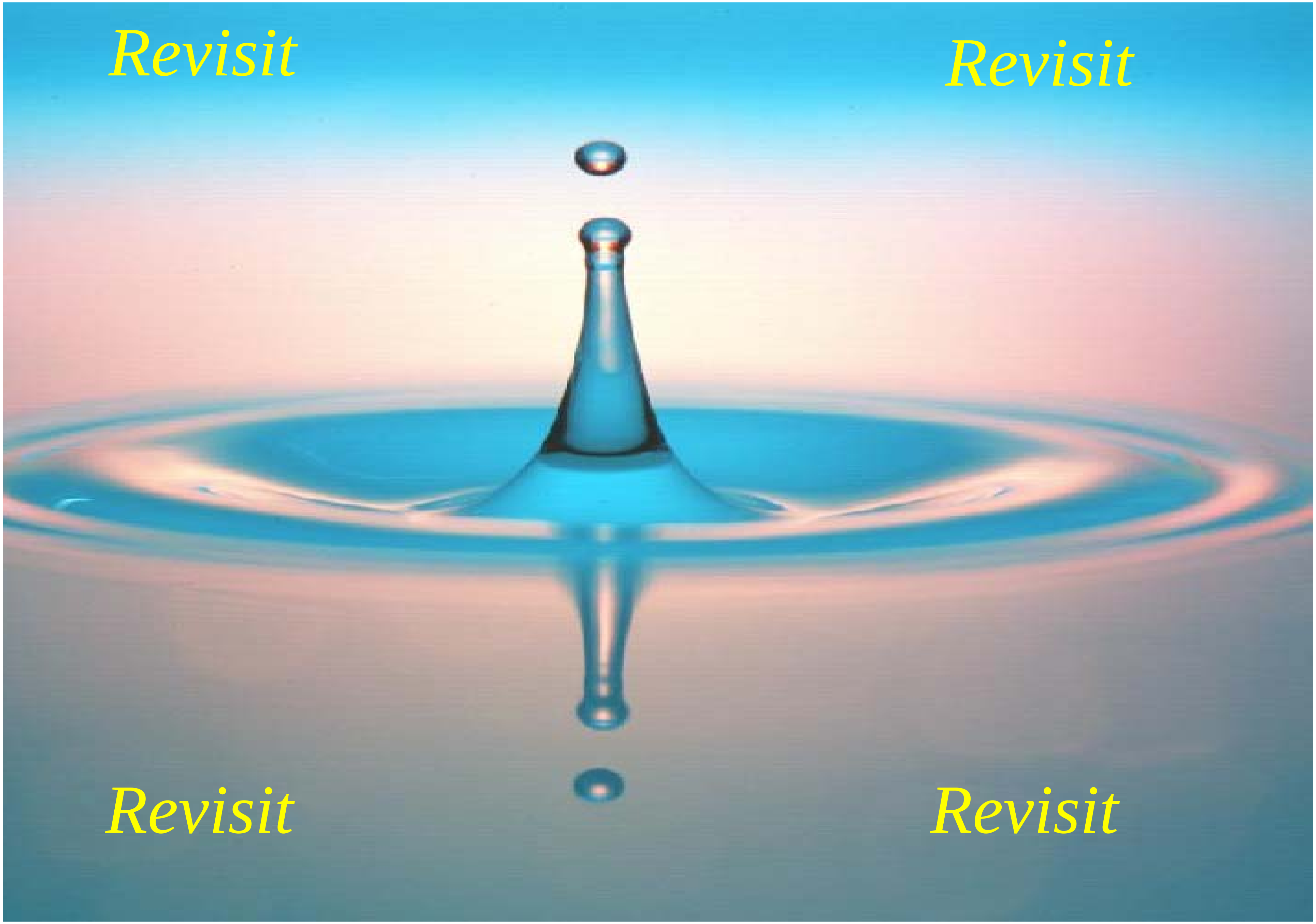
