Network Constrained Coalitional Dynamic Games and Evolution of Network Topologies

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Taxonomy of Networked Systems

Infrastructure / Communication Networks

Internet / WWW
MANET
Sensor Nets
Robotic Nets
Hybrid Nets: Comm, Sensor, Robotic and Human Nets

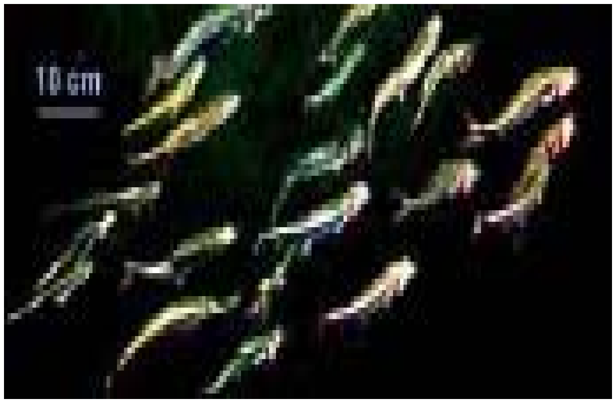
Social / Economic Networtks

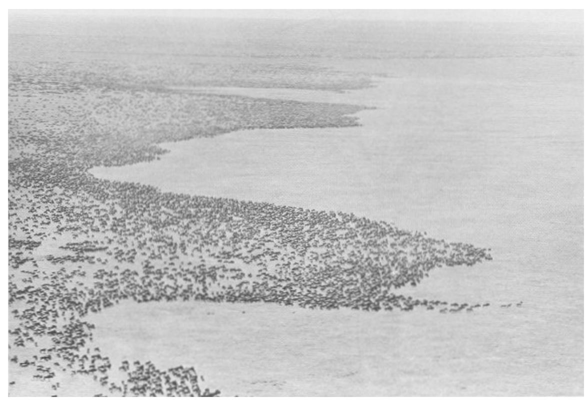
Social Interactions
Collaboration
Social Filtering
Economic Alliances
Web-based social systems

Biological Networks

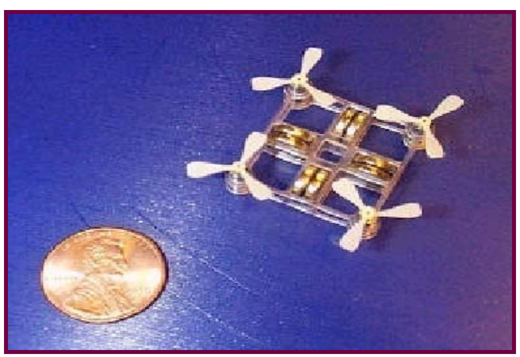
Community
Epiddemic
Cellular and Sub-cellular
Neural
Insects
Animal Flocks

Biological Swarms

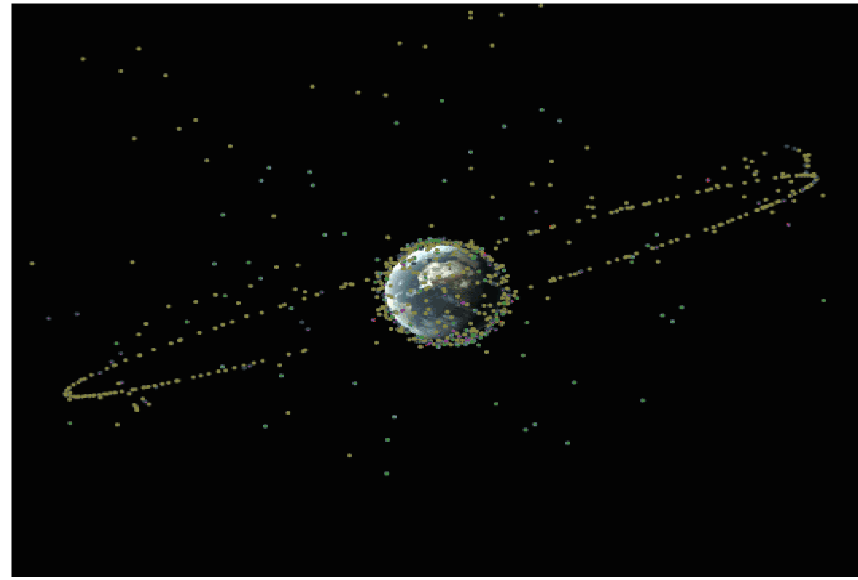


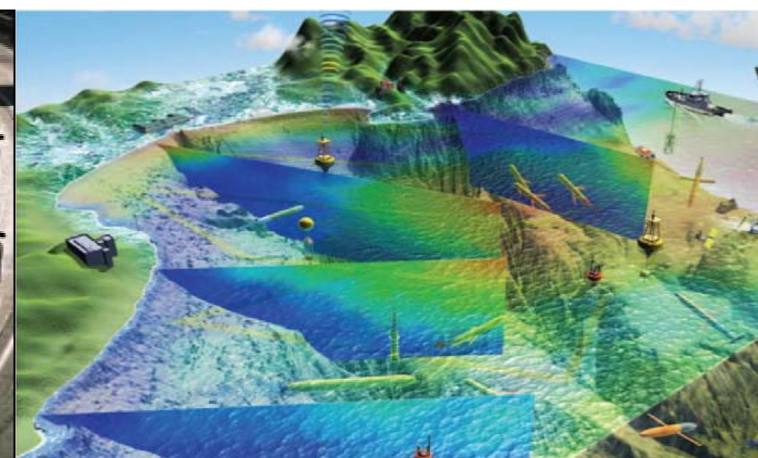


Collaborative Robotic Swarms

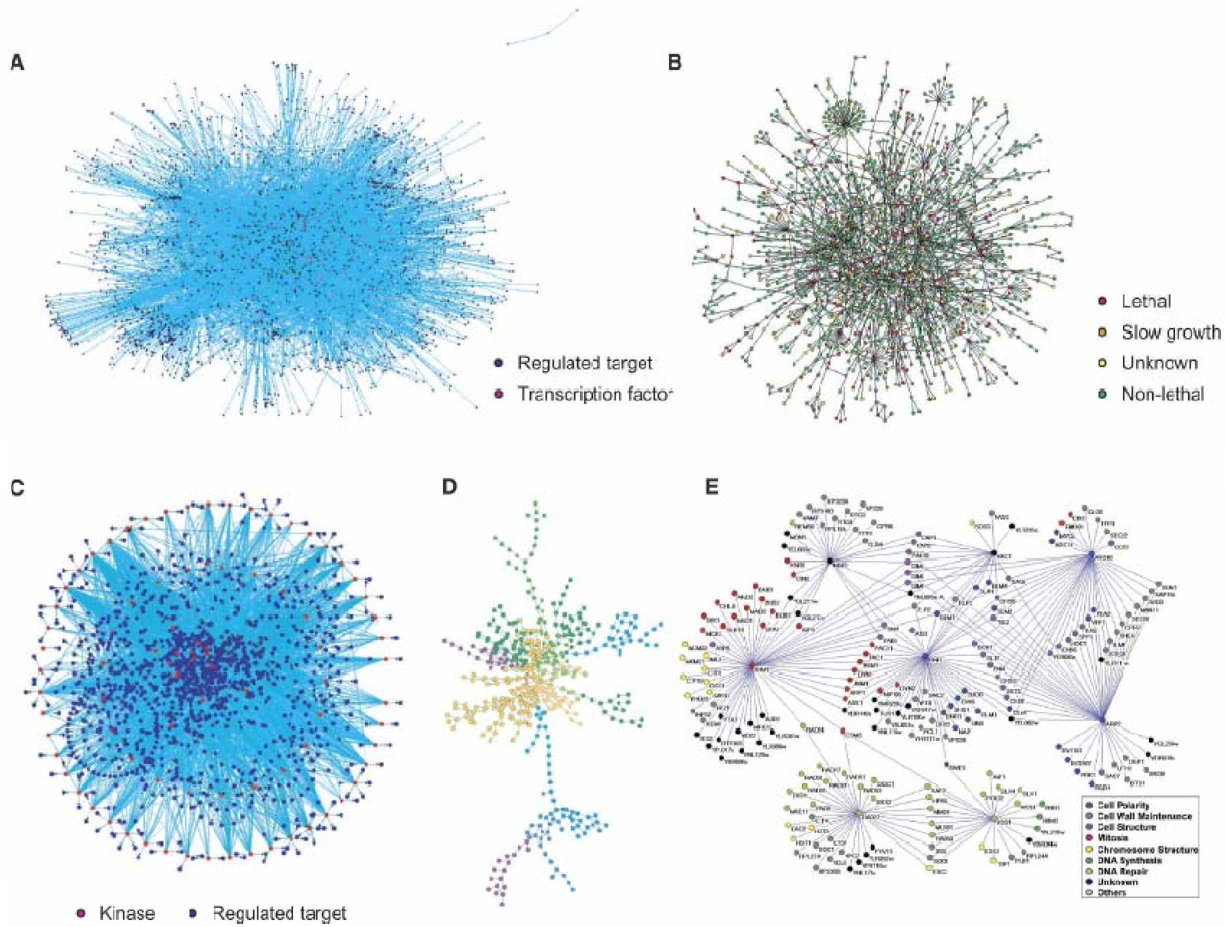


Autonomous Swarms – Networked Control





Biological Network Types

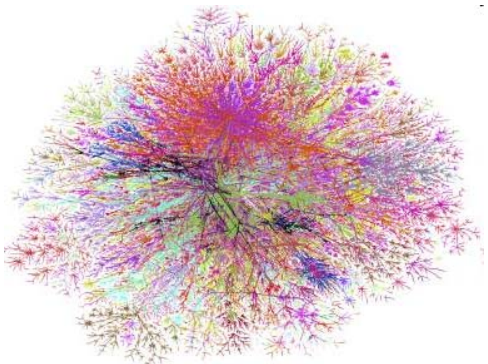


Examples of biological networks: [A] Yeast transcription factor-binding network; [B] Yeast protein -protein interaction network; [C] Yeast phosphorylation network ; [D] *E. Coli* metabolic network ; [E] Yeast genetic network ; Nodes colored according to their YPD cellular roles [Zhu et al, 2007]

Biological Networks

- Systematic approaches to study large numbers of proteins, metabolites, and their modification have revealed complex molecular networks
- Significantly different from random networks and often exhibit ubiquitous properties in terms of their structure and organization
- They are actually dynamic, interacting, weighted hypergraphs. Weights exist at nodes and links. Weights can be numerical, logical, ODEs, rules, etc. (various annotations).
- Analyzing these networks provides novel insights in understanding basic mechanisms controlling normal cellular processes and disease pathologies
- Indispensable component of Systems Biology

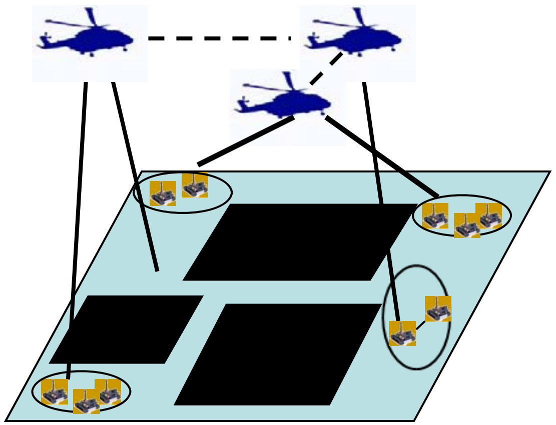
Networks and Networked Systems



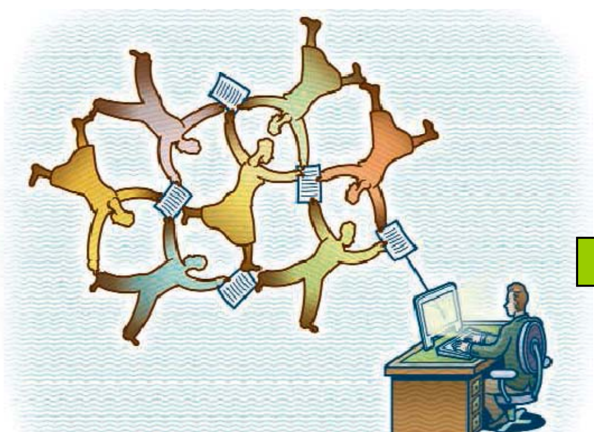
Internet backbone

(Lumeta Corp.)

