CSCAMM

Oct. 25 / 2010

Driven Motion of Interfaces

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joint work with T. Sasamoto, Chiba Univ.

S. Prolhac, TUM + Saclay

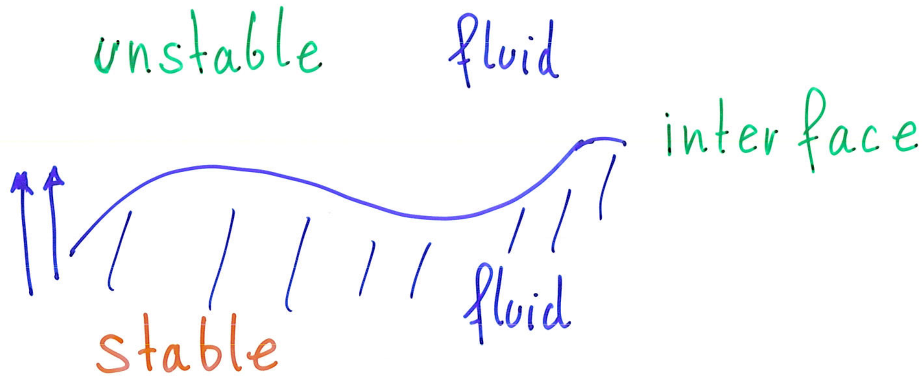
central theme

- nonequilibrium (driven)
- macroscopic evolution (pattern formation, instabilities)
- fluctuations

my talk

2D droplet growth

much simpler



- bulk nucleation is on a longer time scale

2D thin film 1D interface

// shape fluctuations //

- experiment on turbulent liquid crystal, Takeuchi, Sano 2010
- theory

universal probability density functions

1. experiment by Takeuchi and Sano   Tokyo Univ.

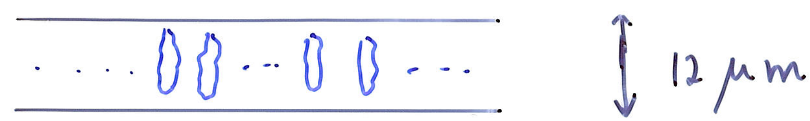
- thin film of turbulent liquid crystal

  25°C, voltage 26V at 250 Hz

  non equilibrium steady state

  ⇒ cell size 16 mm × 16 mm × 12 μm

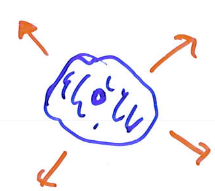
- in-plane isotropic



- two phase coexistence

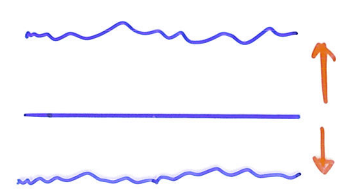
DSM 1	unstable	grey
DSM 2	stable	black

⇒ point seed of DSM 2

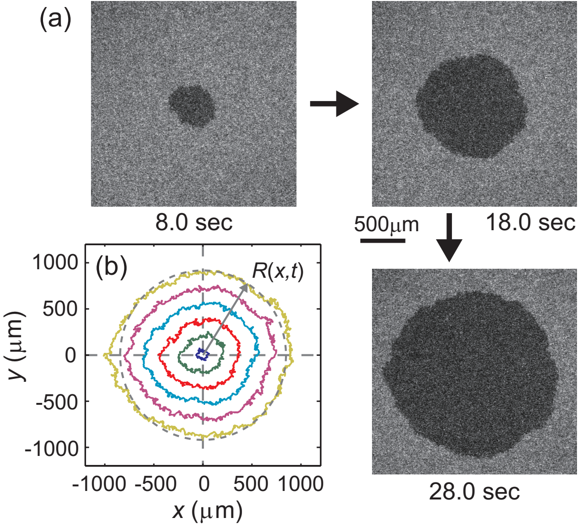


"droplet"

