A new 2D model for 3D Euler

Prof. R. Kerr
Warwick University, Coventry, UK

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Abstract:

- Two-dimensional models have been proposed in recent years.
- These contain aspects of the underlying dynamics of the three-dimensional incompressible Euler equations
- while being more tractable.
- This presentation will introduce a new model in this class.
- It inspired by fully three-dimensional solutions as well as
- a new conditional restriction upon Euler [Gibbon *et al.*(2006)] that shows that require symmetrical alignments if there is to be a singularity.
- Model: Equation for growth of vorticity and curvature
- plus the usual advection equation.
- Goals: Encourage mathematicians to study it.
- Provide a setting to test the particular numerical issues currently being contested.

- Outstanding modern mathematical problem, $1 million prize:
- For 3D incompressible **Navier-Stokes** with finite energy, etc. Either:
  - Find an example of a singularity of 3D incompressible Navier-Stokes
  - Prove that Navier-Stokes is regular.
  - Folk-belief: Navier-Stokes is regular
- Related: 3D incompressible **Euler**, what is known:
  - Beale, Kato, Majda (1984), $\int \|\omega\|_\infty dt \rightarrow \infty$
  - Numerical work: Kerr (1993) anti-parallel vortices, tests:

$$\text{Is } \frac{1}{\|\boldsymbol{\omega}\|_\infty} \sim (T-t)? \qquad \frac{1}{\|e_{yy}\|_\infty}? \qquad \frac{1}{\int dV \omega_i e_{ij} \omega_j}?$$

  - Folk-belief: Unknown.
- Constantin, Fefferman, Majda (1996) and Deng, Hou, Yu (2005): relations on time integrals of curvature and velocity:

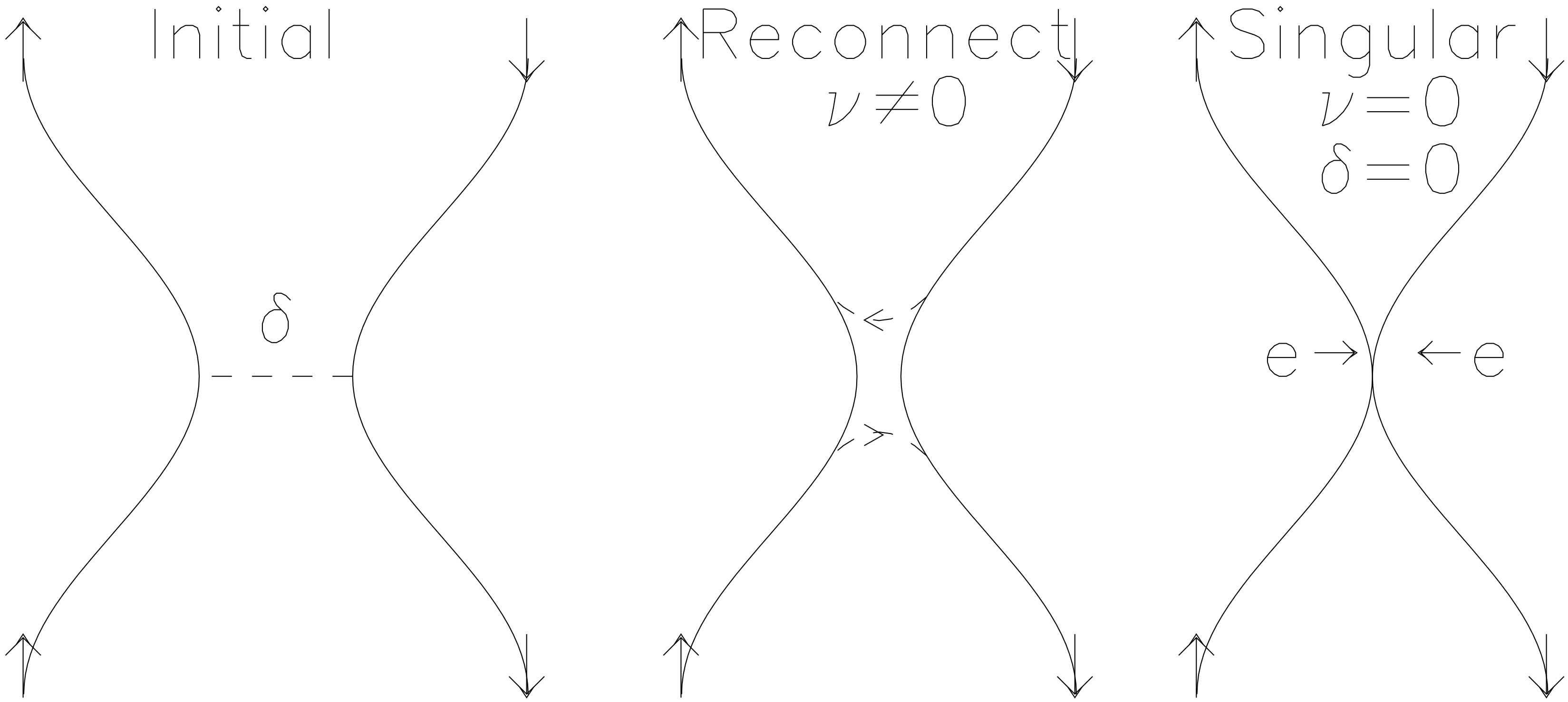
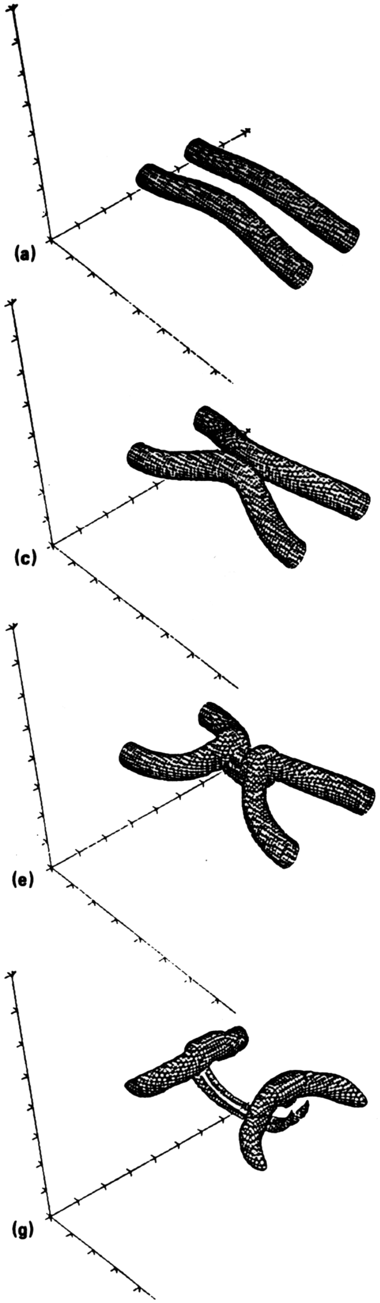
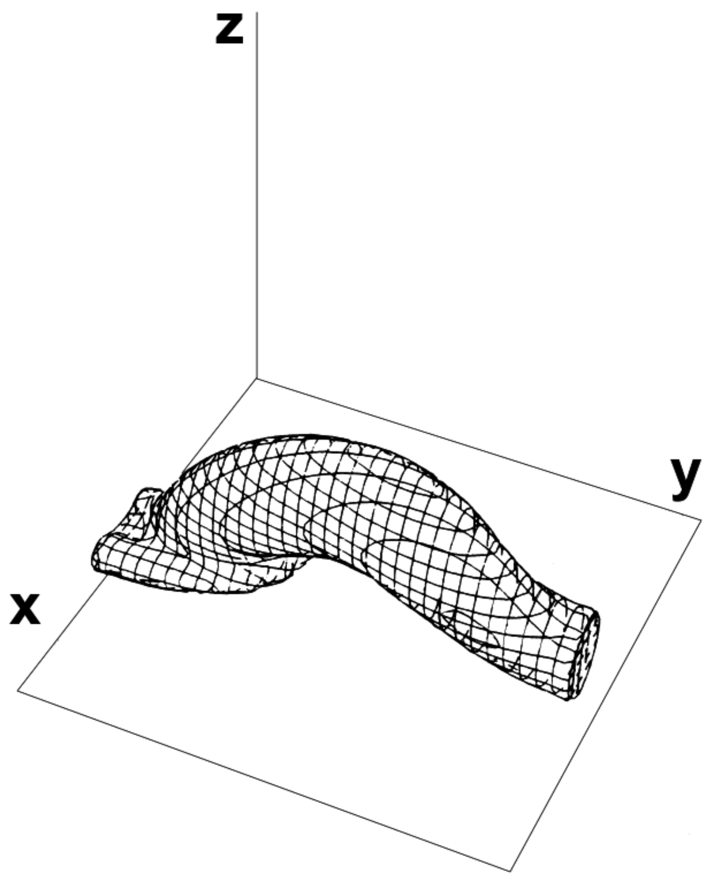
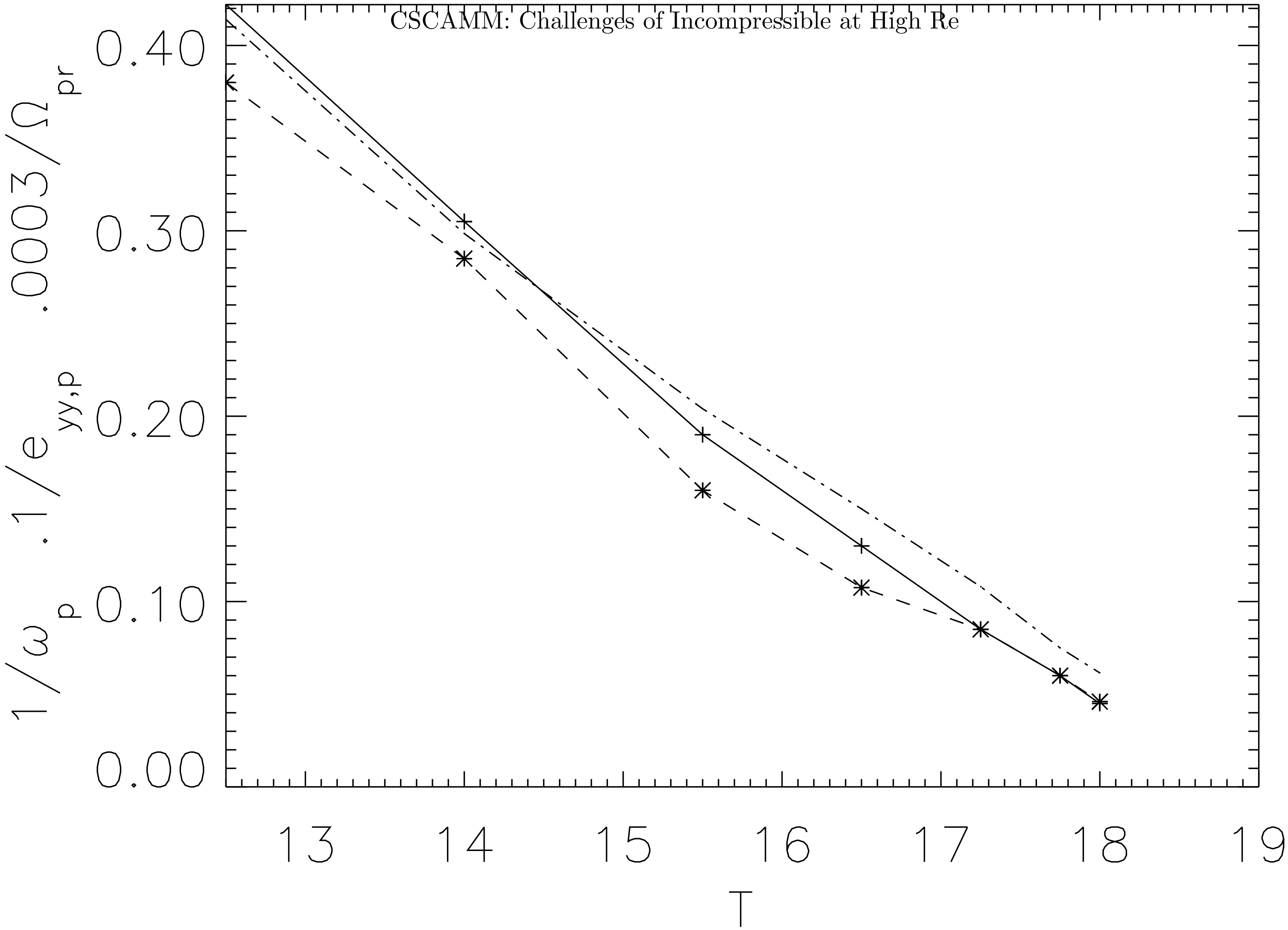


Diagram of the interaction of anti-parallel vortices. From an initial condition of anti-parallel vortices separated at their closest approach by δ, if $\nu \neq 0$ there is reconnection that forms new vortices indicated by the dashed curves. However, if $\nu = 0$, a singularity can form when $\delta = 0$ if the vortices are pushed together by the self-induced strain indicated by e.