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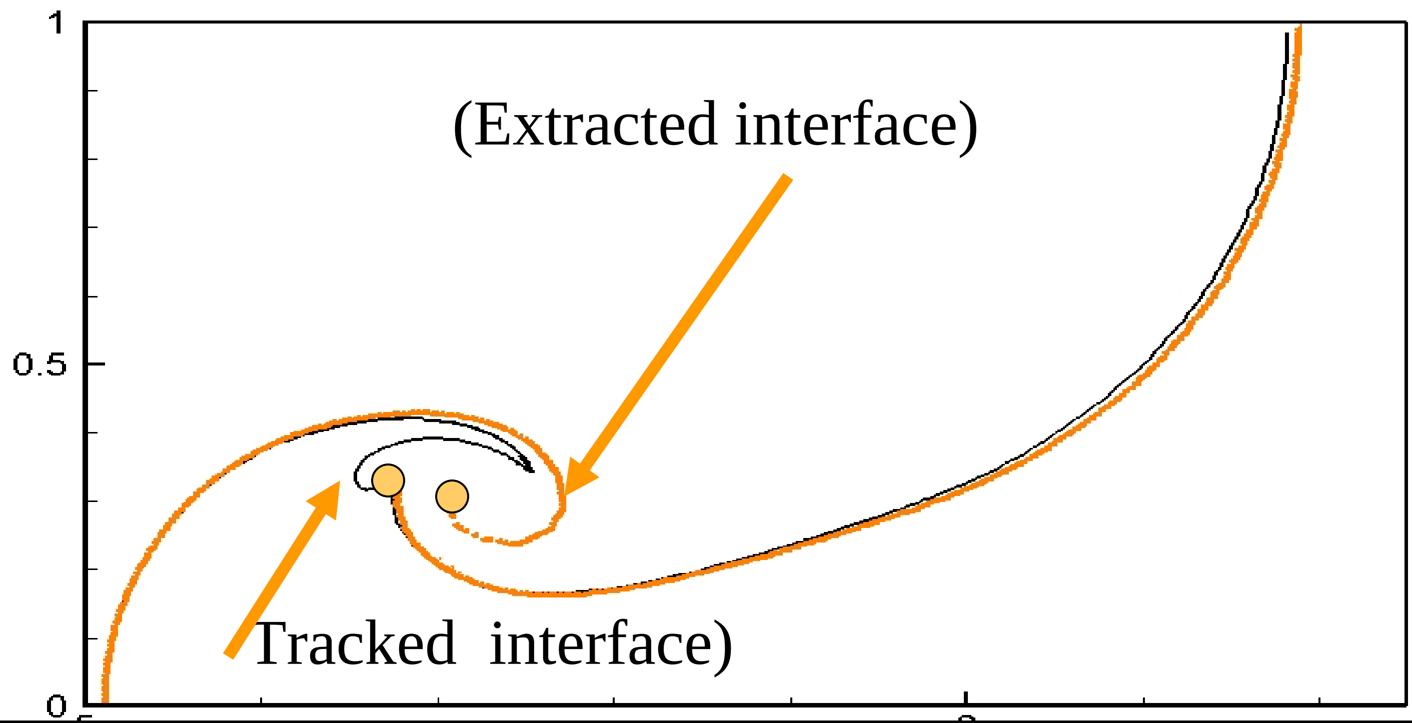