⇒ line seed of DSM 2

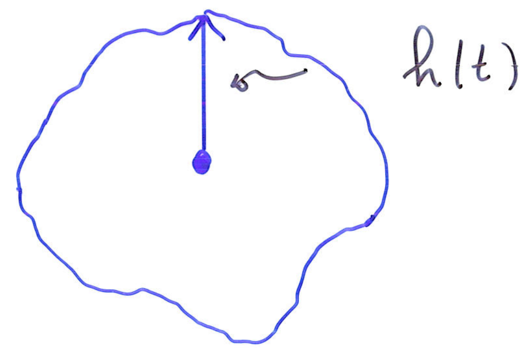


"flat"


(a)
8.0 sec
18.0 sec
28.0 sec
500μm
(b)
R(x,t)
1000
500
0
-500
-1000
y (μm)
-1000
-500
0
500
1000
x (μm)

statistics of shape fluctuations (height) 1100 repeats

$h(t)$ height (radius) along fixed direction



• nonuniversal properties (KPZ theory)

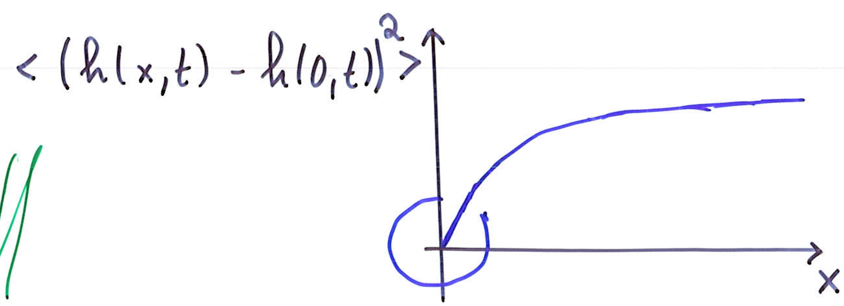
$$h(t) = v_\infty t + c_2 t^{1/3} \chi$$