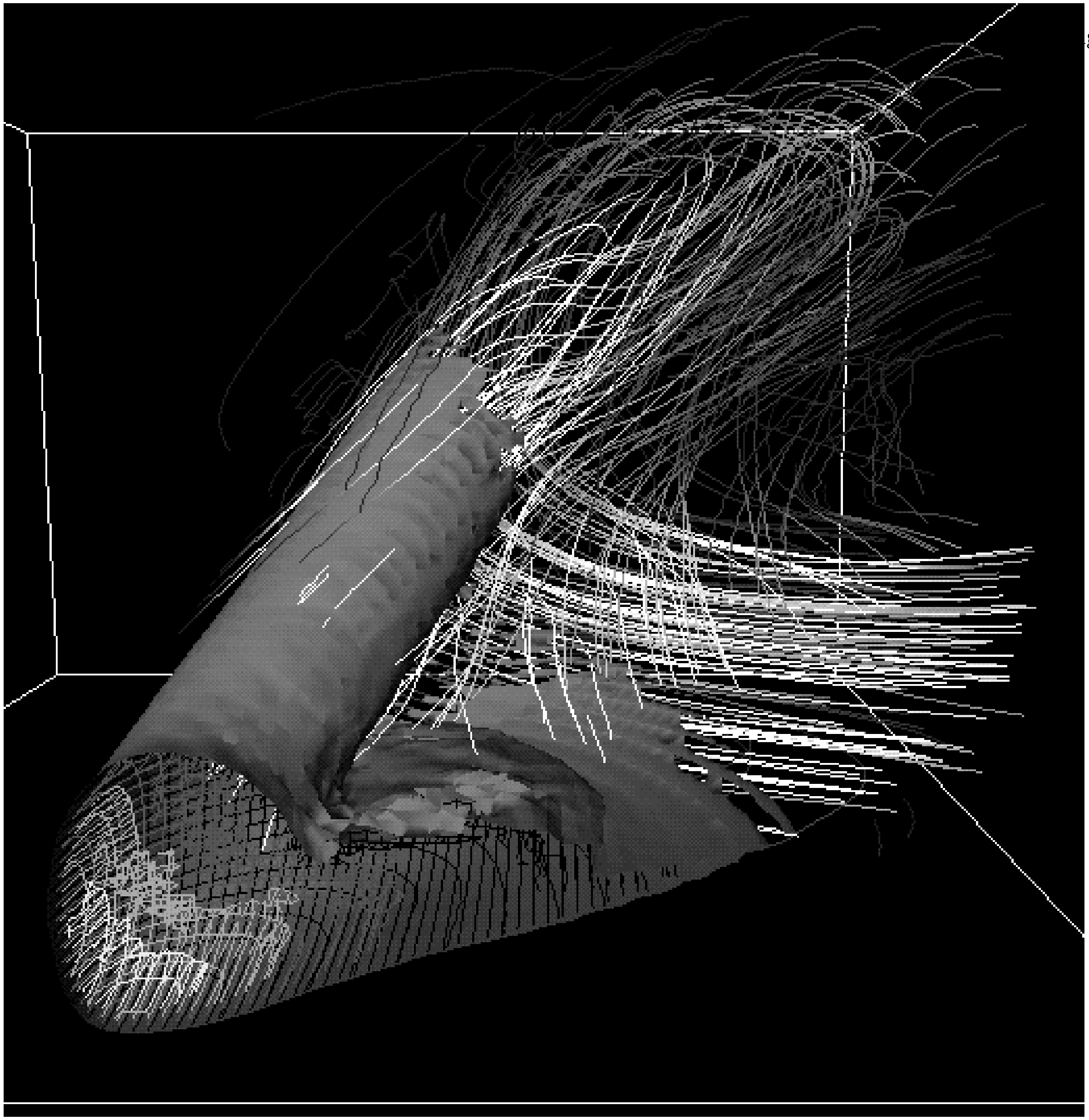




Steps in vortex reconnection taken from a low resolution, low Reynolds number calculation Melander and Hussain, 1989. From top to bottom, the first two frames show the anti-parallel vortex tubes being pushed together by self-interaction through the law of Biot-Savart. The third frame shows that reconnection has progressed to form two new tubes orthogonal to the original tubes. In the bottom frame the new tubes are separating.



Dependence of $1/\|\omega\|_\infty$, $1/\|e_{yy}\|_\infty$ and $1/\int dV\omega_i e_{ij}\omega_j$ on time from the anti-parallel Euler calculation Kerr, 1993 showing convergence to a singular time of about $T = 18.7$.



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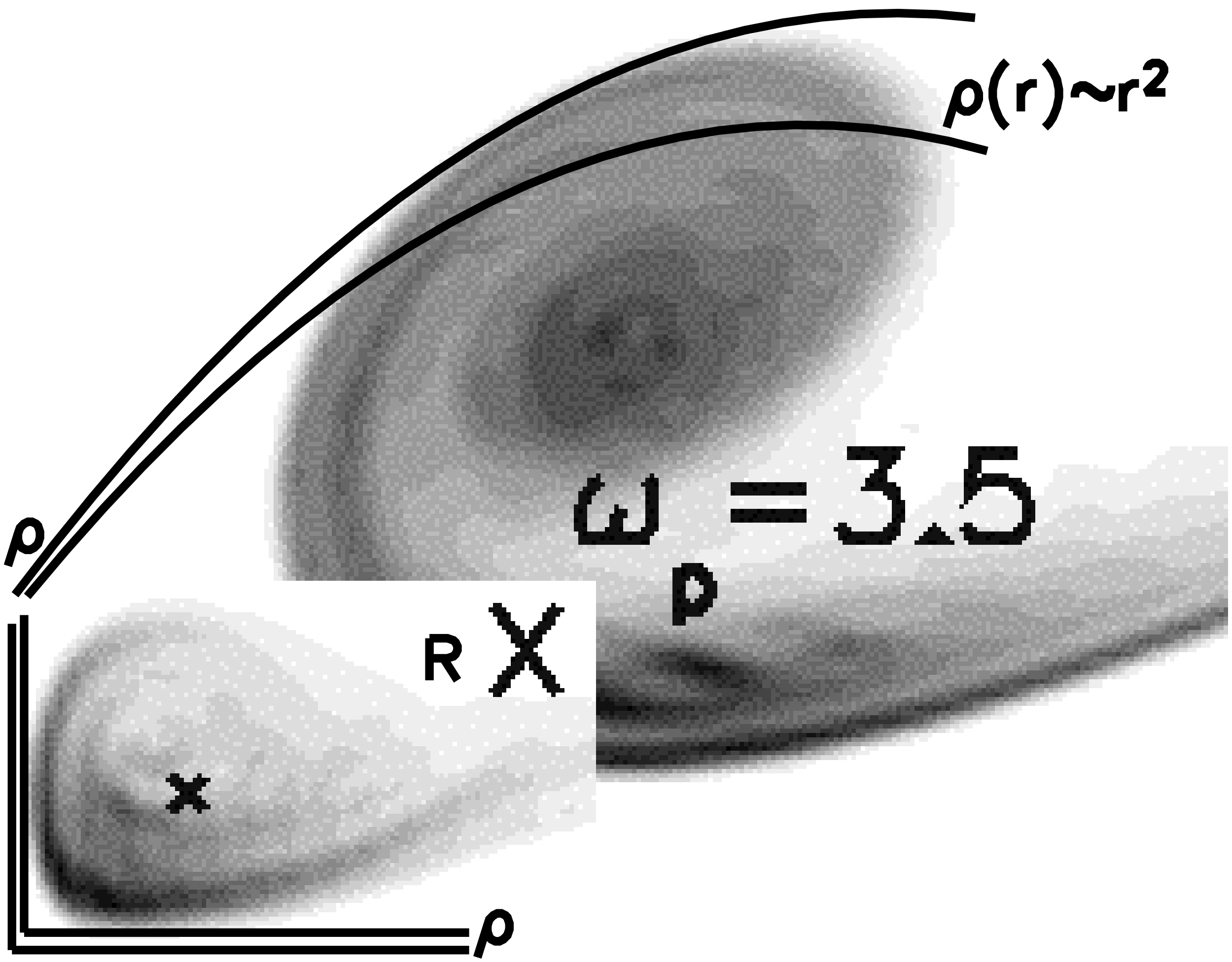
Three-dimensional visualization of the singular collapse of anti-parallel vortex tubes in the incompressible Euler equations at $t = 17$. One half of one of the anti-parallel vortices, cut through the symmetry plane of maximum vorticity with z expanded by 4 is shown. This is a a black and white version of the 1996 color cover figure of Nonlinearity [51]. Three visualization procedures are used: mesh lines with shading, an isosurface, and vortex lines. This illustrates how the physical space structure can be divided into three regions, inner, intermediate, and outer by the length scales $R \sim (T_c - t)^{1/2}$ and $\rho \sim (T_c - t)$. The inner region within a distance $\rho \sim (T_c - t)$ of $\|\omega\|_\infty$ is visualized with bright lines. $\|\omega\|_\infty$ is among the brightest lines. The dominant feature is an isosurface set at $0.6\|\omega\|_\infty$ indicating the region out to R, the extent of the intermediate region. Beyond the isosurface is an outer region indicated by swirling vortex lines that originate from within the surface.