Physical



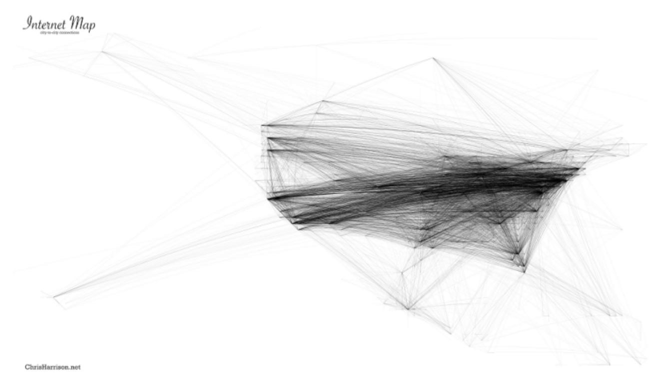
Vehicle, robot networks



Trust

(J Golbeck - Science, 2008)

Logical



Internet: North American cities

(Chris Harrison)

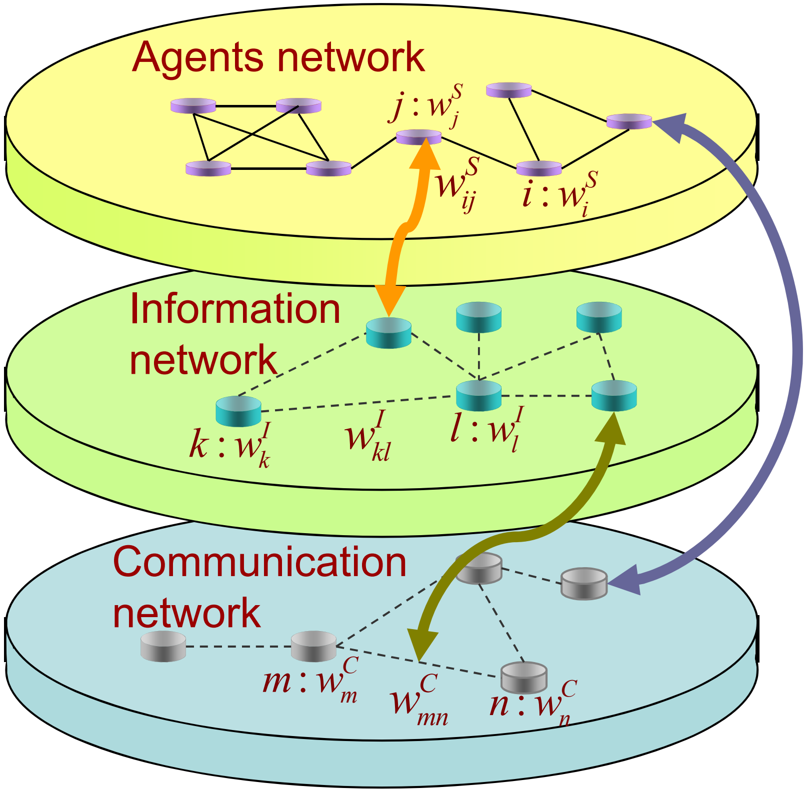
Outline

- Multiple interacting dynamic hypergraphs – three challenges
- Networks and Collaboration

  Constrained Coalitional Games
- Trust and Networks
- Topology Matters
- Conclusions and Future Directions

Multiple Interacting Dynamic Hypergraphs

- Multiple Interacting Graphs
  - *Nodes*: agents, individuals, groups, organizations
  - Directed graphs
  - *Links*: ties, relationships
  - Weights on links : value (strength, significance) of tie
  - Weights on nodes : importance of node (agent)
- **Value directed graphs with weighted nodes**
- **Real-life problems: Dynamic, time varying graphs, relations, weights, policies**



Networked System architecture & operation

Three Fundamental Challenges

- **Multiple interacting dynamic hypergraphs involved**
  - ***Collaboration hypergraph***: who has to collaborate with whom and when.
  - ***Communication hypergraph***: who has to communicate with whom and when
- **Effects of connectivity topologies**:
  Find graph topologies with favorable tradeoff between performance improvement (**benefit**) of collaborative behaviors *vs* **cost** of collaboration
  - **Small word graphs** achieve such **tradeoff**
  - **Two level algorithm** to provide efficient communication
- Need for **different probability models** – the classical Kolmogorov model is **not correct**
  - Probability models over logics and timed structures
  - Logic of projections in Hilbert spaces – not the Boolean of subsets

Outline

- Multiple interacting dynamic hypergraphs – three challenges
- Networks and Collaboration
  Constrained Coalitional Games
- Trust and Networks
- Topology Matters
- Conclusions and Future Directions

What is a Network …?

- **In several fields or contexts:**
  - social
  - economic
  - communication
  - sensor
  - biological
  - physics and materials

A Network is …

- A collection of nodes, agents, …

  that **collaborate** to accomplish actions, gains, …

  that cannot be accomplished with out such collaboration

- Most significant concept for **dynamic autonomic networks**

The Fundamental Trade-off

- The nodes **gain** from collaborating
- But collaboration has **costs** (e.g. **communications)**
- **Trade-off: gain from collaboration *vs* cost of collaboration**

  Vector metrics involved typically

  → **Constrained Coalitional Games**

- **Example 1**: **Network Formation** -- Effects on Topology
- **Example 2**: **Collaborative robotics, communications**
- **Example 3**: **Web-based social networks and services**
- **Example 4**: **Groups of cancer tumor or virus cells**

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Example: Autonomic Networks

- Autonomic: self-organized, distributed, unattended
  - Sensor networks
  - Mobile ad hoc networks
  - Ubiquitous computing
- Autonomic networks depend on **collaboration** between their nodes for all their functions
  - The nodes gain from collaboration: e.g. multihop routing
  - Collaboration introduces cost : e.g. energy consumption for packet forwarding

Example: Social Webs

- In August 2007, there were totally 330,000,000 unique visits to social web sites. (Source: Nielsen Online)
  - 9 sites with over 10,000,000 unique visits
  - MySpace, Facebook, Windows Live Spaces, Flickr, Classmates Online, Orkut, Yahoo! Groups, MSN Groups
- Main types of social networking services
  - directories of some categories: e.g. former classmates
  - means to connect with friends: usually with self-description pages
  - **recommender systems** linked to **trust/reputation**

Game Theoretic Approach

- The conflict between the benefit from collaboration and the required cost naturally leads to game-theoretic studies.
  - Nodes strategically decide the degree to which they volunteer their resources for the common good of the network.
  - Nodes attempt to maximize an objective function that takes the form of a payoff, which depends on the pattern of collaboration
- We study collaboration based on the notion of coalitions.
  - In coalitions, users connect to (join) each other, and are able to acquire access to each other.
  - The notion of coalitions can be well captured by coalitional game theory (*aka* cooperative game theory).

Coalitional Games

- The central concept is that of **coalition formation**: subsets of users that join their forces and decide to act together.
  - Players form coalitions to obtain the optimal payoffs
  - Players can negotiate collectively
  - The coalitional game model fits better to the practical scenarios, where agents naturally form coalitions, such as soldiers in the same group.
- Coalitional Games in **characteristic function form**
  - The coalitional game $G = \{N, v\}$, where $N = \{1, 2, \ldots, n\}$ is the set of all nodes
  - Characteristic function $v : 2^N \rightarrow R$, on all subsets S (coalitions) of N, represents the total payoff of a coalition

Network Model

- The communication structure of the network is represented as an **undirected** graph G .
  - Undirected links: the willingness of both nodes is necessary to establish and maintain a link.
  - In wireless networks, reliable transmissions require that two nodes interact to avoid collisions and interference.

- If i and j agree to collaborate with each other, the link $ij \in G$.
  - Add link ij to the existing graph g: $G + ij$;
  - Sever link ij from g : $G - ij$.

- A **coalition** of G is a subgraph $G' \subseteq G$, where $\forall i \in G'$ and $j \in G'$
  - there is a path in G' connecting i and j ;
  - $ij \in G$ implies $ij \in G'$.

Gain

- Users gain by joining a coalition.
  - **Wireless networks**
    - The benefit of nodes in wireless networks can be the rate of data flow they receive, which is a function of the received power

$$B_{ij} = f(P_j l(d_{ij}))$$

P_j is the power to generate the transmission and $l(d_{ij}) < 1$ is the loss factor

e.g: $B_{ij} = \log(1 + (P_j l(d_{ij}) / N_0))$

  - **Social connection** model (Jackson & Wolinsky 1996)

$$B_{ij} = \sum_{j \in g} V \delta^{r_{ij} - 1} \quad \text{or} \quad w_i(G)$$

- r_{ij} is # of hops in the shortest path between i and j
- $0 \leq \delta \leq 1$ *is the connection gain depreciation rate*

Cost

- Activating links is costly. $$c_i(G) = \sum_{j \in N_i^t} C_{ij}$$
  - **Wireless networks**
    - Energy consumption for sending data: $C_{ij} = RSd_{ij}^{\alpha}$

      RS depends on transmitter/receiver antenna gains and system loss not related to propagation

      α : path loss exponent
    - Data loss during transmission

      v_i is the environment noise and I_{ij} is the interference

      $$C_{ij} = h(v_i, I_{ij}) > 0$$
  - **Social connection model**
    - The more a node is **trusted**, the lower the cost to establish link

      e.g.suppose that the trust i has on j is s_{ij} (between 0 and 1), we can define the cost as the inverse of the trust values

      $$C_{ij} = 1 / s_{ij}$$

Pairwise Game and Convergence

- **Payoff** of node $\boldsymbol{i}$ from the network G is defined as

$$v_i(G) = \text{gain} - \text{cost} = w_i(G) - c_i(G)$$

- **Iterated process**
  - Node pair $\boldsymbol{ij}$ is selected with probability $\boldsymbol{p_{ij}}$
  - If link $\boldsymbol{ij}$ is already in the network, the decision is whether to sever it, and otherwise the decision is whether to activate the link
  - The nodes act **myopically**, activating the link if it makes each at least as well off and one strictly better off, and deleting the link if it makes either player better off
  - **End**: if after some time, no additional links are formed or severed
  - **With random mutations** , the game converges to a unique Pareto equilibrium (underlying Markov chain states)

Pairwise Game

- Pairwise game is modeled as an ***iterated process***
  - Individual nodes activate and delete links based on the improvement that the resulting network offers them relative to the current network
- A **strategy** of node *i* is a vector defined as

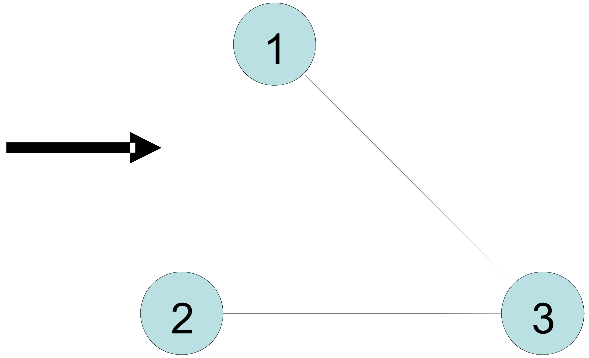
$$\gamma_i = (\gamma_{i,1}, \cdots, \gamma_{i,i-1}, \gamma_{i,i+1}, \cdots, \gamma_{i,n}).$$

  - $\gamma_{i,j} = 1$ (or 0) : node ***i*** wants (or does not) to form a link with node ***j***
  - A link *ij* is formed only if $\gamma_{i,j} = 1$ and $\gamma_{j,i} = 1.$
- A **strategy profile** $\gamma^{(t)} = (\gamma_1^{(t)}, \cdots, \gamma_n^{(t)})$ at time period *t* corresponds to the network $G^{(t)}$ at time *t*.

$\gamma_1 = \{0,1\}$

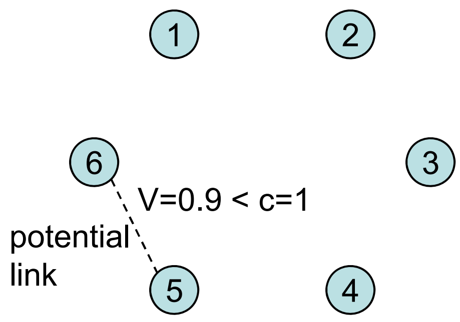
$\gamma_2 = \{0,1\}$

$\gamma_3 = \{1, 1\}$

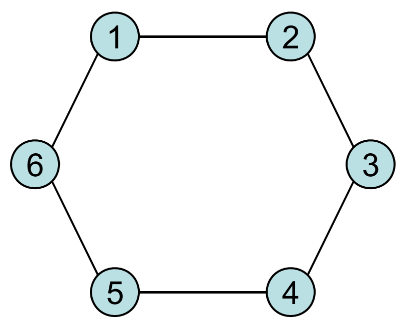


Convergence of the Iterated Pairwise Game

- **Pairwise stability**
  - No more link is added and no existing link is deleted
- Lemma: the iterated pairwise game converges to a pairwise stable network or a cycle of networks.
  - The converging pairwise stable network may be **inefficient**



pairwise stable



pairwise stable and better payoff

Cost between neighboring nodes is 1

$V = 0.9, \delta = 0.3$

  - Random mutations are introduced, the game converges to a unique Pareto equilibrium (Markov chain states strategy profiles γ)

    Intent of players is carried out with probability 1- ε

Stochastic Stability

- Dynamic process is now a finite state, aperiodic, irreducible Markov chain (graph process)-- steady-state distribution, $\Pi(g, \varepsilon)$.
- A network *g* is **stochastically stable** if $\Pi(g, \varepsilon)$ is bounded below as the error rate, ε, tends to zero;

  $\Pi(g, \varepsilon) \rightarrow a > 0$, as $\varepsilon \rightarrow 0$.