χ ← random amplitude

(1) asymptotic growth velocity v_∞

(2) coupling strength (nonlinearity) $\lambda \overset{!}{=} v_\infty$ isotropic

(3) stationary height-height correlations at small distances



NO adjustable parameters //

probability densities are known from random matrix theory

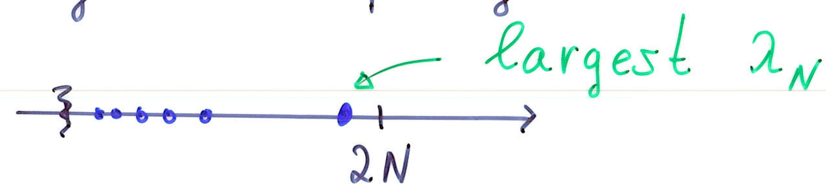
Tracy - Widom 1994

droplet GUE $\beta = 2$ Gaussian Unitary Ensemble

A is $N \times N$ hermitean matrix

$$\frac{1}{Z_N} e^{-\frac{1}{2N} \operatorname{tr} A^2}$$

eigenvalue spacing 1

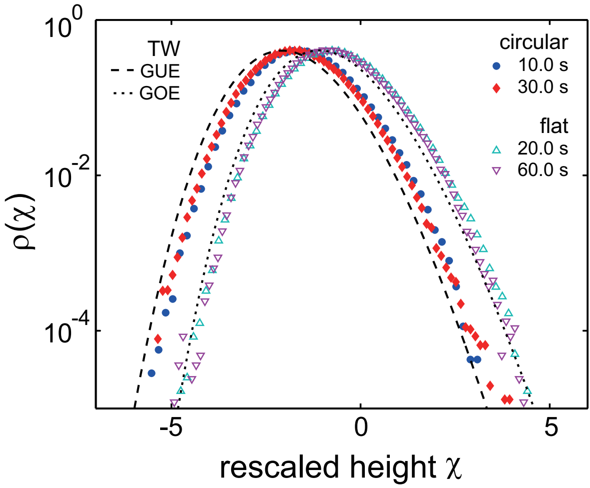


$$\lambda_N \cong 2N + N^{1/3} \chi_2$$

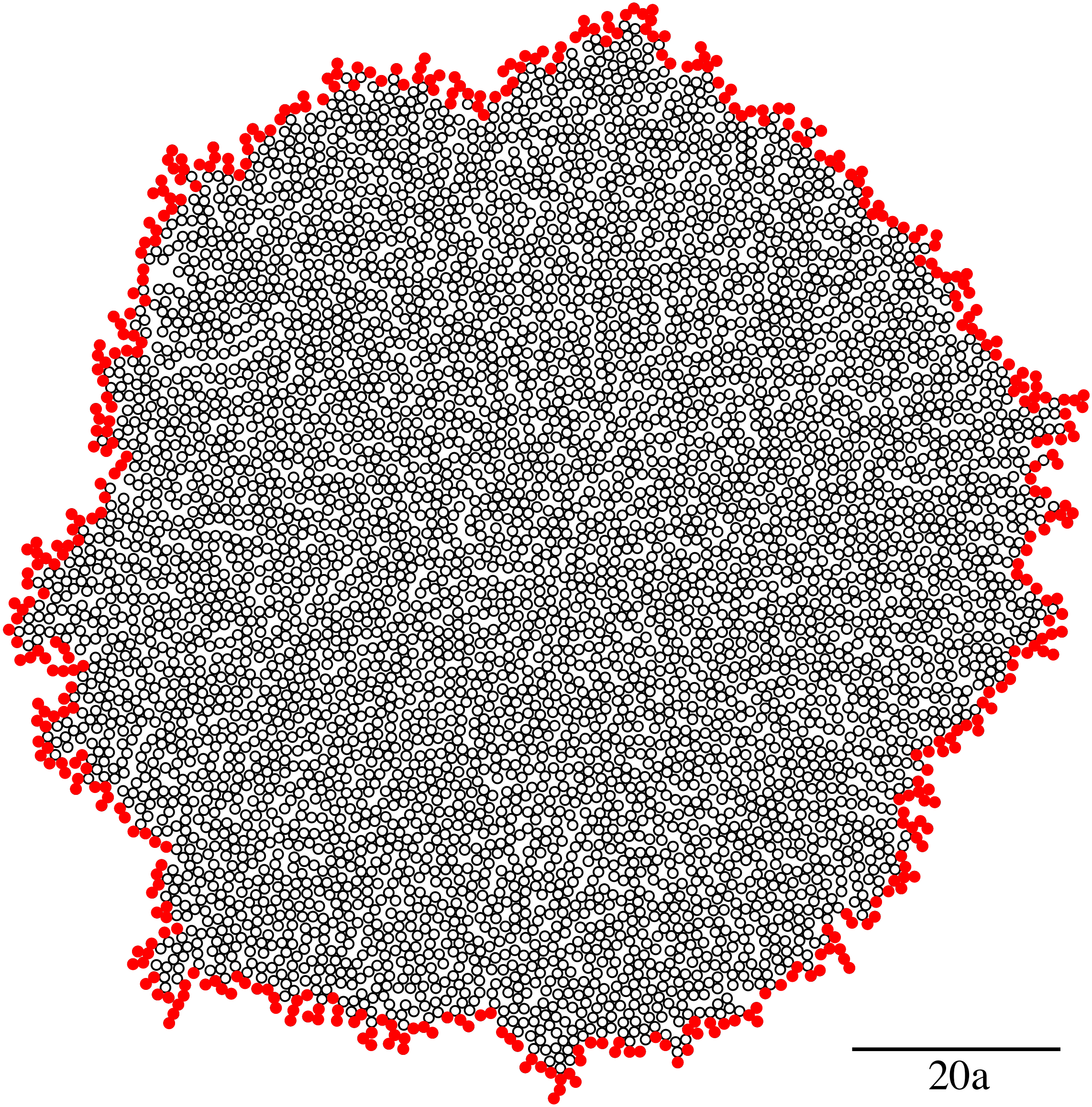
flat GOE $\beta = 1$ Gaussian Orthorgonal Ensemble