Computational Challenge

- Thin but long structures localized in two directions.
- Slower collapse in the third direction.

Diagram of the scaling of the structure formed by overlaying a plane through the symmetry plane and a plane through the intermediate swirling region.



For small $r < R$, vorticity growth is confined to the two nearly perpendicular vortex sheets represented by the pairs of vertical and horizontal lines separated by ρ and of extent R. For $r > R$, where vorticity is no longer growing, the residual vorticity is found in swirling regions whose width increases as $\rho(r) \sim r^2$.

HISTORY OF EULER

Method, then YES or NO on singularity

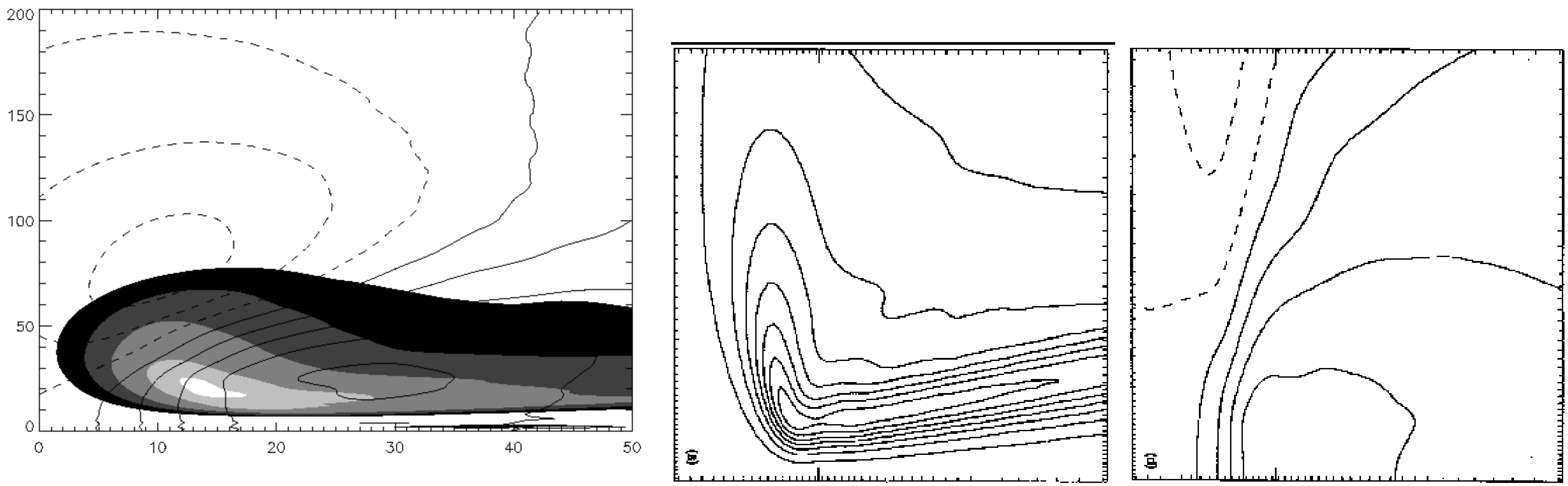
- 1975 Early Taylor-Green. Inconclusive.
- 1979 Pade of Taylor-Green. Yes
- 1983 DNS of Taylor-Green for Euler No
- 1984 Beale-Kato-Majda. Bounds for Euler
- 1986 Chorin/Siggia. Vortex filaments. Yes
- DNS = Direct numerical simulation
  - 1987 Early: NO too much flattening
  - 1989 Spectral: YES too crude
  - 1990 Nested DNS: NO bad numerics
  - 1993 Filtered initial conditions: YES $\|\boldsymbol{\omega}\|_\infty \approx 18/(T-t)$
  - 1998 Cylindrical vortex [Grauer *et al.*(1998)] YES with $\|\boldsymbol{\omega}\|_\infty \approx 18/(T-t)$
  - 2006 Hou and Li, filtered spectral: NO
  - 2006 Orlandi and Carnevale, new claims of singular behavior with unresolved problems