  - Stochastically stable networks must be pairwise stable networks or networks of closed cycles
  - Stochastic stability identifies the most “robust” or easy to reach networks in a particular sense (the most mutations needed to get “unstuck”).
  - The above example converges to a Pareto efficient pairwise stable network by considering all the possible dynamic paths between the left and right networks.

Network Formation Dynamics

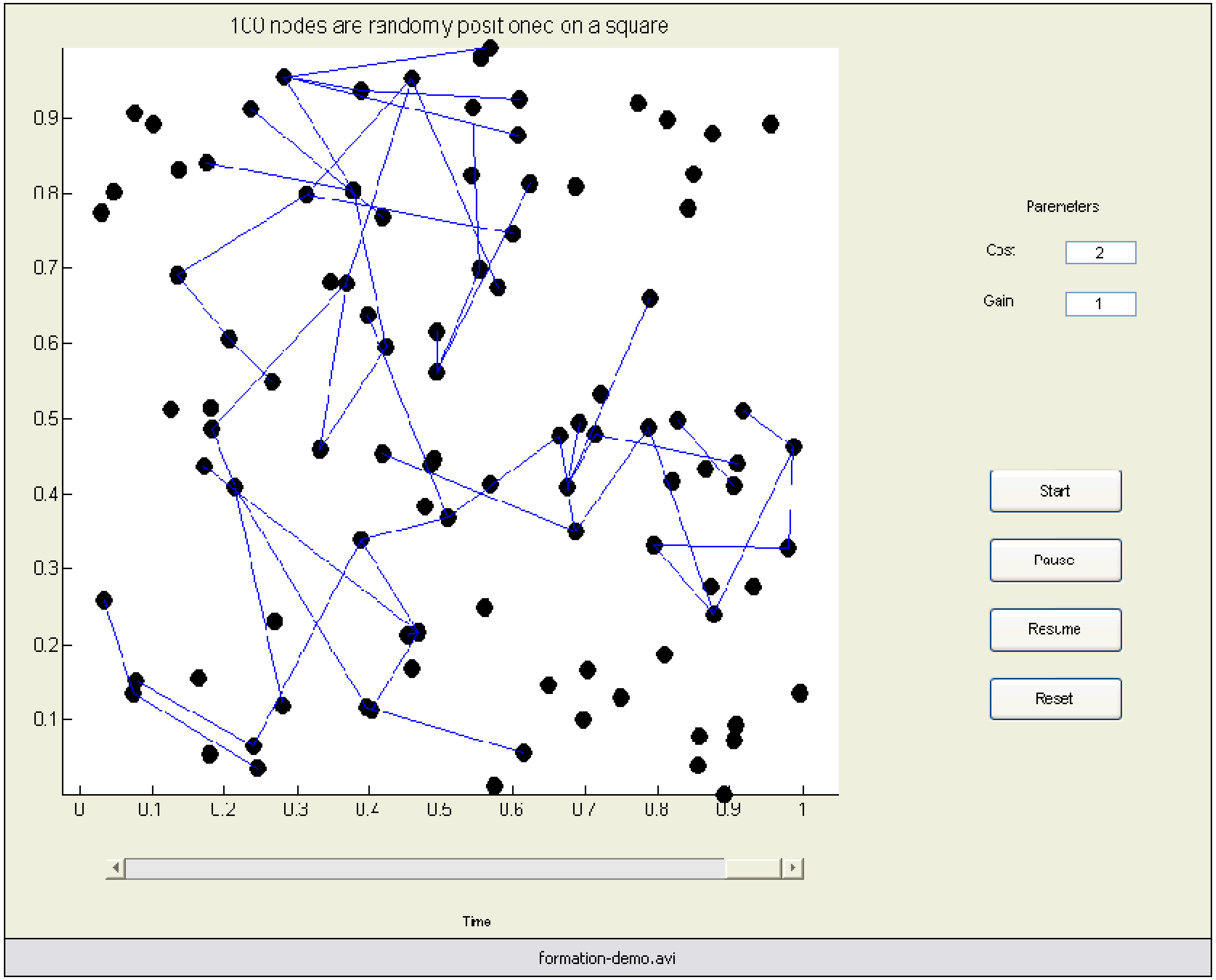
Parameter Values:

$\delta = 0.2$

$\alpha = 2$

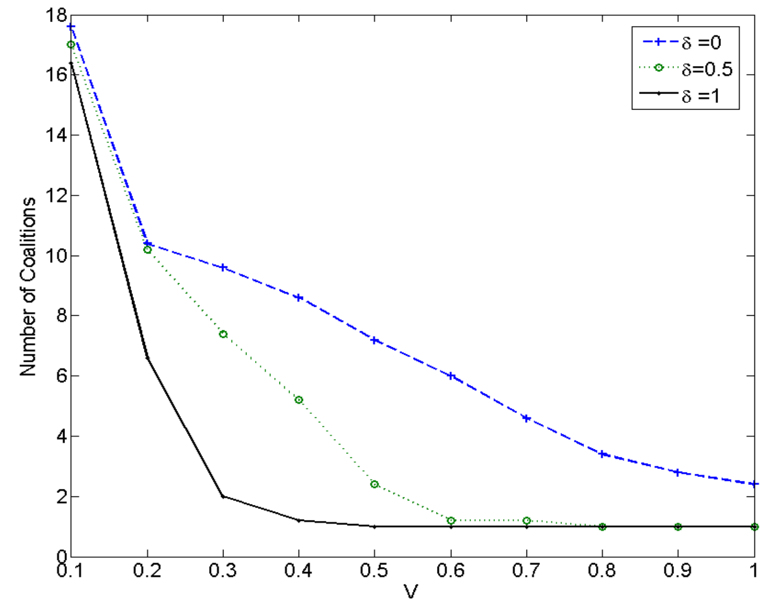
$V = 1$

$P = 2$



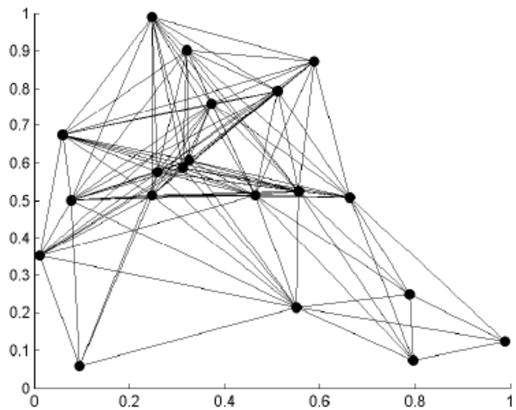
Coalition Formation at the Stable State

- The cost depends on the physical locations of nodes
  - Random network where nodes are placed according to a uniform Poisson point process on the [0,1] x [0,1] square.
- Theorem: The coalition formation at the stable state for n→∞
  - Given $\delta = 0, \quad V = P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$ is a sharp threshold for establishing the grand coalition (number of coalitions = 1).
  - For $0 < \delta \leq 1$, the threshold is less than $P\left(\frac{\ln n}{n}\right)^{\frac{\alpha}{2}}$.

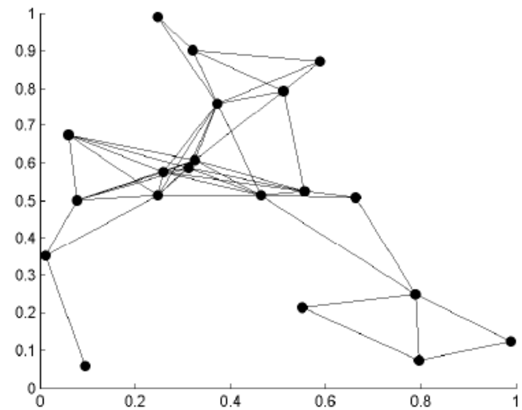


n = 20

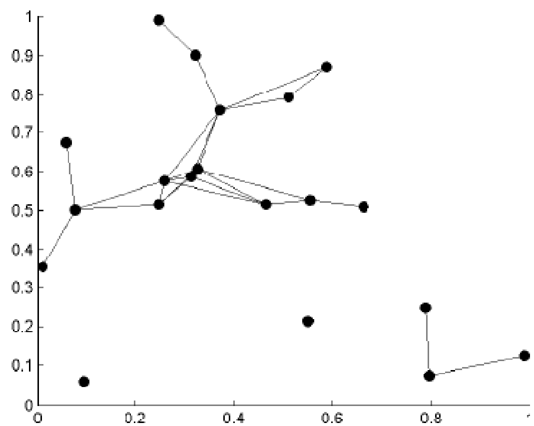
Topologies Formed



(a) P = 0.5 (low cost); complete graph



(b) P = 2 (middle cost); small world topology



(c) P = 4 (high cost); partitioned network

Stability of Coalitions

- **Core stability**
  - A network G is core stable if there is no subset of nodes S who prefer another network $\hat{G}$ to G and who can change the network from G to $\hat{G}$ without the cooperation from the rest of the set of nodes $N \setminus S$.

$$x_i(\hat{G}) \geq x_i(G) \text{ for all } i \in S \text{ and there is at least one strict inequality}$$

$$\text{If } ij \in \hat{G} \text{ but } ij \notin G, \text{ then } i, j \in S$$

$$\text{If } ij \notin \hat{G} \text{ but } ij \in G, \text{ then } i \in S \text{ and/or } j \in S$$

  - Core stability allows that a node is able to interact and coordinate with any other nodes in the same coalition.

- Core stability is stronger than pairwise stability.

Formation Topology

- Conditions under which the formation game converges to a network with small-world properties
- Network model:
  - All nodes are **equally** placed on a circle
  - **Benefit**: $B_{ij} = \sum_{j \in g} V \delta^{r_{ij}-1}$
  - **Cost**: C_{d_r} (cost of establishing a link between two nodes that are r hops away)
- Formation process
  - Initial network where nodes only connecting to their immediately neighbors, i.e.,

$$C_{d_1} < B < \frac{C_{d_2}}{1-\delta^{\left\lfloor \frac{n}{2} \right\rfloor -1}}$$

  - Direct connections between nodes that are at least r hops away on the circle if

$$B > \frac{C_{d_r}}{1-\delta^{\left\lfloor \frac{n}{2} \right\rfloor -r+1}}$$

Formation Topology (cont.)

- We investigate the effect of shortcuts following the perturbation approach to small worlds proposed by Higham (Higham, 2003)
  - ε represents the probability that a shortcut is added to the initial network.
  - Assume the shortcuts are added to nodes that are at most r hops away on the circle

- Proposition: Let $r\varepsilon = K/n^{\beta}$, where $K > 0$ and $\beta \geq 0$. For $\beta > 2$, the effect of shortcuts on convergence rate is negligible. $\beta = 2$ is the threshold. For $\beta < 2$, the shortcuts are dominantly decreasing the SLEM, thus the small-world topology appears.

- Given that $B > \dfrac{C_{d_r}}{1-\delta^{\left\lfloor \frac{n}{2} \right\rfloor - r}}$, small-world topology appears if more than K/n shortcuts are added.

Time-dependent Game

- The game is time-dependent
  - The payoff players receive **varies over time**.
  - The dynamics of the game can be separated in rounds of successive coalition expansions (or contractions).

- The dynamic coalition formation process is described as an iterated game
  - x_i^t : the action *i* chooses at time *t*.
  - $v_i\left(x^t\right)$: the payoff of user *i* at time *t*.
  - $q^t(x)$: players' probability of playing action *x* at time *t*.
  - C_i^t : the set of users that form the coalition user *i* belongs to at time *t*.
  - user *i* and user *j* decide to activate link *ij* at time *t:*

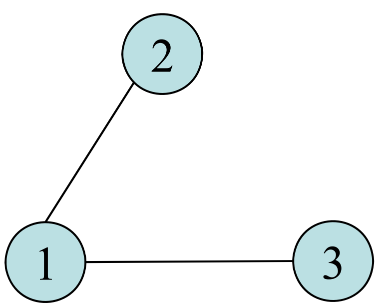
$$C_i^t = C_j^t = C_i^{t-1} \cup C_j^{t-1}$$

Value Function

- Value function for coalition C (component-wise additive value function)

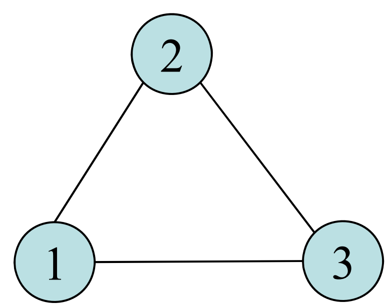
$$v(C) = \sum_{i \in C} v_i(g)$$

- Value function depends on topology



Same coalition C={1,2,3} with different topology

$v(\{12,13\}) \neq v(\{12,23,13\})$



Stability of Game Dynamics

- **Nash equilibrium**
  - Player action probability q is a Nash equilibrium if no player i can deviate from q and achieve a higher payoff

$$\forall i, \sum_{x_{-i} \in X_{-i}} v_i(x'_i, x_{-i}) \prod_{j \neq i} q_j(x_j) \leq \sum_{x \in X} v_i(x) \prod_j q_j(x_j).$$

- **Core stability**
  - A network g is **core stable** if there is no subset of nodes S who prefer another network g' to g and who can change the network from g to **without the cooperation** from the rest of the set of nodes $N \backslash S$.