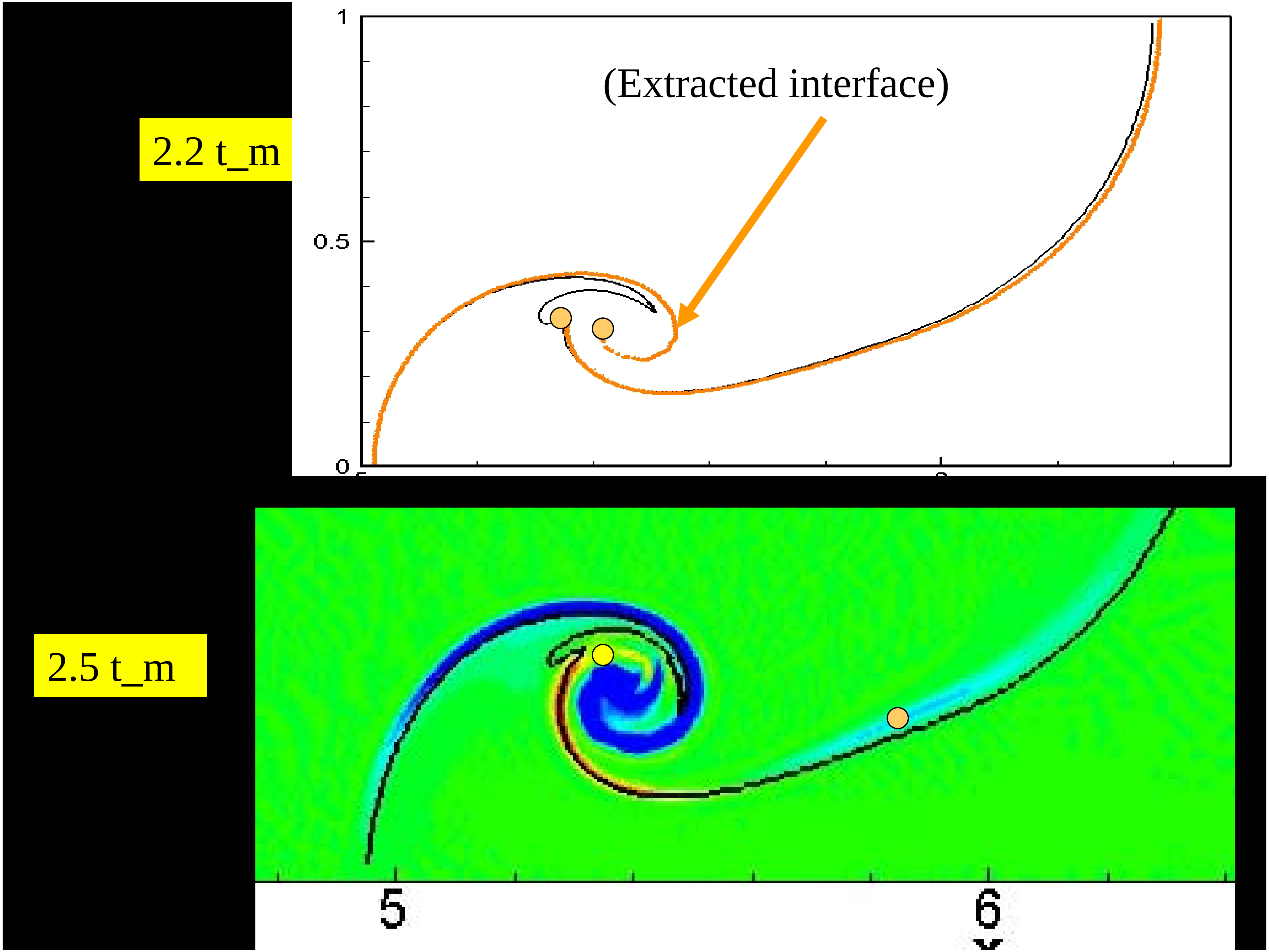
Revisit
Revisit