GUIDELINES FOR SIMULATIONS

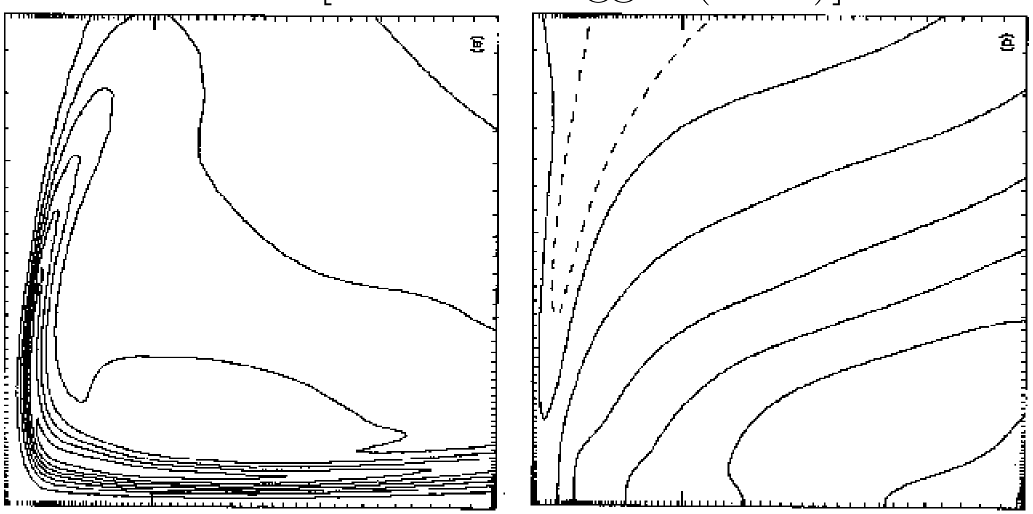
Generally agreed upon at these meetings:

- IUTAM Symposium on Topological Fluid Dynamics, Cambridge, England, August 1989.
  - U. Frisch, F. Hussain, R.M. Kerr, A. Pumir E.D. Siggia.
- Program on Topological Fluid Dynamics, Institute for Theoretical Physics, Santa Barbara, California, Fall 1991.
  - R.M. Kerr, R. Pelz, A. Pumir E.D. Siggia, N. Zabusky.
- Research Institute in Mathematical Sciences, Kyoto, Japan, October 1992.
  - R.M. Kerr, A. Majda.
- Institute for Advanced Studies, Princeton, March 2003,
  - A. Bhattacharee, U. Frisch, R.M. Kerr, N. Zabusky.
  - This meeting was instigated by the untimely death of our friend and collegue Rich Pelz.

- These are some of the guidelines:
  - Run only Euler. Do not try to reach conclusions about Euler using a series of decreasing viscosity Navier-Stokes calculations.
  - Use refined meshes.
  - Complementary pseudo-spectral calculations can still be useful to confirm the numerical method.
  - In addition to the quantities already listed, positions of maxima should collapse.
- Suggestions based on simulations is to look for:
  -
$$x_p - X(T) \sim T - t \quad , \quad z_p \sim T - t$$
  - $\sup(|v|^2) \sim T - t$ where v is the axial velocity in the direction of vorticity in the symmetry plane.
  - Curvature blowup as $\kappa^{-2} \sim (T - t)$.



$\|\boldsymbol{\omega}\|_\infty$ and α contours in the symmetry plane from (left) $t = 15$ [Kerr (1993)] and for $t = 7.2$ from [Pumir & Siggia (1990)] while I think it is still singular.



$\|\boldsymbol{\omega}\|_\infty$ and α contours in the symmetry plane for $t = 8.3$ from Pumir & Siggia (1990) after I think numerical effects are smoothing the flow.

2D models

- Stretched two & one-half dimensional Euler system proposed by [Gibbon *et al.*(1999)], calculated by [Ohkitani & Gibbon (2000)].
  - All three velocity components are included.
  - Variation in one spatial direction is at worst linear.
  - $\{u_1(x,y,t),\ u_2(x,y,t),\ z\gamma(x,y,t)\}$.
  - It was shown subsequently [Constantin (2000)] that this is a Ricatti system
  - And there is **singular behavior in γ.**
- Surface quasi-geostrophic model.
  - $q_{,t} + J(\psi, q) = 0 \qquad q = -(-\triangle)^{\alpha}\psi,\ \alpha = \frac{1}{2}$ (2D Euler is $\alpha = 1$)
  - Bounds on the curvature of active lines restricting singularities [Constantin *et al.*(1994)]
  - **Probably not singular.**
  - **Contour dynamics** version probably is **singular**.

Stretched 2.5D

$$\boldsymbol{U}(x,y,z,t)=\begin{pmatrix}u\\v\\w\end{pmatrix}=\begin{pmatrix}u(x,y,t)\\v(x,y,t)\\z\gamma(x,y,t)+W(x,y,t)\end{pmatrix}$$

$$\gamma=u_{z,z}\neq\alpha=\boldsymbol{\omega}\cdot\boldsymbol{S}\boldsymbol{\omega}\qquad\text{because}\qquad\hat{\boldsymbol{\omega}}\cdot\hat{\boldsymbol{a}}\neq 1$$

$$\boldsymbol{u}_\perp=(u,v)\qquad\nabla_{(x,y)}=(\partial_x,\partial,y)\qquad\nabla_{(x,y)}\times\boldsymbol{u}_\perp=\omega\hat{\boldsymbol{z}}\qquad\nabla_{(x,y)}\cdot\boldsymbol{u}_\perp=-\gamma$$

$$\frac{D\gamma}{Dt}+\boldsymbol{u}_\perp\cdot\nabla_{(x,y)}\gamma=-\gamma^2-\boldsymbol{P}_\gamma(t)$$

$$-\boldsymbol{P}_\gamma=-p_{,zz}=2<\gamma^2>=C(t)$$

$$\frac{DW}{Dt}+\boldsymbol{u}_\perp\cdot\nabla_{(x,y)}\gamma=-\gamma W$$

$$\frac{D\omega_z}{Dt}+\boldsymbol{u}_\perp\cdot\nabla_{(xy)}\gamma=\gamma\omega_z$$

$$-\frac{\partial p}{\partial z}=z\left(\frac{\partial\gamma}{\partial t}+\boldsymbol{u}_\perp\cdot\nabla_{(xy)}\gamma+\gamma^2\right)+\left(\frac{\partial W}{\partial t}+\boldsymbol{u}_\perp\cdot\nabla_{(xy)}W+\gamma W\right)$$

$$-p(x,y,z,t)=\tfrac{1}{2}z^2\left(\frac{\partial\gamma}{\partial t}+\boldsymbol{u}_\perp\cdot\nabla_{(xy)}\gamma+\gamma^2\right)+z\left(\frac{\partial W}{\partial t}+\boldsymbol{u}_\perp\cdot\nabla_{(xy)}W+\gamma W\right)+P(x,y,t)$$

Symmetry Plane

(x, y) is in the symmetry plane, z is out-of-plane, $(\partial_s = \hat{\boldsymbol{\omega}} \cdot \nabla_{(xy)})$,
$\boldsymbol{\kappa} = \kappa\hat{\mathbf{n}} = \hat{\boldsymbol{\omega}}_{,s} = (\hat{\boldsymbol{\omega}} \cdot \nabla_{(xy)})\hat{\boldsymbol{\omega}}$.