A is $N \times N$ real symmetric

$$\lambda_N = 2N + N^{1/3} \chi_1$$

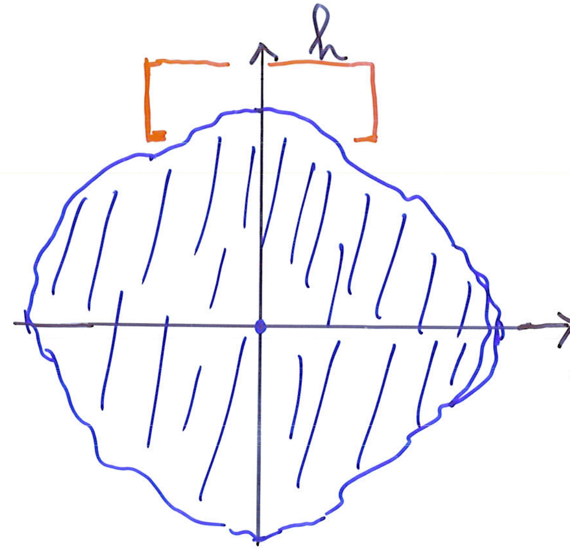


TW
GUE
GOE
circular
10.0 s
30.0 s
flat
20.0 s
60.0 s
10^0
10^-2
10^-4
ρ(χ)
-5
0
5
rescaled height χ


20a

2. theory of Kardar, Parisi, Zhang 1986

- top part only, height function $h(x,t)$



$$\frac{\partial}{\partial t} h = \frac{1}{2} \lambda \left(\frac{\partial h}{\partial x}\right)^2 + \frac{1}{2} \frac{\partial^2}{\partial x^2} h + W$$

nonlinearity $\lambda > 0$

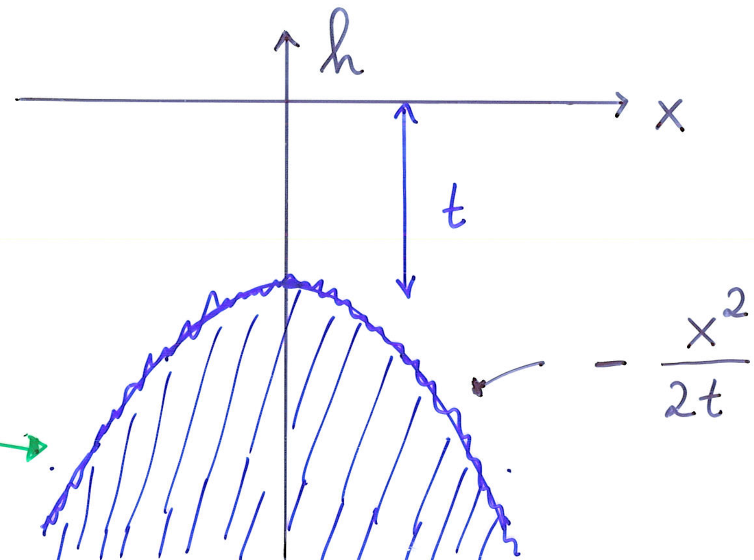
Gaussian white noise $W(x,t)$

$$\langle W(x,t) W(x',t') \rangle = \delta(x-x')\,\delta(t-t')$$

- sharp wedge initial conditions

$$h(x,0) = -\frac{1}{a}|x|$$

$a \to 0$



equivalently

- noisy Burgers equation $\frac{\partial}{\partial x} h = u$

$$\frac{\partial}{\partial t} u + \frac{\partial}{\partial x}\left[-\lambda u^2 - \frac{1}{2}\frac{\partial}{\partial x} u - W\right] = 0$$

- Cole-Hopf, directed polymer — construction of solution

$\lambda = 1$

$$Z(x,t) = e^{h(x,t)}$$

$$\frac{\partial}{\partial t} Z = \frac{1}{2}\frac{\partial^2}{\partial x^2} Z + W Z \qquad Z(x,0) = \delta(x)$$