Extraction Algorithm

- |GRAD rho| >0.1 {max|GRAD rho|_0]}
- LAPLACE (density) = 0

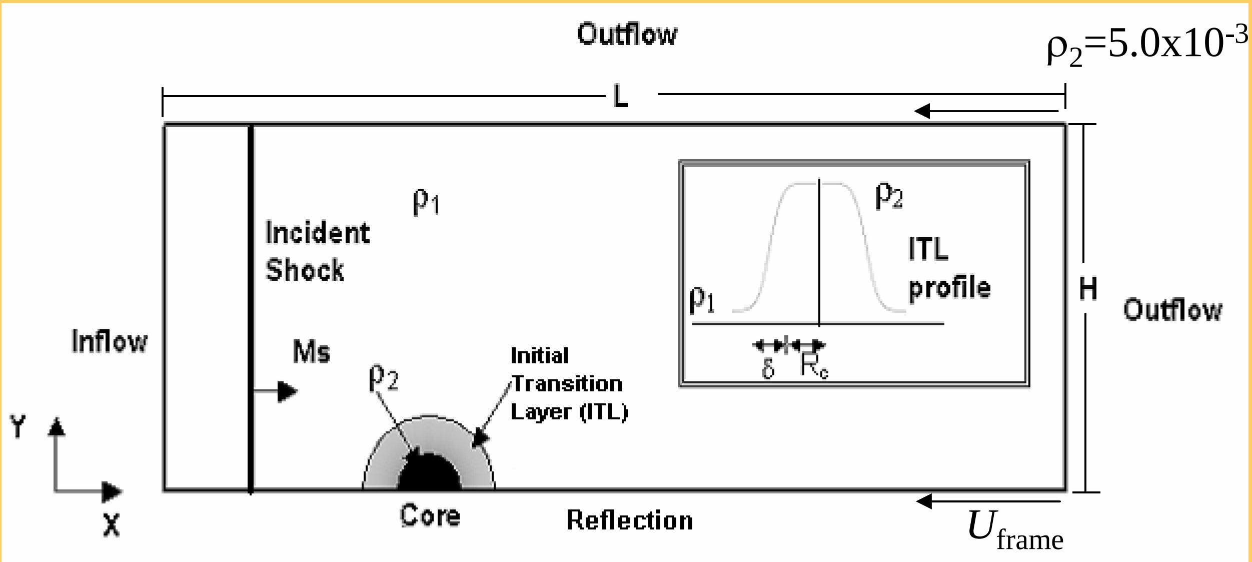


2.2 t_m
(Extracted interface)
1
0.5
0
2.5 t_m
5
6

Initial domain

ρ_1=1.0x10-3 (Air)

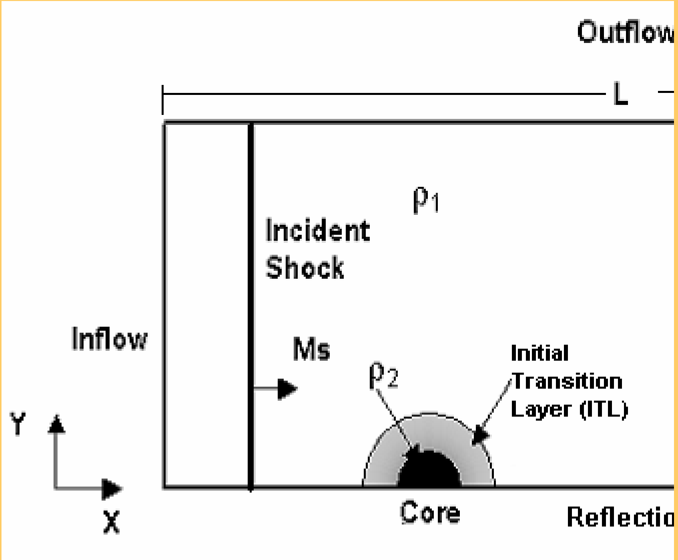
ρ_2=5.0x10^{-3} (SF_6)



Initial domain

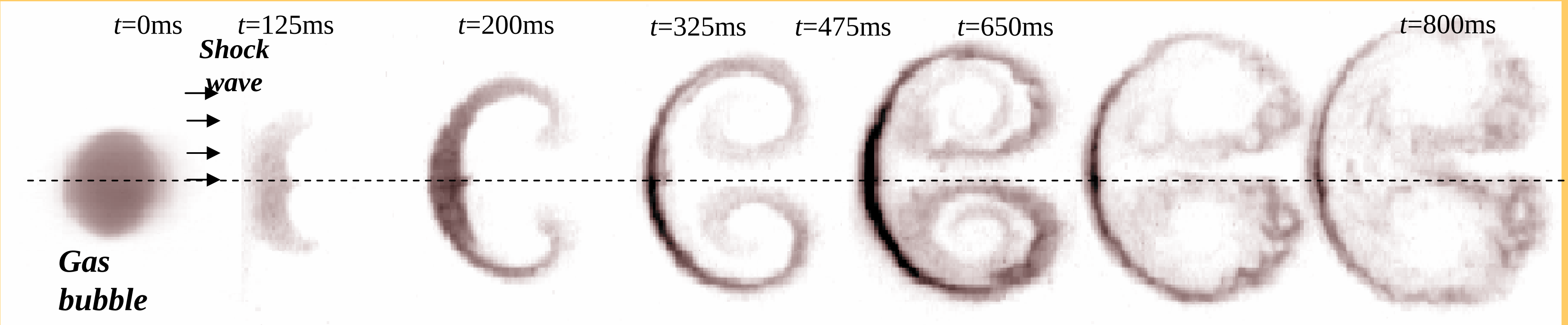
ρ_1=1.0x10-3 (Air)

ρ_2=5.0x10^{-3} (SF_6)