Learning via Regret

- Question:
  - Are there simple strategies that lead our coalition formation game to equilibrium?
- **Solution**:
  - Players stochastically adjust their strategies by a reinforcement learning rule guided by “**regret**”.
- **Learning strategy**:
  - At each period, a player may either choose to continue playing the same action as in the previous period, or switch to other actions with probabilities that are **proportional to how much higher his accumulated payoff would have been if he had always made that change in the past.**

Regret Matching Strategy

- Average payoff through time *t* for user *i*

$$\overline{v}_i^t = \frac{1}{t} \sum_{1 \le s \le t} v_i(x_i^t, x_{-i}^t).$$

- Average **regret** from not having played *x'*i

$$\overline{r}_{i,x'_i}^t == \frac{1}{t} \sum_{1 \le s \le t} v_i(x'_i, x_{-i}^t) - \overline{v}_i^t.$$

- **Regret matching strategy**
  - at each time period *t* + 1, the player *i* plays either action activate or not activate with a probability proportional to the nonnegative part of his regret up to time *t*

$$q_i^{t+1}(x_i) = \frac{(\overline{r}_{i,x_i}^t)^+}{\sum_{x'_i} (\overline{r}_{i,x'_i}^t)^+}.$$

Results on Stability

- Nash equilibrium
  - $\phi^t(x)$: the proportion of time up to t that each action-tuple x was played.
  - ϕ : the empirical distribution.
  - Results: Given that all players use the **regret matching strategy**, the empirical distribution converges almost surely to the set of Nash equilibria.

- Core existence
  - If $\forall i, j,\ v_{ij} + v_{ji} \geq 0$, the core of the formation game is **nonempty**

Simple Case Study

- ***Gain***: users benefit from connecting to as many other users as possible, directly (one-hop) or indirectly (multi-hop, through other users)

$$b_i(g) = |C^t(i) - 1|$$

- ***Cost***: random variable with an exponential probability distribution with parameter λ.
- Result 1: All coalitions formed at the Nash equilibrium are trees.
- **Result 2:** The probability that the game has nonempty core is greater than or equal to

$$(1 - e^{-4\lambda})^{N(N-1)}.$$

Coalitions, Networks and Constraints

- **Cooperative Game** in **characteristic function form** $\Gamma = \{\mathcal{N}, v\}$, $\mathcal{N} = \{1, 2, \ldots, N\}$, $v : 2^N \rightarrow \mathbf{R}$, on all subsets $\mathcal{S}$ (***coalitions***) of $\mathcal{N}$
- **All coalitions cannot be formed**
- **To collaborate agents need to communicate**
- **Communication Network (*N*, *L*)**
  - **Edges – links between payers**
  - ***i* and *j* directly connected**
  - ***i* and *j* path connected**
- **Cooperation components**
- **Links between players in $\mathcal{S}$, L($\mathcal{S}$)**
- **Network ($\mathcal{S}$, L($\mathcal{S}$)) induces a partition of $\mathcal{S}$**

Constrained Coalitional Games

- **Network-restricted cooperation game** or **constrained coalition** $\{\mathcal{N}, v^L\}$
- $\{\mathcal{N}, v, L\}$ communication situation

- Characteristic function

$$v^L(S) = \sum_{C \in S/L} v(C) \qquad \text{for each} \quad S \subseteq \mathcal{N}$$

- **Myerson value** : Shapley value of $\{\mathcal{N}, v^L\}$

Network Formation

- Form links pairwise
- Iterative game
- Better understanding of topologies – dynamics – topology control
- **Network formation with costs** for establishing links
- $\{\mathcal{N}, v, L, c\}$ $\{\mathcal{N}, v^{L,c}\}$

$$v^{L,c}(S) = \sum_{C \in S/L} v(C) - c\,|L(S)| \qquad \text{for each} \quad S \subseteq \mathcal{N}$$

- ***Stability*** vs ***efficiency*** of the resulting network
- Small world graphs, expander graphs …

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  Constrained Coalitional Games
- Trust and Networks
- Topology Matters
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Networks and Trust

- **Trust and reputation critical for collaboration**
- Characteristics of trust relations:
  - *Integrative* (Parsons1937) – main source of social order
  - *Reduction of complexity* – without it bureaucracy and transaction complexity increases (Luhmann 1988)
  - *Trust as a lubricant for cooperation* (Arrow 1974) – rational choice theory
- **Social Webs, Economic Webs**
  - MySpace, Facebook, Windows Live Spaces, Flickr, Classmates Online, Orkut, Yahoo! Groups, MSN Groups
  - e-commerce, e-XYZ, services and service composition
  - **Reputation** and **recommender** systems

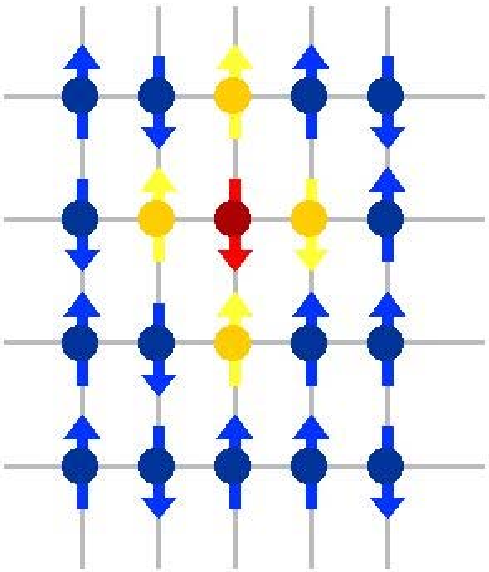
Ising and Spin Glass Models

- Statistical Physics models for magnetization

  Orientation of each particle's spin depends on its **neighbors**

  Ising Model: behavior of simple magnets

  Spin Glass Model: complex materials



- Interpretation:

  s = $\{s_1, s_2, \ldots, s_n\}$ is a configuration of n particle spins -- s_j = 1 or -1 (up or down)

  Energy for configuration ***s***

$$H(\mathbf{s}) = -\frac{1}{T}\sum_{\substack{i\in V \\ j\in N_i}} J_{ij}s_is_j - \frac{mH}{T}\sum_i s_i$$

  – **Ising Model**: $J_{ij} = J$ for all i, j

  – **Spin Glass Model**: J_{ij} depend on i,j and can be **random**

Ising/SG Models and Games

- Ising/SG models can be interpreted as dynamic (repeated) games:
  - The value of s_i represents whether node i is willing to cooperate or not
  - each particle selects spin to **maximize** its own *payoff*

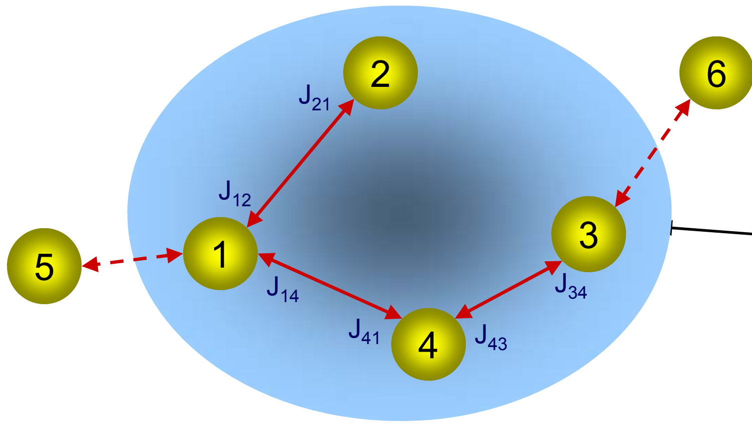
$$\pi_i = \Big(\sum_{j \in N_i} J_{ij} s_i s_j\Big) / T$$

  - Ising model ($J_{ij} = J > 0$) : align its spin with the majority of neighbors spin
    - High T, conservative agents, not willing to change, small payoffs
    - Low T, aggressive agents, larger payoffs
  - Collection of local decisions reduces the total energy of the interacting particles

- Inspires an approach where **trust** is an **incentive for cooperation**
  - J_{ij} can be interpreted as the **worth of player *j* to player *i***
  - decide to cooperate or not based on benefit from cooperation **and** trust values of neighbors

Spin Glass Cooperative Game

- Spin glass model as a cooperative game (***spin glass game***)
  - ***S*** $\subseteq$ ***N*** = {1, 2, …, *n*} is a ***coalition***, in which all nodes cooperate
  - *Interaction topology* (J_{ij}'s) *moderates* effects pos. and neg. feedback
  - *v*(***S***) **:** value of the *characteristic function of the game* , *v*: $2^N \rightarrow R$, which is the maximum payoff ***S*** can get without cooperation from other nodes ***N*** /***S***.



$$v(S) = \sum_{i \in S} \pi_i = \sum_{i,j \in S} J_{ij} - \sum_{i \in S, j \notin S} J_{ij}$$

Subset *S* = {1,2,3,4}

$v(S) = J_{12}+J_{21}+J_{14}+J_{41}+J_{43}+J_{34} - J_{36} - J_{15}$

  - The cooperative game is denoted as $\Gamma = (N, v)$
- **Object**: to find **what form or policy** for J_{ij} can induce all (or most) nodes to cooperate: **maximize the coalition**

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Spin Glass Cooperative Game Properties

- Spin Glass game is a convex and superadditive game iff (net pos. effects)

$$\forall\, i, j,\ J_{ij} + J_{ji} \geq 0$$

- ***Shapley value*** : $\varphi(v)_i = \sum_{j \in \mathcal{N}} J_{ij}$ in the core
- Not well understood in the regime of both negative and positive net effects
- Effects of interaction matrix structure (sparsity, neighborhood structure, range of interactions, strength of interactions) not well understood; Topology effects in network analog
- Oriented Spin Glass Game $\Gamma(\mathcal{N},v)$ where v now depends on both an **interaction matrix** $\boldsymbol{J}$ and a **preference vector** $\boldsymbol{L}$; a pair of char. fcns

$$v_{\pm}(S) = \sum_{i,j \in S} J_{ij} - \sum_{i \in S, j \notin S} J_{ij} \pm \sum_{i \in S} L_i$$

- **Replica method** can be used to analyze various problems under various models and constraints on J and L

Cooperative Games with Negotiation

- **Theorem**: $\Gamma = (\mathcal{N}, v)$ has a nonempty core if $J_{ij} + J_{ji} \geq 0, \forall i, j$. The payoff allocation to node i, $x_i = \sum_{j \in N_i} x_{ij}$, where x_{ij} is computed as follows

$$x_{ij} = \begin{cases} J_{ij}, & \text{if } J_{ij} \geq 0, J_{ji} \geq 0 \\ J_{ij} + \lambda_{ji} J_{ji}, & \text{if } J_{ij} \leq 0, J_{ji} > 0 \\ (1 - \lambda_{ij}) J_{ij}, & \text{if } J_{ij} > 0, J_{ji} \leq 0 \end{cases}$$

with $0 \leq \lambda_{ij}, \lambda_{ji} \leq 1$

is a solution in the core.