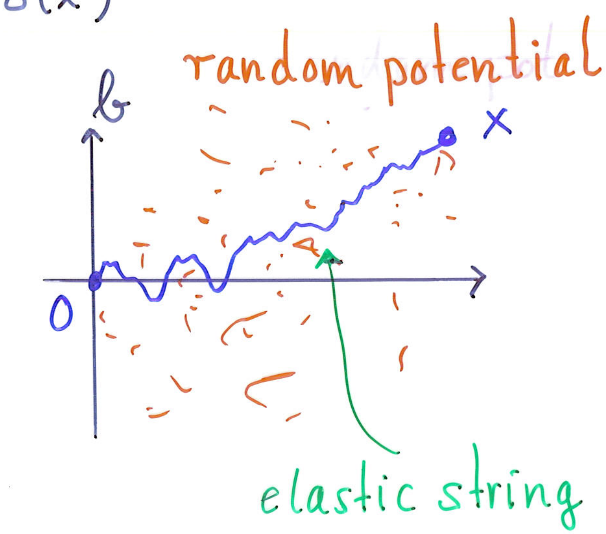
sharp wedge

$$Z(x,t) = \mathbb{E}\left(e^{\int_0^t ds\, W(b(s),s)}\, \delta(b(t)-x)\right)$$

Brownian motion

random

point-to-point



3. generating function

Sasamoto, H.S. 2010

sharp wedge only

$\lambda = 1$

$$\langle \exp\left[-e^{-s + \left(h(x,t) + t + \frac{x^2}{2t} \right)} \right] \rangle = \det(1 - K_s)$$

↑ white noise

on $L^2(\mathbb{R})$

kernel

$$K_s(x,y) = \frac{e^{t^{1/3}x - s}}{1 + e^{t^{1/3}x - s}} K_{Ai}(x,y)$$

Airy kernel

$$K_{Ai}(x,y) = \int_0^\infty dw \, Ai(x+w) Ai(y+w)$$

- $t \to \infty$

$$h(x,t) = -t - \frac{x^2}{2t} + t^{1/3} \chi_2$$

replace $s \rightsquigarrow a t^{1/3}$

$$\mathbb{P}(\chi_2 \le a) = \det(1 - P_a K_{Ai} P_a)$$

→ { Tracy-Widom, GUE, $\beta = 2$ }

P_a ← projects onto $[a, \infty)$

Probability densities

- fixed t

$$h(x,t) = t + \frac{x^2}{2t} + t^{1/3}\xi_t$$

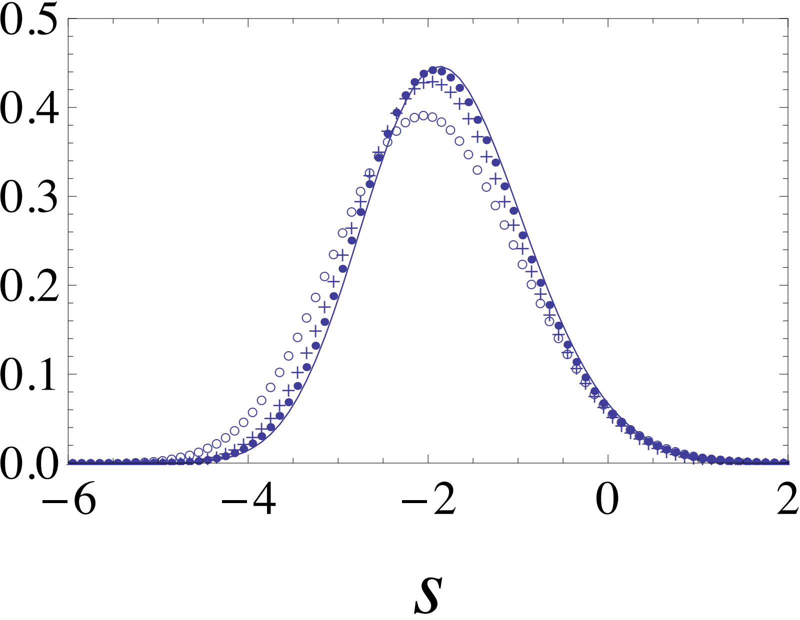
$$\lim_{t\to\infty} \xi_t = \chi_2$$

ξ_t has probability density ρ_t

$$\rho_t(s) = \rho_{GU} * g_t(s)$$

- Gumbel density $\quad t^{1/3} e^{t^{1/3}x} e^{-e^{t^{1/3}x}}$
- g_t is difference of two Fredholm determinants

↑ computable by 100×100 approximation



0.5
0.4
0.3
0.2
0.1
0.0
−6
−4
−2
0
2
s

puzzle of finite time correction

relative to TW

slowest mode: mean with decay $c_0 t^{-1/3}$

sign of c_0 ?