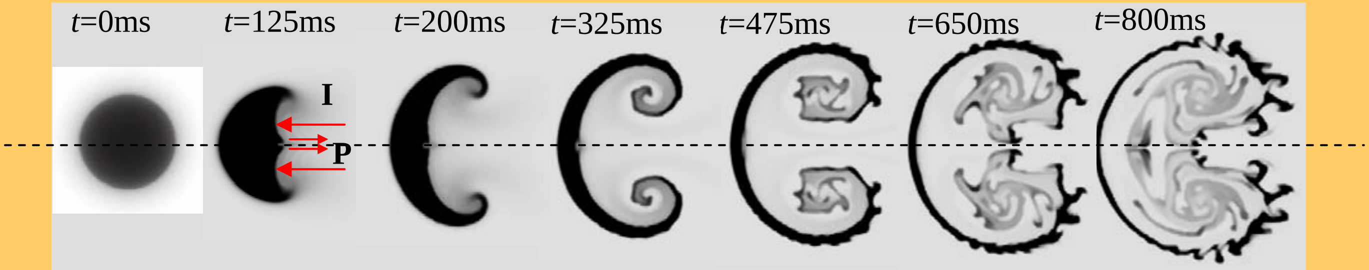


U_{frame}

Visiometrics: uncertainty quantification & numerical validation

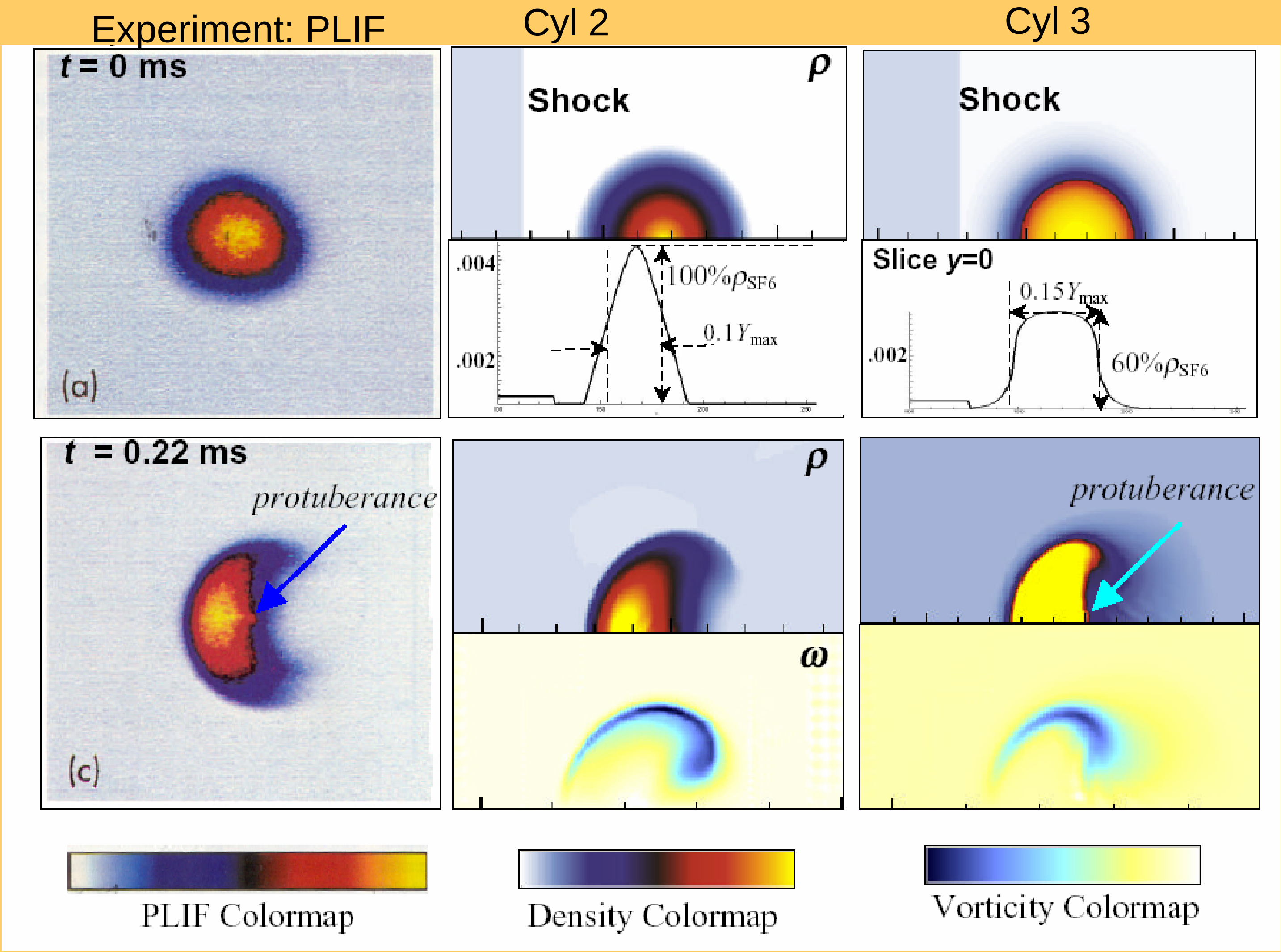


(a). Experimental images (LANL)