$$\boldsymbol{u}_\perp = (u_x, u_y) = (u, v) \neq 0 \qquad u_z = u_3 = 0 \qquad \boldsymbol{\omega} = \omega\hat{\boldsymbol{\omega}} \neq 0$$

$$\boldsymbol{u}_{\perp,z} = 0 \neq 0 \qquad u_{z,z} = \hat{\boldsymbol{\omega}} \cdot \boldsymbol{S}\hat{\boldsymbol{\omega}} = \alpha \neq 0 \qquad \omega_{,z} = 0$$

$$\boldsymbol{u}_{\perp,zz} = \boldsymbol{u}_{\perp,ss} - (\boldsymbol{\kappa} \cdot \nabla_{(xy)})\boldsymbol{u}_\perp \qquad \alpha_{,z} \neq 0 \qquad \omega_{,zz} = \omega_{,ss} - (\boldsymbol{\kappa} \cdot \nabla_{(xy)})\omega$$

3D Biot-Savart

$$\boldsymbol{u}(\boldsymbol{x}) = \int \frac{\boldsymbol{\omega} \times (\boldsymbol{x} - \boldsymbol{y})}{|\boldsymbol{x} - \boldsymbol{y}|^3} d^3y \qquad = \boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla} \times \int \frac{\boldsymbol{\omega}}{|\boldsymbol{x} - \boldsymbol{y}|} d^3y$$

where

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{\Gamma}{4\pi} \oint \frac{\hat{\boldsymbol{\omega}}}{|\boldsymbol{x} - \boldsymbol{y}|} ds \quad \text{with} \quad \boldsymbol{y} = \boldsymbol{x}(s) \qquad (\text{eq} - \text{A})$$

- Define $\hat{\boldsymbol{\omega}}_j = \pm\hat{\boldsymbol{z}}$, $\hat{\mathbf{n}}_j$, and $\hat{\mathbf{b}}_j = \hat{\boldsymbol{\omega}}_j \times \hat{\mathbf{n}}_j$ as the tangent, normal and bi-normals to vortex lines at (x_j, y_j) through the **symmetry plane only**.

- $x_n = \hat{\mathbf{n}}_j \cdot (x - x_j, y - y_j)$, $x_b = \hat{\mathbf{b}}_j \cdot (x - x_j, y - y_j)$, and $x_t = z(\hat{\boldsymbol{z}} \cdot \hat{\boldsymbol{\omega}})$.

Expand this, put in (eq-A) integrate along arclength s to an arbitrary distance ϵ.
Along vortex lines

$$\hat{\boldsymbol{\omega}} = \hat{\boldsymbol{\omega}}_j + \kappa s\hat{\mathbf{n}}_j \qquad \boldsymbol{x} - \boldsymbol{y} = (x_n - \tfrac{1}{2}\kappa s^2)\hat{\mathbf{n}}_j + x_b\hat{\mathbf{b}}_j + (x_t - s)\hat{\boldsymbol{\omega}})$$

$$|\boldsymbol{x}-\boldsymbol{y}|^{-1} = \left((x_n - \tfrac{1}{2}\kappa s^2)^2 + x_b^2 + (x_t - s)^2\right)^{-1/2} \approx (r^2 + s^2 - \kappa x_n s^2 - 2x_t s)^{-1/2}$$
$$\approx (r^2+s^2)^{-1/2}\left[1 + \tfrac{1}{2}\left(\frac{x_n \kappa s^2 + 2x_t s}{r^2+s^2}\right)\right]$$

to get

$$\boldsymbol{A}_j = \frac{\Gamma}{4\pi}\left\{2\hat{\boldsymbol{\omega}}_j \log\frac{\epsilon}{r} + \kappa x_n \hat{\boldsymbol{\omega}}_j \left(\log\frac{\epsilon}{r} - 1\right) + 2\kappa x_t \hat{\mathbf{n}}_j \left(\log\frac{\epsilon}{r} - 1\right)\right\}$$

yields the velocity

$$\boldsymbol{u}_{\perp j} \sim \frac{\Gamma}{2\pi}\left(\frac{x_n}{r^2}\boldsymbol{b}_j - \frac{x_b}{r^2}\boldsymbol{n}_j\right) + \frac{\Gamma}{4\pi}\kappa \log\frac{\epsilon}{r}\boldsymbol{b}_j - \frac{\Gamma}{4\pi}\kappa\left(\frac{x_b^2}{r^2}\boldsymbol{b}_j + \frac{x_n x_b}{r^2}\boldsymbol{n}_j\right)$$

This is the velocity in equation (2.3.9) of Saffman's book.

- **It neglects:**
- Any core effects, out-of-plane velocity.
- The **fixes:**
  - The vector potential gives: $u_z = \hat{\boldsymbol{z}}\cdot\frac{\Gamma}{4\pi}\frac{2\kappa x_b x_t}{r^2}\hat{\boldsymbol{\omega}} = 0,$
  - yielding $u_{z,z} = \alpha = \frac{\Gamma}{4\pi}\frac{2\kappa x_b}{r^2} \neq 0$
  - So that $\nabla_{(xy)}\cdot\boldsymbol{A} = 0$ add to $\boldsymbol{A}$: $\kappa x_n \hat{\boldsymbol{\omega}}_j \frac{x_t^2}{r^2}$

Continuum 2D system

Assume the velocity in the symmetry plane, obeys

$$\boldsymbol{u}_\perp = \nabla_{(x,y)} \times \psi + \nabla_{(x,y)}\phi$$

where $\phi = \phi_a + \phi_b$ and $\nabla^2_{(x,y)}\psi = -\omega$, $\nabla^2_{(x,y)}\phi_a = -\alpha$ and $\nabla^2_{(x,y)}\phi_b = 0$

$$\nabla^2\alpha = \nabla^2_{(x,y)}\alpha + \alpha_{,zz} = -\hat{\boldsymbol{\omega}} \cdot (\nabla_{(x,y)} \times \omega\boldsymbol{\kappa})$$

$$\text{where} \qquad \boldsymbol{\kappa} = \kappa\boldsymbol{n} = \hat{\boldsymbol{\omega}}_{,s} = (\hat{\boldsymbol{\omega}} \cdot \nabla)\hat{\boldsymbol{\omega}} \quad ,$$

This is a set of 4-th order equations. **IF** we know $\alpha_{,zz}$. Assume $\alpha_{,zz} = 0$, then the time derivatives are

$$\frac{D\omega}{Dt} = \alpha\omega \qquad \frac{D\boldsymbol{\kappa}}{Dt} = \nabla_{(x,y)}\alpha + (\boldsymbol{\kappa} \cdot \nabla_{(x,y)})\boldsymbol{u}_\perp - 2\alpha\boldsymbol{\kappa}$$

This comes from the quaternion formulation [Gibbon *et al.*(2006)] and (Gibbon, private communication). Use

$$\frac{D}{Dt}\hat{\boldsymbol{\omega}} = \boldsymbol{\chi} \times \hat{\boldsymbol{\omega}} = 0, \qquad \boldsymbol{\chi} = 0$$

$$\frac{D\boldsymbol{\kappa}}{Dt} = (\boldsymbol{\chi} \times \hat{\boldsymbol{\omega}})_{,s} - \alpha\boldsymbol{\kappa} = \boldsymbol{\chi}_{,s} \times \hat{\boldsymbol{\omega}} + \boldsymbol{\chi} \times \boldsymbol{\kappa} - \alpha\boldsymbol{\kappa}$$

$$\frac{D\boldsymbol{\kappa}}{Dt} = (\boldsymbol{S}\hat{\boldsymbol{\omega}} - \alpha\hat{\boldsymbol{\omega}})_{,s} - \alpha\boldsymbol{\kappa} = (\boldsymbol{S}\hat{\boldsymbol{\omega}})_{,s} - \alpha_{,s}\hat{\boldsymbol{\omega}} - 2\alpha\boldsymbol{\kappa}$$

Then apply the conditions of the symmetry plane.

