Critical Thresholds in Euler-Poisson Equations and Related Models

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We study the questions of global regularity vs. finite time breakdown in Euler-Poisson equations and related models. The Euler-Poisson equations have been applied to several physical problems including semiconductors. The presence of the dispersive forcing makes the system just conditionally stable. To describe the conditional stability, we propose the notion Critical Threshold (CT), where a conditional finite time singularity depends on whether the initial configuration crosses an intrinsic, $O(1)$, critical threshold. Our approach for multi-D setting is based on a main new tool of spectral dynamics, where eigenvalues of velocity gradient and associated eigenpairs (l, r), are traced by a forced Ricatti equation. We begin with the one-dimensional Euler-Poisson equations, precisely characterizing the critical threshold. Next we introduce a $n-$dimensional Restricted Euler-Poisson (REP) system, identifying a set of $[n/2]$ global invariants, which yields a remarkable characterization of 2D initial REP configurations with global smooth solutions. Our study reveals the dependence of the CT phenomena on the initial spectral gap.