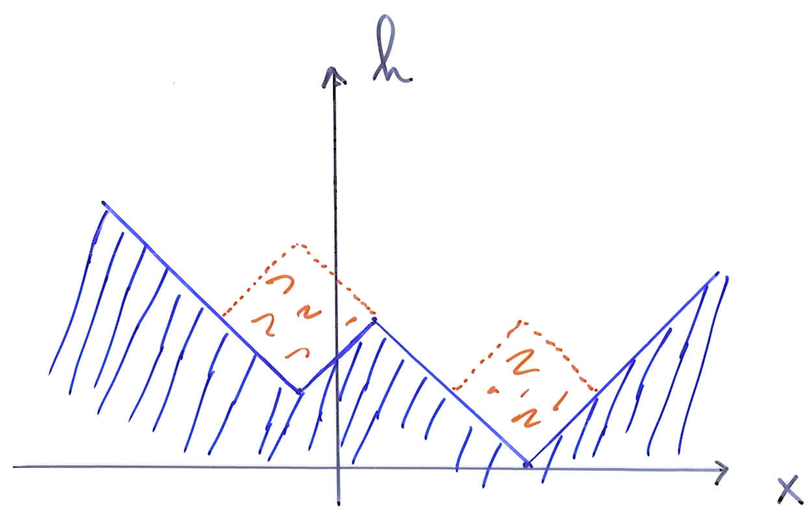
KPZ equation c_0 negative

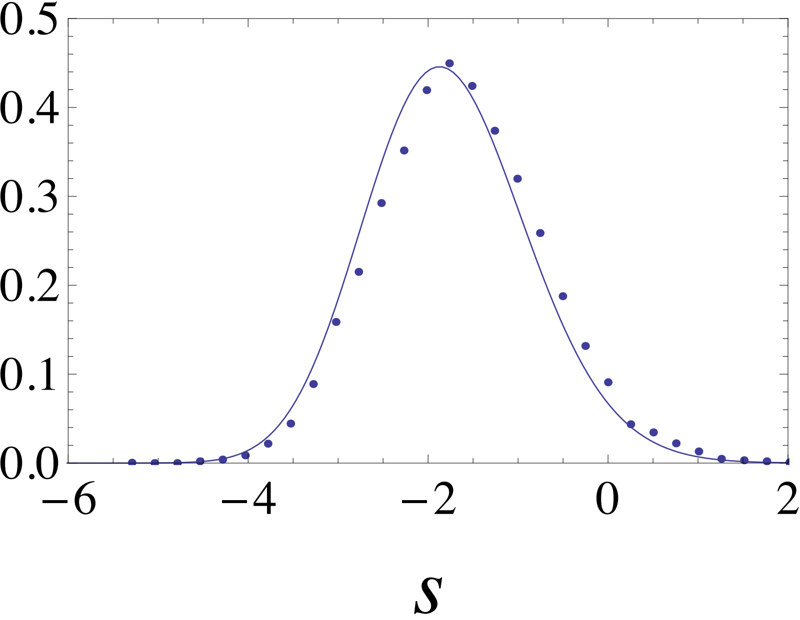
experiment c_0 positive

KPZ holds for weak asymmetry

strong asymmetry

single step growth model




0.5
0.4
0.3
0.2
0.1
0.0
−6
−4
−2
0
2
s

4. method / generalizations

- approximation through weakly asymmetric single step growth

  Sasamto, H.S. 2010

  independently Amir, Corwin, Quastel 2010

  yields density $\rho_t(s)$

  based on Tracy, Widom 2009

- replica method, Kardar 1987

  moments $\langle Z(x,t)^n \rangle$ = attractive δ-Bose gas on the line

  divergent series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} e^{n^3}$

  independently Calabrese, Le Doussal, Rosso 2010

  Dotsenko 2010

  yields generating function

two-point function — Prolhac, H.S. 2010

sharp wedge

generating function + shift

$$\langle \exp\left[-e^{-s_1+h(x_1,t)} - e^{-s_2+h(x_2,t)}\right]\rangle$$

‖ same time ‖

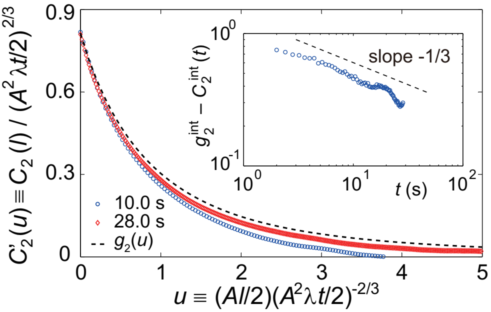
stationary, depends only on $x = x_2 - x_1$