Calculation with two vortex filaments with finite cores

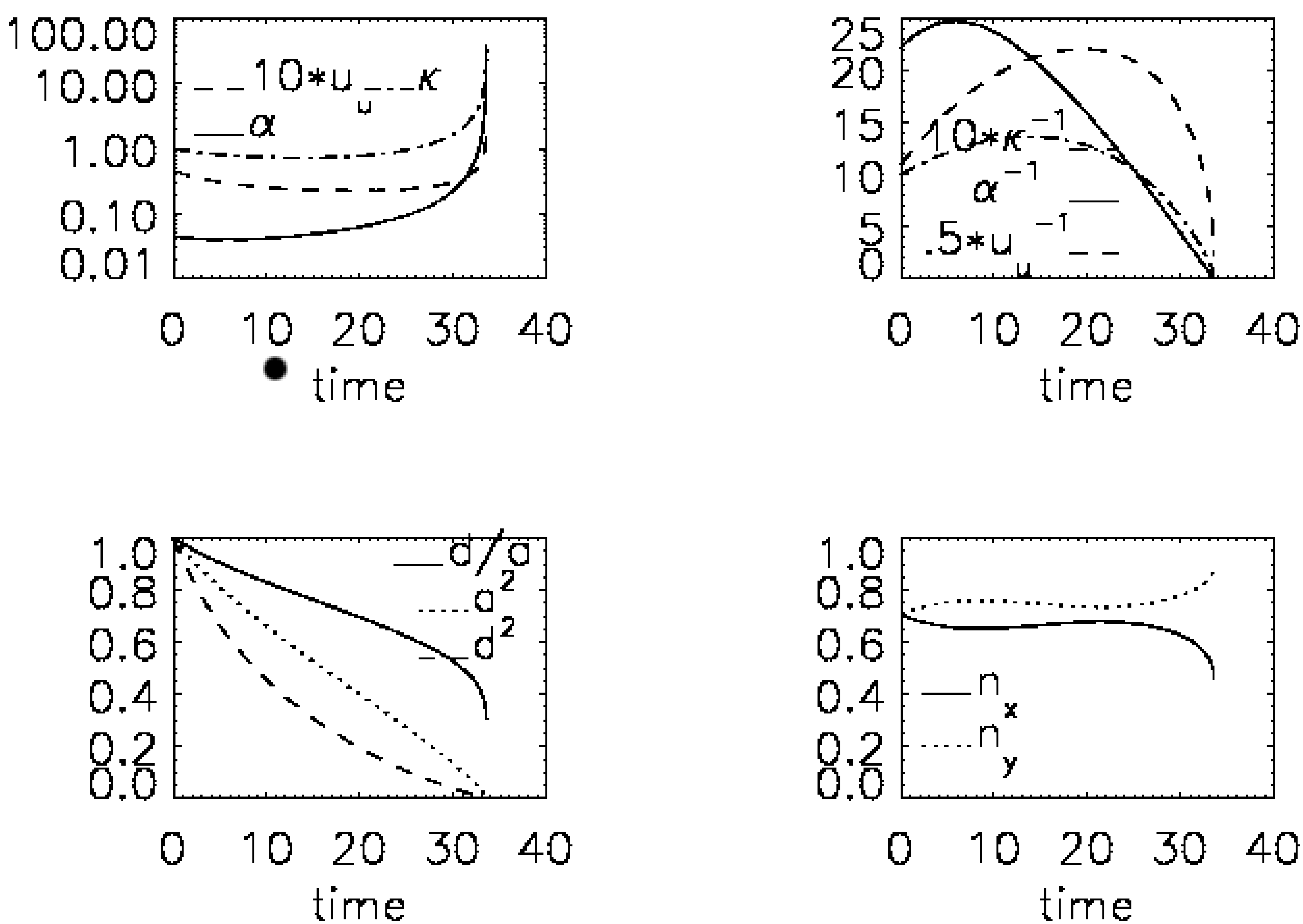
Assume two 3D vortex filaments that are mirrored across a dividing plane.

- $\hat{\boldsymbol{\omega}}_1 = (0,0,-1)$, $\hat{\boldsymbol{\omega}}_2 = (0,0,1)$ and $\omega_2 = \omega_1 = \omega$ $\qquad \dot{\omega} = \alpha\omega$
- If $\boldsymbol{x}_1 = (x_1, y_1) = (x, y)$ then $\boldsymbol{x}_2 = (x_2, y_2) = (x, -y)$, $\qquad \dot{\boldsymbol{x}} = \boldsymbol{u}_\perp$
- If $\boldsymbol{\kappa}_1 = (\kappa_x, \kappa_y)$ then $\boldsymbol{\kappa}_2 = (\kappa_x, -\kappa_y)$ $\qquad \dot{\boldsymbol{\kappa}} = \nabla_{(x,y)}\alpha + (\boldsymbol{\kappa}\cdot\nabla_{(x,y)})\boldsymbol{u}_\perp - 2\alpha\boldsymbol{\kappa}$
- x can be neglected.
- Set $y = d$ with $x_n = -2d,\ \ x_b = 2d,\quad r^2 = 4d^2$
- Rosenhead regularization of core with thickness a. $\qquad \dot{a} = -\frac{\alpha_2}{2}a$
- Velocity due to vortex $\hat{\boldsymbol{\omega}}_2$ using a.