  - This payoff allocation indicates a way to **encourage cooperation**
  - **Players with positive gain can negotiate with their neighbors by sacrificing certain gain** (offering their partial gain $\lambda_{ij} x_{ij}$)

Trust as Mechanism to Induce Collaboration

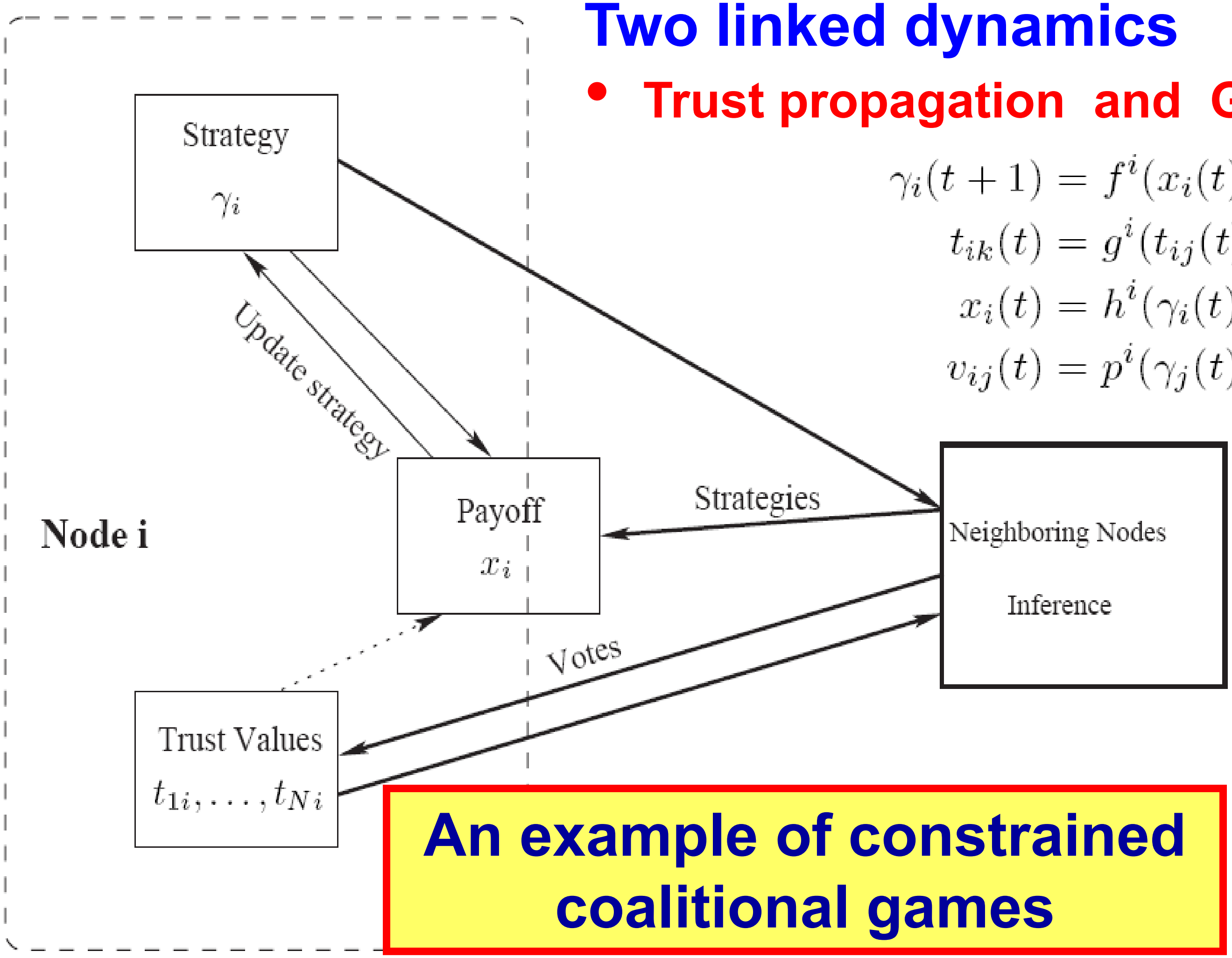
- Trust **is an incentive** for collaboration
  - Nodes who refrain from cooperation get lower trust values
  - Eventually penalized because **other nodes tend to only cooperate with highly trusted ones**.
- For node i **loss for not cooperating** with node j is a nondecreasing function of J_{ji}, $f(J_{ji})$,
- New characteristic function is

$$\nu(\mathcal{S}) = \sum_{i,j\in\mathcal{S}} J_{ij} - \sum_{i\in\mathcal{S},j\notin\mathcal{S}} f(J_{ij})$$

- **Theorem** : if $\forall i,j,\ J_{ij} + f(J_{ji}) \geq 0$, the core is nonempty and $x_i = \sum_{j\in N_i} J_{ij}$ is a feasible payoff allocation in the core.

By introducing a trust mechanism, all nodes are induced to collaborate without any negotiation

Dynamic Coalition Formation



Two linked dynamics

- **Trust propagation and Game evolution**

$$\gamma_i(t+1) = f^i(x_i(t), \gamma_i(t), \gamma_j(t), t_{ij}(t))$$
$$t_{ik}(t) = g^i(t_{ij}(t), v_{jk}(t)) \quad \forall k \in N$$
$$x_i(t) = h^i(\gamma_i(t), \gamma_j(t))$$
$$v_{ij}(t) = p^i(\gamma_j(t), t_{ji}(t))$$

An example of constrained coalitional games

Stability of dynamic coalition

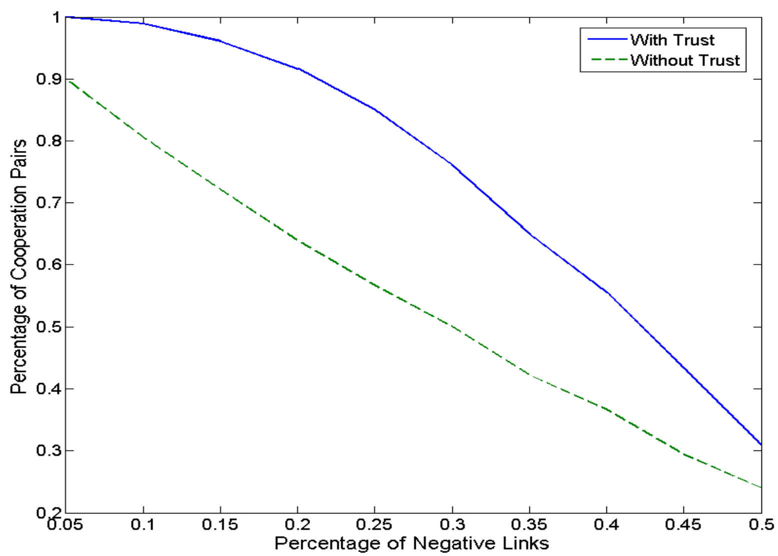
Nash equilibrium

Game Evolution

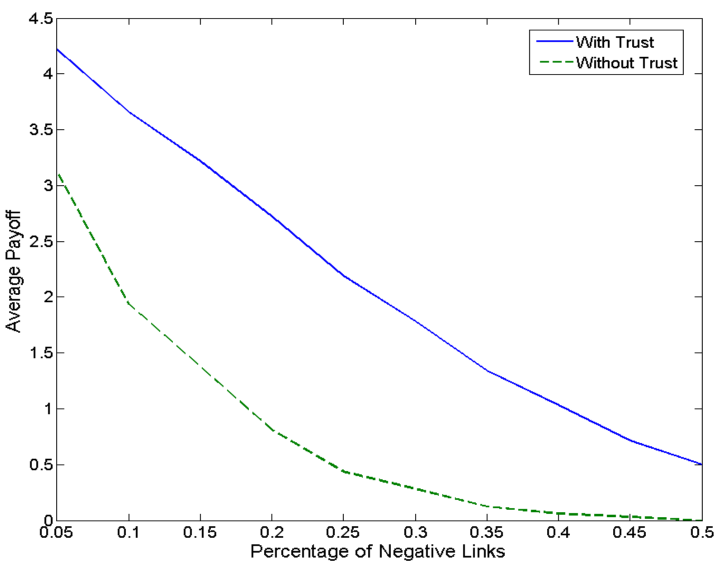
- Strategy of node *i*: $s_{ij} \in \{-1, 1\}, \forall j \in N_i$
  - s_{ij}= 1 (= -1) represents that *i* **cooperates** (does not cooperate) with its neighbor *j*
- Payoff for node *i* when interacting with *j* : $x_{ij} = J_{ij} s_{ij} s_{ji}$
  - $x_{ij} > 0$ (< 0) **positive** link (negative link)
  - **Node selfishness** → cooperate with neighbors on **positive** links
- Strategy updates: node *i* chooses s_{ij}= 1 only if all of the following are satisfied:
  - Neighbor *j* is trusted
  - $x_{ij} > 0$, or the cumulative payoff of *i* is less than the case when it unconditionally conducts s_{ij}= 1.
- Trust evaluation:
  - The deterministic voting rule
  - **Reestablishing period** τ : once a node is not trusted, in order to reestablish trust it has to cooperate for τ consecutive time steps

Results of Game Evolution

- **Theorem**: $\forall i \in N_i$ and $x_i = \sum_{j \in N_i} J_{ij}$, there exists τ_0, such that for a reestablishing period $\tau > \tau_0$
  - terated game converges to Nash equilibrium;
  - In the Nash equilibrium, all nodes cooperate with all their neighbors.
- Compare games with (without) trust mechanism, strategy update:



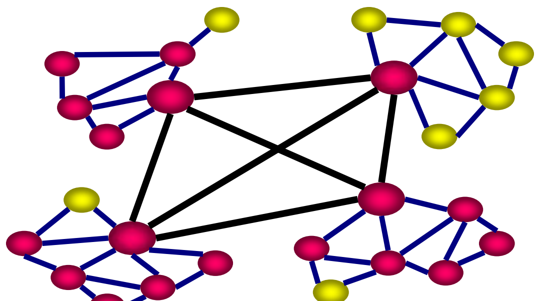
Percentage of cooperating pairs *vs* negative links



Average payoffs *vs* negative links

Next Generation Trust Analytics

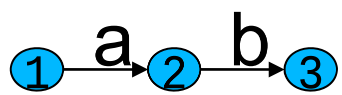
- Trust evaluation, trust and mistrust dynamics
  - Spin glasses (from **statistical physics**), phase transitions



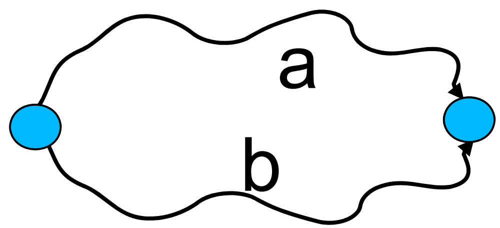
$$s_i(k+1) = f\left(\hat{J}_{ji}, s_j(k) \mid j \in N_i\right)$$

- **Indirect** trust; reputations, profiles; Trust computation via 'linear' **iterations in ordered semirings**

$$a \otimes b \leq a, b$$



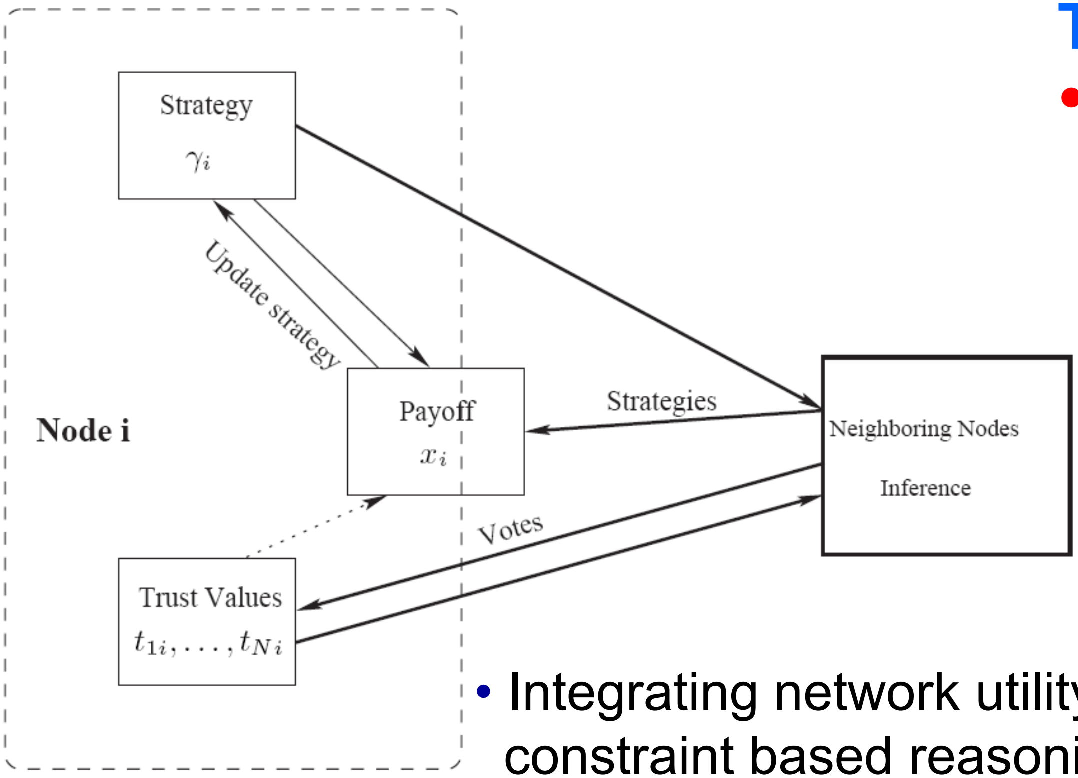
$$a \oplus b \geq a, b$$



2007 IEEE Leonard Abraham prize
***New Book* "Path Problems in Networks" 2010**

- **Direct trust: Iterated pairwise games on graphs** with players of many types

Constrained Coalitional Games: Trust and Collaboration



Two linked dynamics

- **Trust / Reputation propagation and Game evolution**

- Integrating network utility maximization (NUM) with constraint based reasoning and coalitional games