$$= \det(1 - K_{s_1,s_2,x}) \quad \leftarrow L^2(\mathbb{R})$$

$$H = -\frac{d^2}{du^2} + u$$

$$K_{s_1,s_2,x}(u,v) = \frac{e^{t^{1/3}u-s_1} + e^{t^{1/3}v-s_2}}{1+e^{t^{1/3}u-s_1}+e^{t^{1/3}v-s_2}} \left(e^{-t^{-2/3}|x|H}\right)(u,v) \int_0^\infty dw\, e^{-t^{2/3}|x|w} \mathrm{Ai}(u+w)\mathrm{Ai}(v+w)$$

Airy process, Dyson's Brownian motion

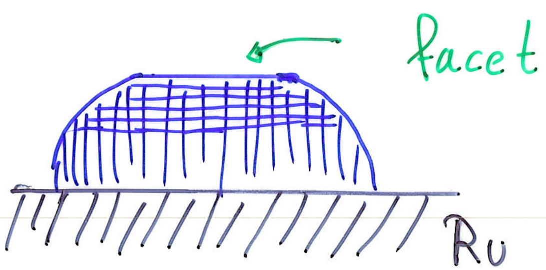


0.9
0.6
0.3
0
0
1
2
3
4
5
$C'_2(u) \equiv C_2(l) / (A^2 \lambda t/2)^{2/3}$
$u \equiv (Al/2)(A^2 \lambda t/2)^{-2/3}$
10.0 s
28.0 s
$g_2(u)$
10^0
10^{-1}
10^0
10^1
10^2
slope -1/3
$g_2^{\rm int} - C_2^{\rm int}(t)$
t (s)

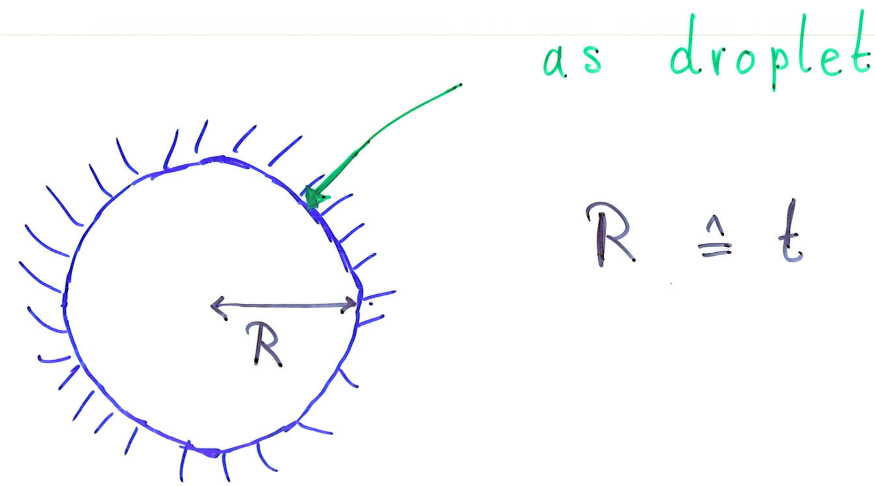
same as facet edge fluctuations

experiment T. Einstein, Williams et al 2006

(1,1,1) facet of Pb on Ru



top



$R \hat{=} t$

5. Summary / Outlook

‖ droplet growth — fluid / fluid interface

DRIVEN (stable/unstable)

- experiment turbulent liquid crystal
- exact solution of 1D KPZ equation

future

flat initial conditions $h(x,0)=0$

exact solution?