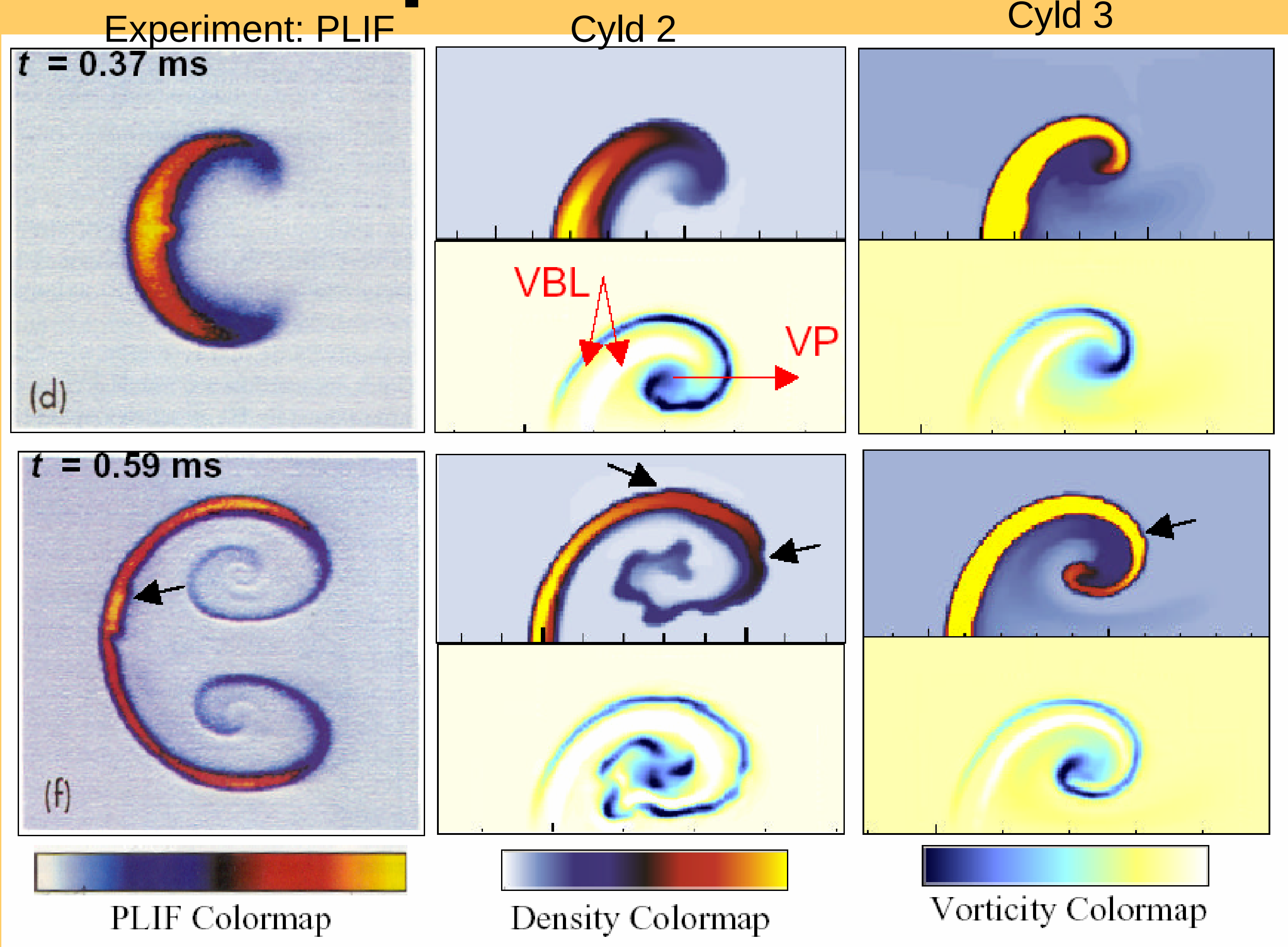


(c). Simulation with visiometrics (SZhang@Vizlab)