- Beyond linear algebra and weights, semirings of constraints, constraint programming, soft constraints semirings, policies, agents
- Learning on graphs and network dynamic games: behavior, adversaries
- Adversarial models, attacks, constrained shortest paths, …

Outline

- Multiple interacting dynamic hypergraphs – three challenges
- Networks and Collaboration

  Constrained Coalitional Games
- Trust and Networks
- Topology Matters
- Conclusions and Future Directions

Networks: Different Linked Views

Networks:

- as **distributed, asynchronous, feedback (many loops), hybrid automata (dynamical systems)**
- as **distributed asynchronous active databases** and **knowledge bases**
- as **distributed asynchronous computers**

Distributed Algorithms in Networked Systems and Topologies

- Distributed algorithms are essential
  - Group of agents with certain abilities
  - Agents **communicate with neighbors,** share/process information
  - Agents **perform local** actions
  - **Emergence** of global behaviors
- **Effectiveness** of distributed algorithms
  - The **speed** of convergence
  - **Robustness** to agent/connection failures
  - Energy/ communication **efficiency**
- **Group topology affects** group performance
- **Design problem**:

  Find graph topologies with favorable tradeoff between performance improvement (**benefit**) *vs* **cost** of collaboration
- **Example: Small Word graphs** in consensus problems

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The Importance of Being Well-Connected

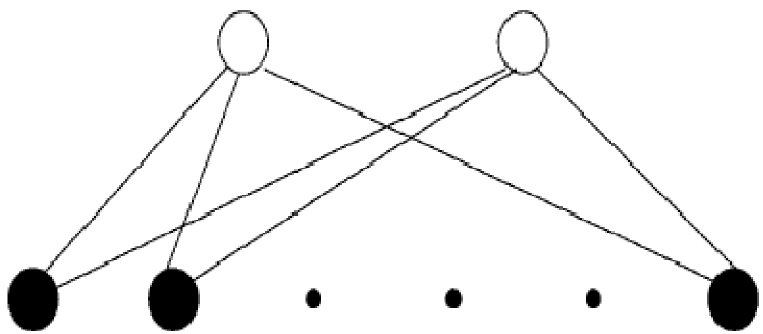
University of Maryland 1856

- **Local majority voting** (Peleg ’96)
  - Each of *n* citizens has an opinion about voting **Yes** or **No**
  - **Rule**: Each citizen’s vote is based on the **majority** of its neighbors, including itself
  - **What is the minimum number of No-voters that can guarantee a No result**?
  - A few number of well connected nodes can determine the outcome of the process!

The Importance of Being Well-Connected (cont.)

White circles: **NO** voters

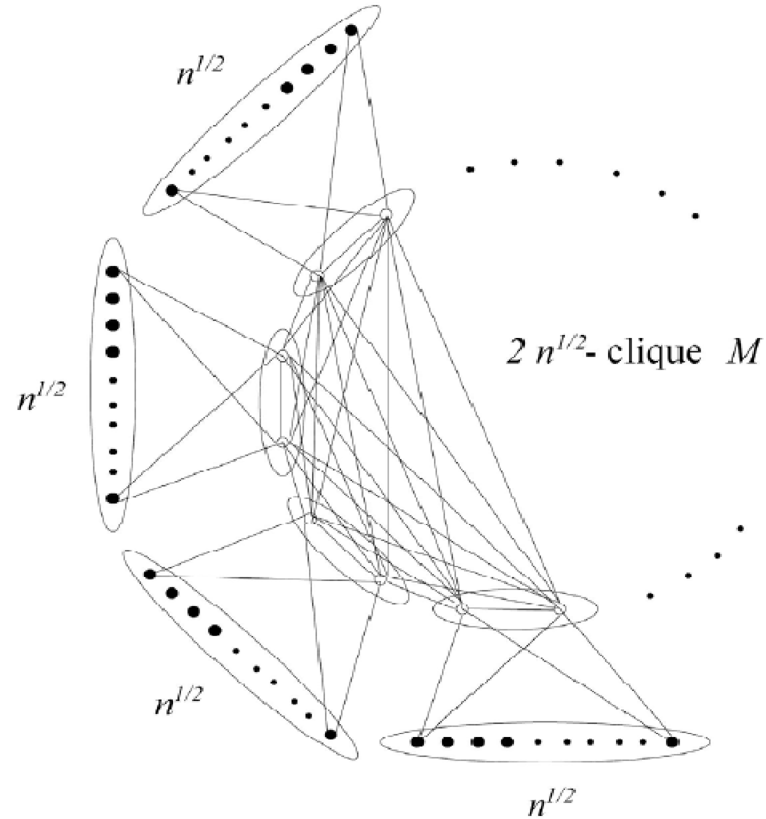
Black circles: **YES** voters



Order of voting matters!

Iterative polling : Oscillation or

If NO voters do not follow the protocol, then 2 NO voters, are sufficient to change the other n-2 YES voters' opinion.



Even if **NO** voters follow the protocol a small minority of $2\sqrt{n}$ can result in one step convergence to **NO**

Consensus problems

- A Simple model:

$$\theta_i(t+1) = f_{ii}(t)\theta_i(t) + \sum_{j \in N(i)} f_{ij}(t)\theta_j(t)$$

$$\forall i \in \{1, ..., n\} : \qquad \sum_j f_{ij} = 1$$

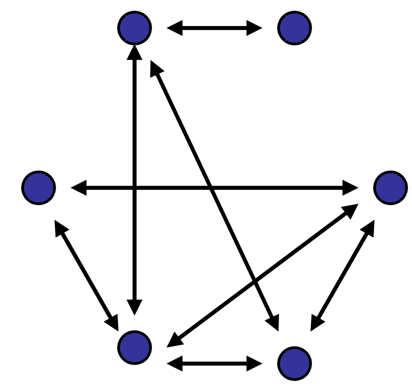
$$\forall i, j \in \{1, 2, ..., n\} : \qquad f_{ij} \geq 0$$

$$\forall i \in \{1, ..., n\} : \qquad f_{ii} \geq \alpha > 0$$

Vicsek's model
(Vicsek et al., Jadbabaie et al.)

- A flock of *n* agents moving at the same speed *s*, but with different headings
- Each agent updates its heading angle as an average of its neighbors including itself
- *D* is diagonal matrix of nodes' degrees
- *A* is adjacency matrix

$$G = (V, E)$$



$$\theta_i(t+1) = <\theta_i(t)> = \frac{1}{1+n_i(t)}[\theta_i(t) + \sum_{j\in N_i(t)} \theta_j(t)]$$

$$\theta(t+1) = F_{\sigma(t)}\theta(t)$$

$$\mathbb{G} = \{G_0, \dots G_{M-1}\}$$

$$\mathbb{F} = \{F_0, \dots F_{M-1}\}$$

$$F_p = (I + D_p)^{-1}(A_p + I)$$

$$\mathbb{M} = \{0, \dots, M-1\}$$

$$\sigma : \mathbb{N}\bigcup\{0\} \rightarrow \mathbb{M}$$

Design of information flow

$$x(k+1) = F(k)x(k)$$

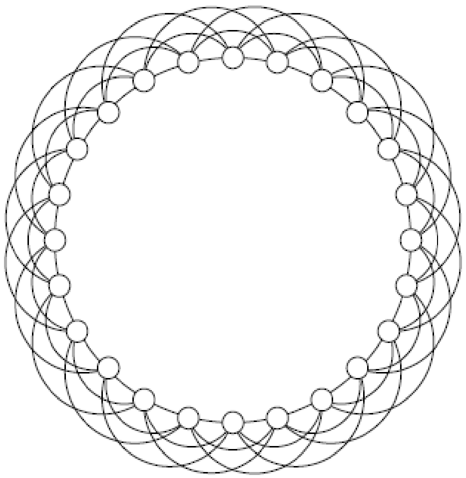
$$F(k) = (I + D(k))^{-1}(A(k) + I)$$

$$F(k) = I - hL(k)$$

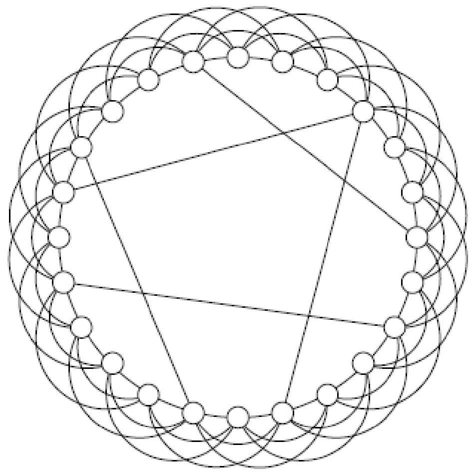
Symmetric communication

- **Fixed graphs**: Geometric convergence with rate equal to Second Largest Eigenvalue Modulus (SLEM)
- How does **graph topology affect** location of eigenvalues?
- How can we **design graph topologies** which result in good convergence speed?

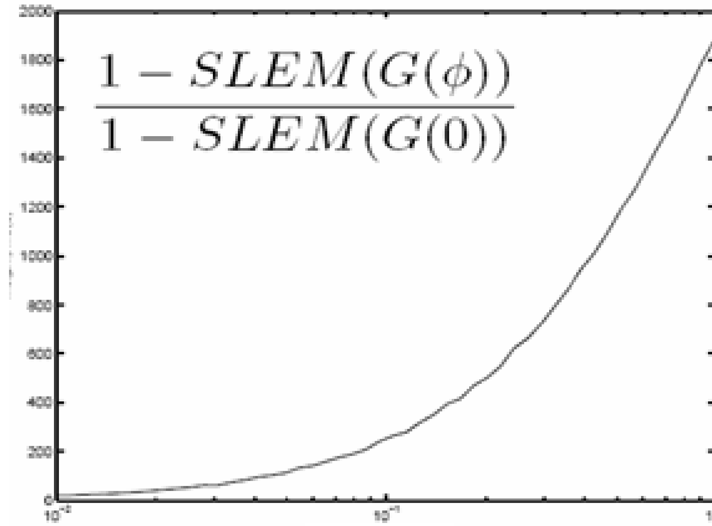
Small World Graphs



Simple Lattice
C(n,k)



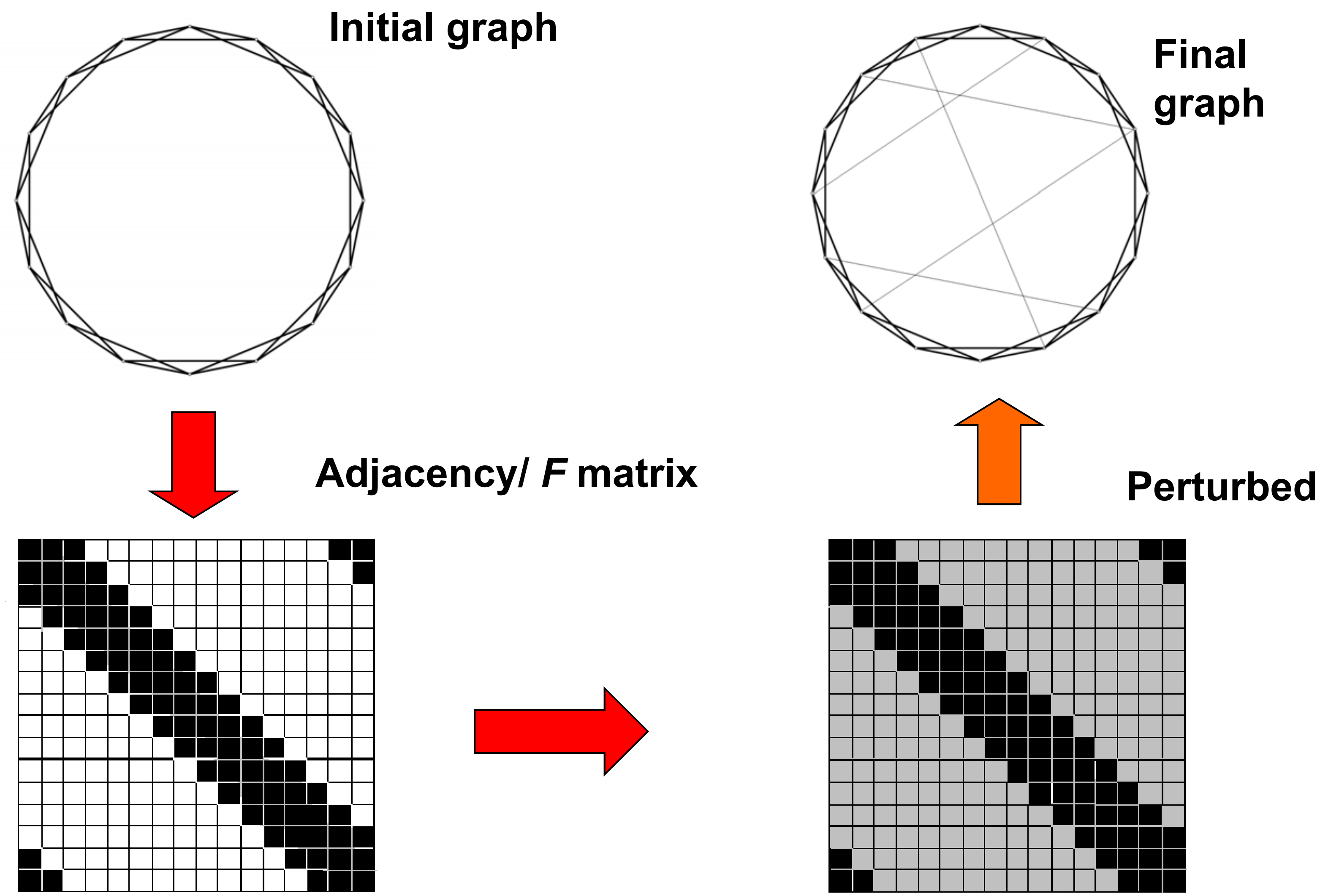
Small world: Slight variation adding $nk\Phi$