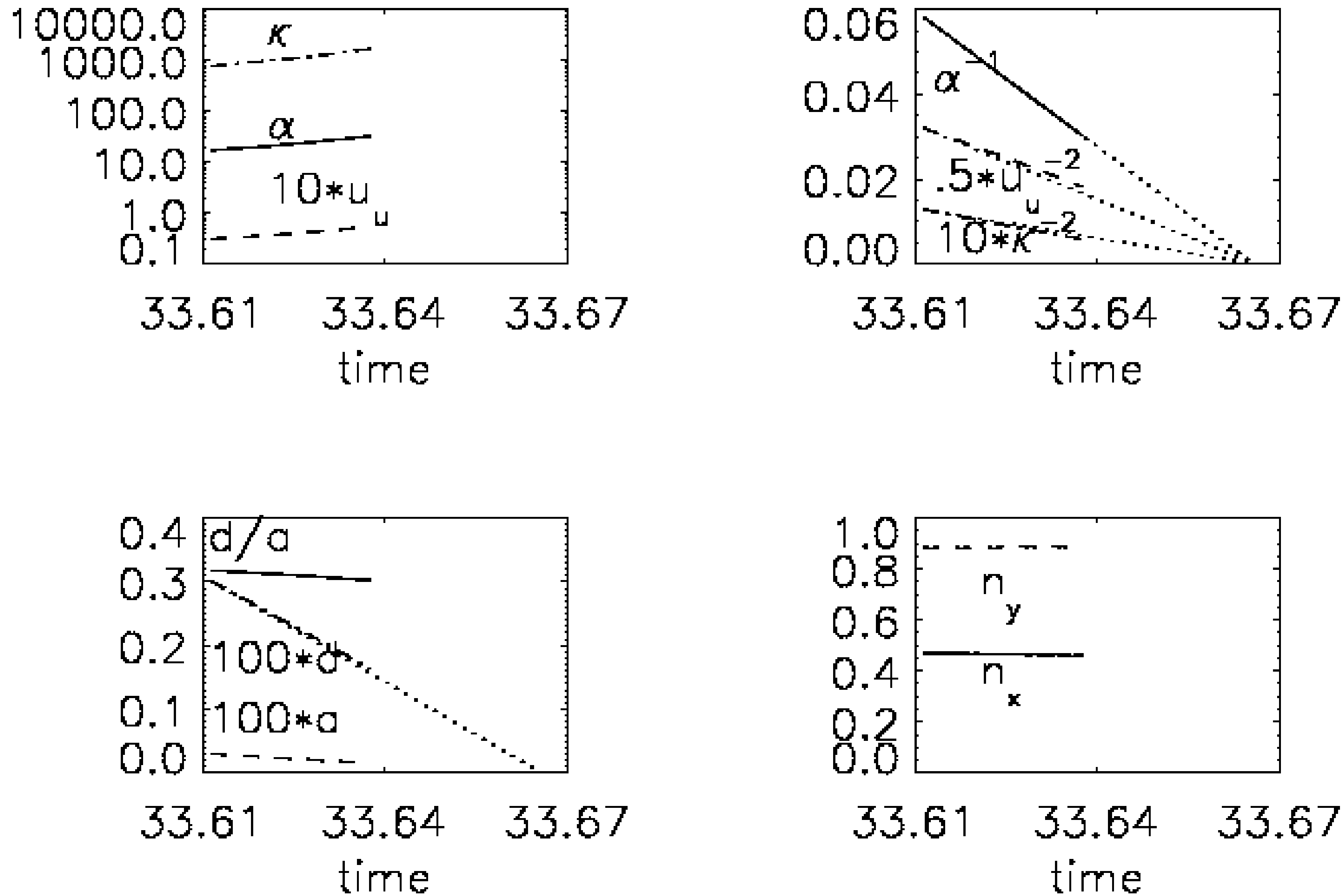
$$\boldsymbol{u}_{2\perp} = \frac{\Gamma}{2\pi}\left(\frac{x_n\hat{\mathbf{b}}_2}{r^2+a^2} - \frac{x_b\hat{\mathbf{n}}_2}{r^2+a^2}\right) + \frac{\Gamma}{4\pi}\kappa\frac{1}{2}\log\frac{\epsilon^2}{r^2+a^2}\hat{\mathbf{b}}_2 - \frac{\Gamma}{4\pi}\kappa\left(\frac{x_b^2+a^2}{r^2+a^2}\hat{\mathbf{b}}_2 + \frac{x_n x_b}{r^2+a^2}\hat{\mathbf{n}}_2\right)$$

- This is used to calculate total velocity and $\nabla_{(x,y)}\boldsymbol{u}_\perp$ needed for $\dot{\boldsymbol{\kappa}}$.
  - Total velocity in y , self-induced plus that to $\hat{\boldsymbol{\omega}}_2$: $\qquad u_y = \dfrac{\Gamma\kappa n_x}{8\pi}\log\dfrac{a^2}{4d^2/a^2}$
- Stretching: $\qquad \alpha_2 = \dfrac{\Gamma}{4\pi}\dfrac{2\kappa x_b}{r^2+a^2} = \dfrac{\Gamma}{4\pi}\dfrac{4\kappa d n_x}{4d^2+a^2}$
- Curvature $\qquad \dot{\boldsymbol{\kappa}} = [\nabla_{(x,y)}\alpha + (\boldsymbol{\kappa}\cdot\nabla)\boldsymbol{u}_{\omega\perp}] \quad + [(\kappa\cdot\nabla)\boldsymbol{u}_{\kappa\perp} - 2\alpha\boldsymbol{\kappa}]$
- stretching $\qquad\qquad$ decay

$$
\begin{aligned}
\nabla_{(x,y)}\alpha + (\boldsymbol{\kappa}\cdot\nabla)\boldsymbol{u}_{\omega\perp} &= \tfrac{\Gamma\kappa}{2\pi(4d^2+a^2)}\left[n_y\tfrac{8d^2}{4d^2+a^2}, \qquad n_x\tfrac{2a^2}{4d^2+a^2}\right] \\
(\kappa\cdot\nabla)\boldsymbol{u}_{\kappa\perp} - 2\alpha\boldsymbol{\kappa} &= \tfrac{2d\Gamma\kappa^2}{4\pi(4d^2+a^2)}\left[-n_x^2 - 4n_x^2 - n_y^2\tfrac{4d^2-a^2}{4d^2+a^2}, \quad 2n_xn_y - 4n_x, n_y\right] \\
&= \tfrac{\Gamma\kappa^2 d}{2\pi(4d^2+a^2)}\left[-5n_x^2 - n_y^2\tfrac{4d^2-a^2}{4d^2+a^2}, \qquad -2n_xn_y\right]
\end{aligned}
$$



Upper left and right: α, κ and u_y blowing up. $\alpha \sim 1/(T-t)$.
Lower left: $d/a \rightarrow \approx .3$. Lower right: $n_y \rightarrow \approx .9$, $n_x \rightarrow \approx .4$.

- 

Upper right: α, u_y^2, $\kappa^2, a^2, d^2 \sim 1/(T-t)$. Dotted lines are extentions to $T = 33.658$. Comparing α and a^2, $\omega = \Gamma/a^2 \approx (.048/.003)/(T-t) \approx 16/(T-t)$. (labels $100a, 100a$ should be $100a^2, 100d^2$)

Conclusions

- Equation for analysis
- There is stretching and potential for singularities due to:
- Agreement with expectations for vortex filaments.
- Could be used for testing regularizations of filaments.

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