Compare Jacobs' Experiment

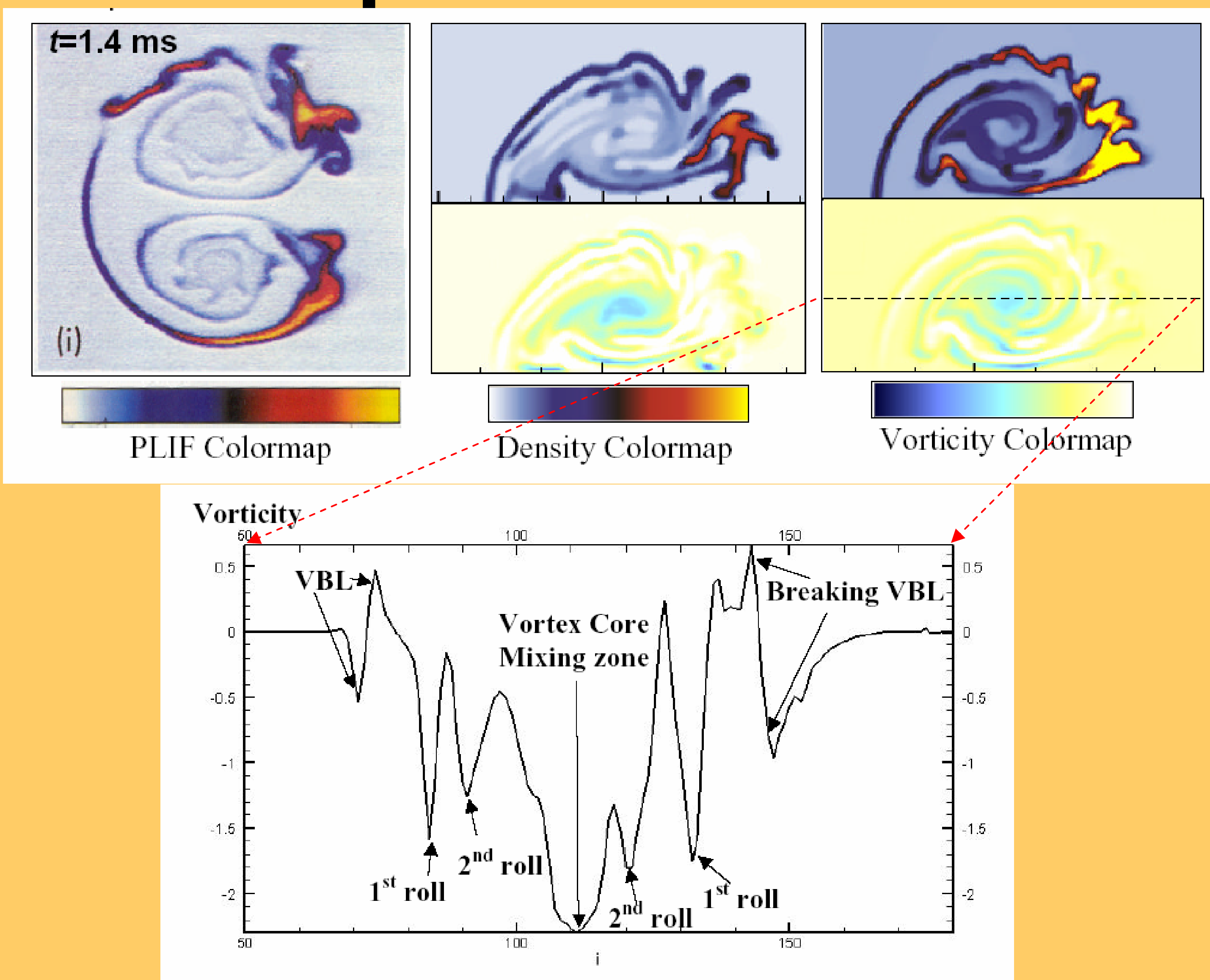


Comparing with Jacobs' Experiment cont.