\phi
C(1000,5)

Adding a **small portion** of well-chosen links → **significant increase** in convergence rate

Mean Field Explanation and Perturbation Approach



Watts-Strogatz Small World networks

- Random graph approach
  (e.g. Durrett 2007, Tahbaz and Jadbabaie 2007)
- **Perturbation** approach (Higham 2003)
  - Start from lattice structure $G_0=C(n,k)$ $\longleftrightarrow$ F_0
  - Perturb zero elements in the positive direction by $\varepsilon = \frac{K}{n^{\alpha}}$ for fixed $K > 0$ and $\alpha > 1$.
  - Perturb the formerly nonzero elements equally, such that the stochastic structure of the F matrix is preserved F_{ε}
  - Analyze the SLEM as a function of the perturbation as α varies

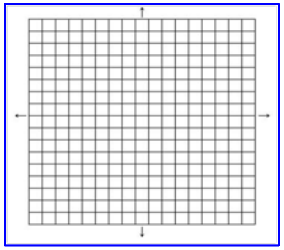
1- D case …

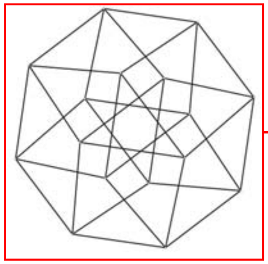
- Refer to the perturbations as **ε-shortcuts**
- In the limit of large *n* :
  - For $\alpha > 3$ the effect of ε-shortcuts on convergence rate is negligible
  - For $\alpha = 3$ the effect of ε-shortcuts on convergence rate starts (spectral gap gain perturbation of same order)
  - **For $\alpha = 2$ the shortcuts dominantly decrease SLEM**
  - For $\alpha = 1$ SLEM is very small
- ε-shortcuts are loosely analogous to the shortcuts in Small World networks

- ***α* = 3** can be considered as the onset of small world effect with **small world effect happening** at ***α* = 2**

Analysis of W-S model

- A graph is small-worldizable if $\frac{SLEM(F)}{1-SLEM(F)} >> \frac{1}{n\varepsilon}$.
- For the ring type structure, in the limit of large *n*:
  – For $\alpha > 3$ the effect of ε-shortcuts is negligible
  – For $\alpha = 3$ the effect of ε-shortcuts starts (spectral gap gain perturbation of same order)
  – For $\alpha = 2$ the shortcuts dominantly decrease SLEM
- *α=3* onset of small world effect; small world effect happening at *α=2.*





$1-\mu(F_0)$	Onset of SW	SW dominant
$O(n^{-1})$	$\varepsilon = O(n^{-2})$	$\varepsilon = O(\frac{1}{n\log n})$
$O\left((\log n)^{-1}\right)$	$\varepsilon = O(\frac{1}{n\log n})$	$\varepsilon = O(\frac{1}{n\log\log n})$

Distributed exploration of the graph structure

- **Self-organization for better performance and resiliency**
- Hierarchical scheme to design a network structure capable of running **distributed algorithms with high convergence speed**
- A two stage algorithm:

  1- Find the most effective choice of **local leaders**

  2- Provide nodes with information about their location **with respect to other nodes and leaders** and the choice of groups to form
- Divide *N* agents into *K* groups with *M* members each

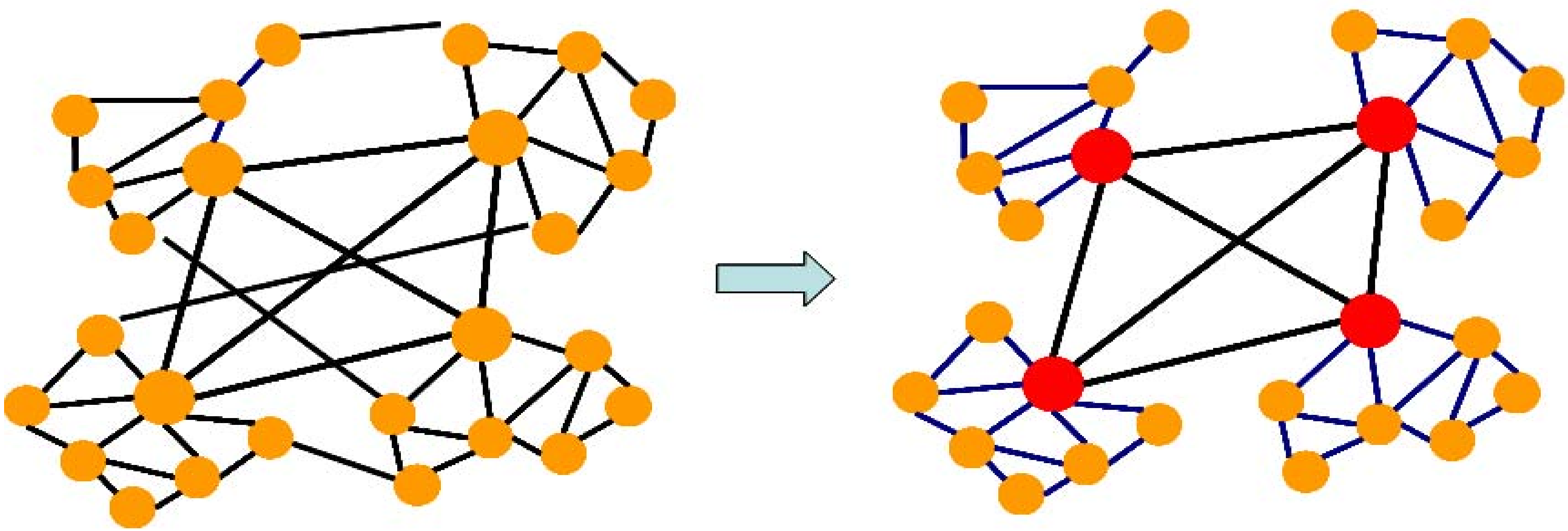
  $N = K \times M, \quad K \leq M \times N$, select 'leaders'

Distributed self - organization

Determine leaders → Find influence → Remove unnecessary links

Semi-decentralized

Decentralized: Dirichlet problem on the graph



Goal: design a scheme that gives each node a vector of compact global information

Social degrees and leaders

- **Social degree** of order 2 : $SD^{(2)}(v)$ = number of neighbors of node v
- Social degree of order $k > 2$: $SD^{(k)}(v)$ = number of cycles of length k passing through node v
- Social degrees of order 2 and 3 can be determined by a simple query
- A node is called a **leader of order *k*** if its social degree of order k is greater than that of its neighbors

Influence vector as a metric for well-connectedness

- *K* local leaders, *N* - *K* nodes regular nodes
- Regular nodes need to **determine how well they are located with respect to local leaders** and how they are **influenced** by them
- Distance to leaders does not include information on how "well-connected" a regular node is to leaders
- Consider a random walk on the graph starting from regular node *i*, with leader nodes as absorbing states, the **influence of leader *k* on regular node *i*** , is the probability that the random walk hits *k* before other leaders

Two stage semi-decentralized algorithm

- **Stage 1: Determining *K* leaders**
  - Each node determines its social degree via local query
  - Dominant nodes in each neighborhood send their degrees to the central authority
  - Central authority computes their social scores

$$SC(k) = \alpha SD^{(2)}(k) + (1-\alpha) SD^{(3)}(k)$$

  Choice of α determines whether leaders in star-like neighborhoods are preferred

  - The central authority selects the *K* nodes with highest scores as social leaders and gives them an arbitrary order

Algorithm (cont'd)

- **Stage 2**: **Determining the influence vectors**
  - Based on its order each leader takes its influence vector to be the fixed vector e_i
  - Regular nodes update their influence vector entries:

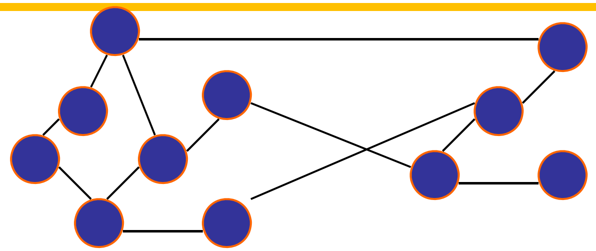
$$x_i^k(t+1) = \frac{1}{n_i + 1}\left[x_i^k(t) + \sum_{j \in N_i(t)} x_j^k(t) \right]$$

- For connected graphs, for t large enough, x_i^k converges to the influence of leader *k* on node *I*

- Upon calculation of influence vectors, each regular node determines its local leader and stops its communication with neighbors who have other leaders

- **Graph decomposes into two level hierarchy with efficient communication pattern**

Reliability and Spanning Trees



$\mathcal{G}(\mathcal{V},\mathcal{E})$

$\mathcal{V} = \{1, 2, \ldots, n\}$

$\mathcal{E} = \{l_1, l_2, \ldots, l_e\}$

p : Constant link loss probability

N_i : # of connected components with i edges

$\tau(\mathcal{G})$: Number of spanning trees

$$\mathrm{Rel}(\mathcal{G}, p) = \sum_{i=n-1}^{e} \mathrm{N_i} (1-p)^i p^{e-i}$$

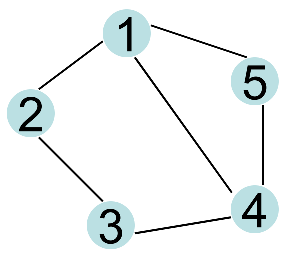
For sufficiently large p:

$$\tau(\mathcal{G})(1-p)^{n-1} p^{e-n+1} \le \mathrm{Rel}(\mathcal{G}, p) \le \tau(\mathcal{G})(1-p)^{n-1}$$

- End to end applications
- Spanning tree as a minimally connected graph
- *T(G)* as a measure of robustness to losses
- *References*: Kelmans, Colbourn

Graph Theory for Robust Network Design

- Goal: Given a base topology add k edges from a set of m candidates such that results in maximum number of spanning trees
- Number of spanning trees $\tau(G) = \frac{1}{n}\prod_{i=2}^{n}\lambda_i(L) = \frac{1}{n}\det(L + \frac{11^T}{n})$
- Incidence vector of an edge shows between which nodes it is



f_i : incidence vector

$f_i = e_\alpha - e_\beta$

$f_{(1,5)} = e_1 - e_5 = [1 \quad 0 \quad 0 \quad 0 \quad -1]^T$

$$f_{15}f_{15}^T = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Graph Laplacian $L = D - A = F_{nm}F_{mn}^T = \sum_{i=1}^{m} f_i f_i^T$

Counting Spanning Trees

- Normalized Laplacian $\mathcal{L} = D^{-1/2} L D^{1/2}$
  - If $G=(V_n,E)$ is connected: $\lambda_n = 0 < \lambda_{n-1} \leq \lambda_{n-2} \leq \ldots \leq \lambda_1 < 2$
- Random walk matrix $P = D^{-1} A = I - D^{-1} L$

$$\lambda_i(P) = 1 - \lambda_{n+1-i}(\mathcal{L})$$

- Matrix-Tree theorem (Kirchhoff)

$$Adj(L) = \tau(\mathcal{G}) L$$

$$\tau(\mathcal{G}) = \frac{1}{n} \prod_{j=2}^{n} \lambda_j(L) = \frac{1}{n} \det(L + \frac{1}{n} J) =$$

$$\prod_{j=2}^{n} \lambda_j(\mathcal{L}) \frac{\prod_{i=1}^{n} d_i}{\sum_{i=1}^{n} d_i} = \det(Q_k) \prod_{i \neq k} d_i, \qquad k = 1, \ldots n.$$

$$J = \frac{1_n 1_n^T}{n}$$

Q_i is the i[th] principal submatrix of $I - P$

Problem Statement

- Goal: Given a base topology add *k* edges from a set of *m* candidates such that results in maximum number of spanning trees

$$\tau(\mathcal{G}) = \frac{1}{n}\prod_{i=2}^{n}\lambda_i(L) = \frac{1}{n}\det(L + \frac{11^T}{n})$$

- Dynamic graph process resulting from adding edges

$$\text{Maximize } \tau(G(t+k))$$

$$\text{Subject to:}$$

$$\begin{cases} G(t+1) = Add(G(t), u(t)), & t = 0, 1, \ldots, k-1 \\ u(t) = e(t+1), & e(t+1) \in S \subseteq E(\bar{G}(t)) \\ G(t) = \mathcal{G}_0 & \end{cases}$$

Formulation and Relaxation

- Goal: Given a base topology add *k* edges from a set of *m* candidates such that results in maximum number of spanning trees (Approach similar to Ghosh and Boyd 06)

Maximize $\tau\left(L_0 + \sum_{i=1}^{m} x_i f_i f_i^T\right)$ or equivalently

$$\log \det \left(L_0 + \frac{1}{n}J + \sum_{i=1}^{m} x_i f_i f_i^T\right)$$

Subject to:

$1^T x = k$

$x \in \{0,1\}^m$

$\log \det \left(L_0 + \frac{1}{n}J + \sum_{i=1}^{m} x_i f_i f_i^T\right)$ is concave in x.

- Relax to $x \geq 0$

$$x_i^* > 0 \Rightarrow \frac{\partial \tau(x^*)}{\partial x_i} \geq \frac{\partial \tau(x^*)}{\partial x_j}, \forall j$$

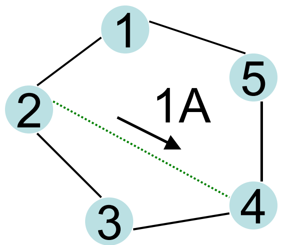
- At maximum $\tau(x)$ has equal derivatives for positive x_i s

Robust Network Design

- Derivative:

$$f_i^T \left(L_0 + \frac{1}{n} J + \sum_{i=1}^{m} x_i f_i f_i^T \right)^{-1} f_i = \lambda, \forall i \in \text{Chosen edge set } (\mathrm{x_i} > 0)$$

$$\mathrm{R_{eff}}(i) = f_i^T \left(L + \frac{1}{n} J \right)^{-1} f_i$$



$$R_{eff}(\alpha, \beta) = V_{\alpha\beta}$$

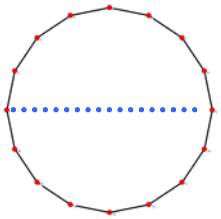
- If feasible, add edges such that the effective resistance distance of all selected edges become equal and greater than the effective resistance distance between non-selected candidates

Special Cases

- Adding 1 edge to a general graph
  - In a given graph which shortcut will result in more spanning trees?
  - If the edge is between nodes *α* and *β*:

$$\tau(G(1)) = \left(1 + R_{eff}(\alpha,\beta)\right)\tau(\mathcal{G}_0)$$

  - Select the edge corresponding to the maximal resistance distance
  - Example: Adding a shortcut to a ring



Special Cases (cont'd)

- Adding 2 edges (α,β) and (γ,δ)

$$\tau(G(2)) = \Big[\big(1+\overbrace{R_{eff}(\alpha,\beta)}\big)\big(1+\overbrace{R_{eff}(\gamma,\delta)}\big) - \Big(\underbrace{(z_{\gamma\alpha}-z_{\gamma\beta})}-\underbrace{(z_{\delta\alpha}-z_{\delta\beta})}\Big)^2\Big]\tau(\mathcal{G}_0),$$

$$Z = [z_{ij}] = \left(L + \frac{1}{n}J\right)^{-1}$$

Maximized by adding edge between high resistance distance nodes

Maximized by adding edge to symmetrize the graph

- Adding 3 or more edges similar: more complex terms due to compromising between symmetrizing the graph and joining nodes with the highest resistance distance

Small World Phenomenon

- Small world phenomenon as the trade-off in adding *k* shortcuts to a base graph such that the number of spanning trees is maximized
- Perturbation based method to model Watts-Strogatz small world networks (based on Higham 03, BarasHovareshti 08)
  - Performance measure: $\tau(\mathcal{G})$

Capture performance measure of G_0, as property of $L(G_0)$	→	Do spectral analysis of G_0, L_0
→ Perturb zero elements of L_0 by $\varepsilon = Kn^{-\alpha}, \rightarrow L_\varepsilon$	→	Analyze $\frac{\tau(\mathcal{G}_\varepsilon)}{\tau(\mathcal{G}_0)}$ for large *n* as a function of ε, as ε varies →
→ Interpret the result as structural perturbation		

Small World (cont'd)

- Consider the ratio of the increase in the number of spanning trees as the result of adding ε weights: $\varepsilon = Kn^{-\alpha}$.

$$r = \frac{\tau(G_\varepsilon)}{\tau(G_0)}$$

- Starting from a ring structure, in the limit of large *n*:
  - For $\alpha > 3$ the effect of ε-shortcuts is negligible
  - For $\alpha = 3$ the effect of ε-shortcuts starts (spectral gap gain perturbation of same order)
  - For $1 < \alpha \leq 3$ the shortcuts dominantly increase the number of spanning trees, i.e. $\lim_{n \to \infty} r = \infty$

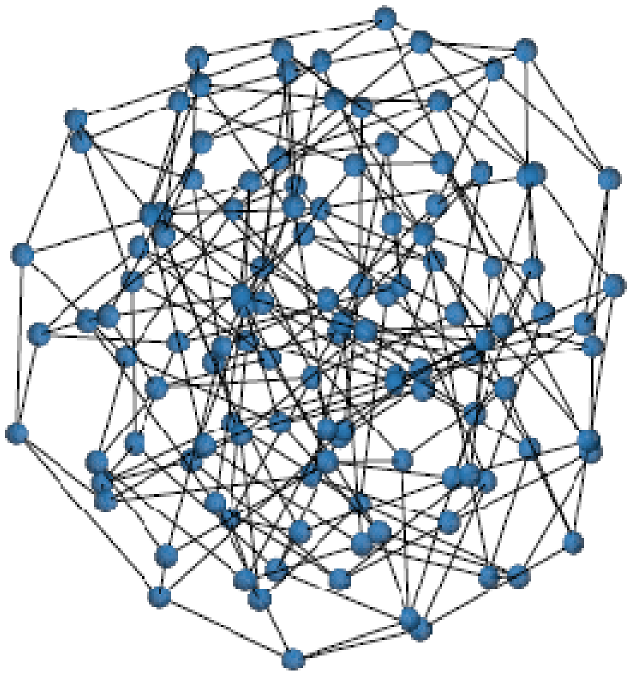
Expander Graphs

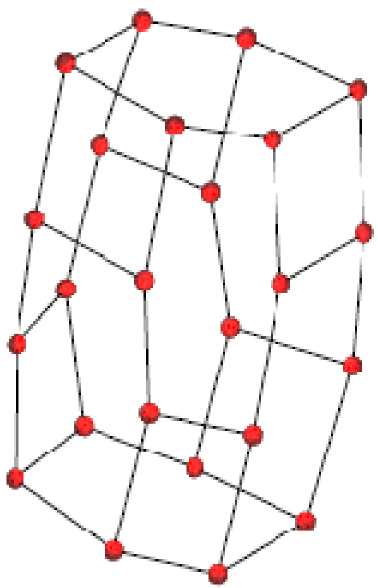
- Fast synchronization of a network of oscillators
- Network where any node is "nearby" any other
- Fast 'diffusion' of information in a network
- Fast convergence of consensus
- Decide connectivity with smallest memory
- Random walks converge rapidly
- Easy to construct, even in a distributed way (ZigZag graph product)
- Graph G, ***Cheeger constant h(G)***
  - All partitions of G to S and S^c,

    $h(G)$=min (#edges connecting S and S^c) /

    (#nodes in smallest of S and S^c)
- (k, N, ε) **expander** : $h(G) > \varepsilon$; **sparse but locally well connected** **(1-*SLEM*(G) increases as $h(G)^2$)**

Expander Graphs – Ramanujan Graphs





Constructing Expander Graphs

- Possible methods:
  - Form a random expander as a *2d*-regular multi-graph in which the set of edges consists of *d* separate Hamiltonian cycles on APs (Law and Siu 2003)
  - Form a union of two spanning trees chosen independently from the uniform distribution over all spanning trees of a complete graph, implementable by a random walk method (Goyal et al. 2009)

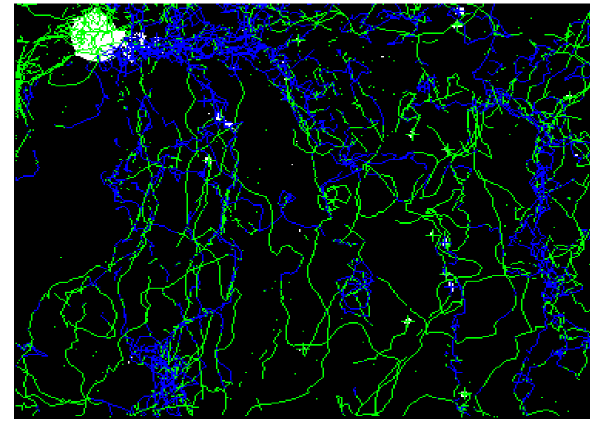
Outline

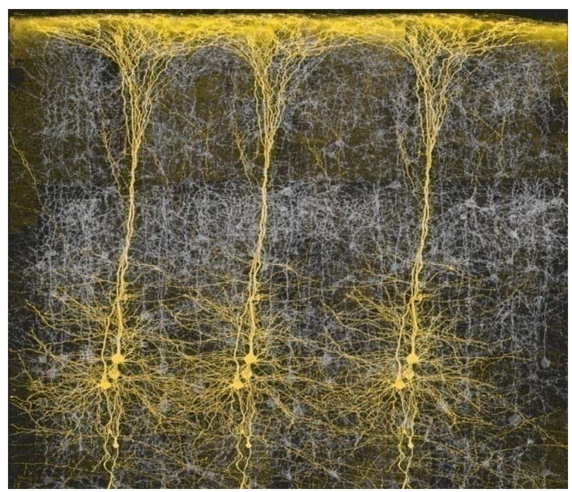
- Multiple interacting dynamic hypergraphs – three challenges
- Networks and Collaboration
  Constrained Coalitional Games
- Trust and Networks
- Topology Matters
- Conclusions and Future Directions

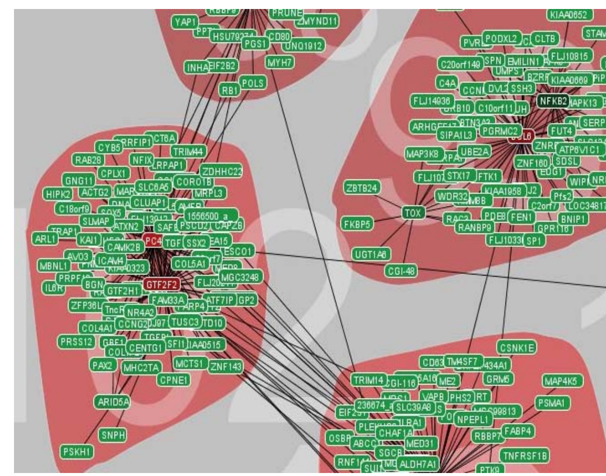
Conclusions

- Fundamental tradeoff between the benefit from collaboration and the required cost for collaboration
- Game theoretic studies for such conflict
- Two-phase coalitional games
- The convergence of the iterated pairwise games
- Phase transition of the coalition formation
- Stability of the formed coalitions
- Trust as a catalyst for collaborations
- Effects of topology on distributed algorithm performance
- Performance vs. efficiency – small world graphs – expander graphs

How Biology Does IT?







Control vs Communication

- Many graphs as **abstractions**
- **Collaboration graph** – or a model of what the system does (**behavior**)
- **Communication graph** – or a model of what the system consist of (**structure**)
- Nodes with **attributes** – several graphs
- **Key question 1**: Given behavior, what structure (subject to constraints) gives best performance?
- **Key question 2**: Given structure (and constraints) how well behavior can be executed?

Lessons Learned -- Future Directions

- **Constrained coalitional games** – unifying concept
- Generalized networks, **flows - potentials**, duality and network optimization (monotropic optimization)
- **Time varying graphs** – mixing – statistical physics
- **Understand autonomy** – better to have self-organized topology capable of supporting (scalable, fast) a rich set of distributed algorithms (small world graphs, expander graphs) than optimized topology
- Given a set of distributed computations **is there a small set of simple rules** that when given to the nodes they can self-generate such topologies?

Thank you!

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Questions?