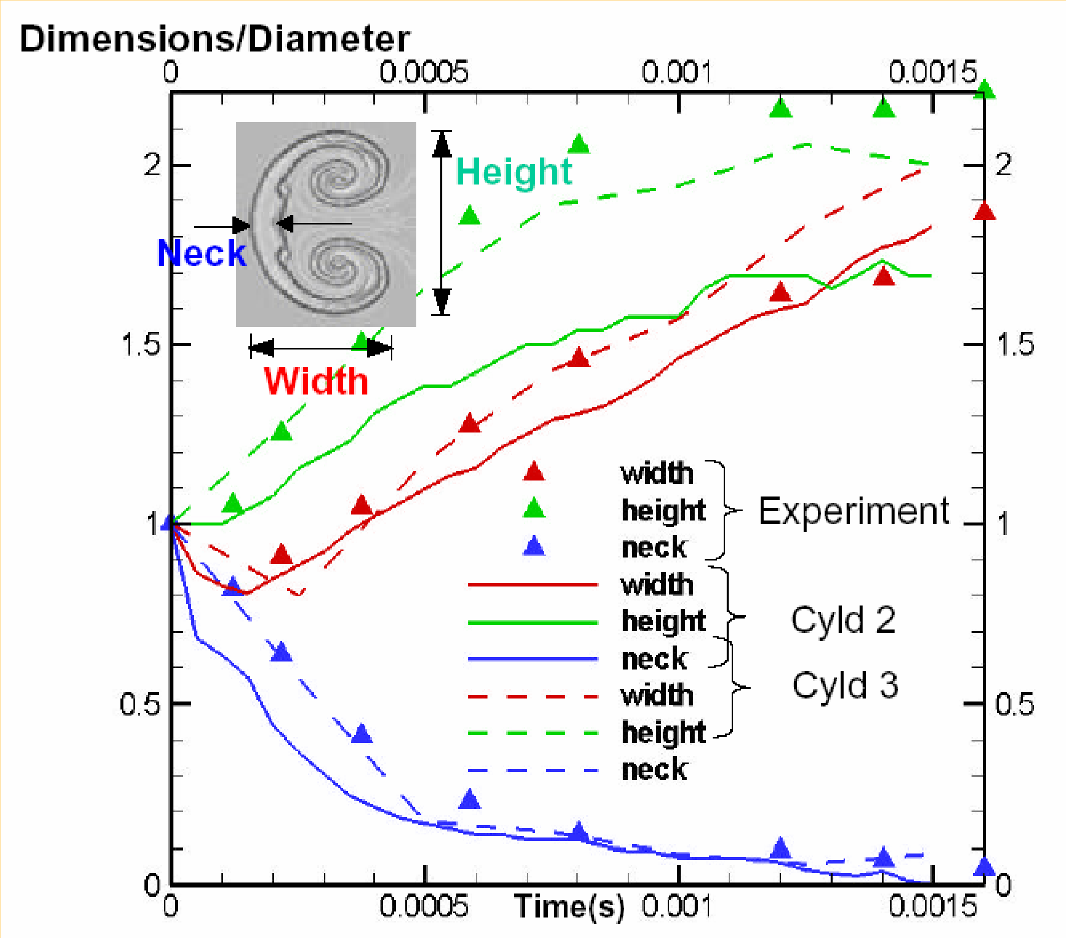


Comparing with Jacobs' Experiment cont.

Experiment PLIF   Cyld 1   Cyld 3

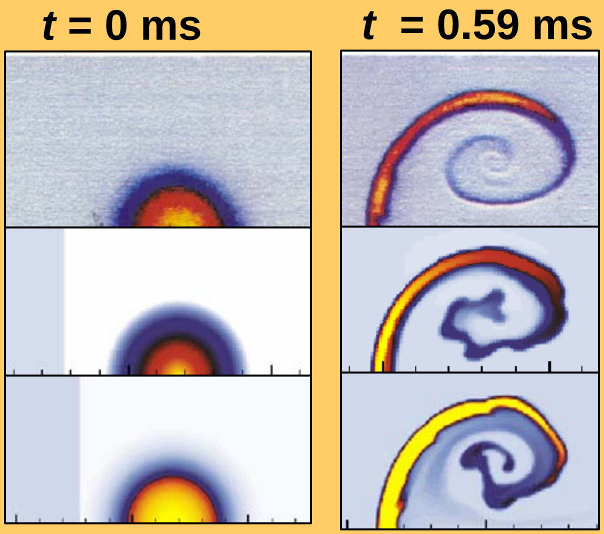


Variation of transition layer thickness: IC uncertainty



Sim I: Linear transition profile;

Sim II: Error function profile, with initial SF6 concentration 60%



Integrated vorticity space (x)- time (t) diagram
(Hawley & Zabusky:PRL 1989)