

Infinite energy singularity formation in the incompressible 3D Euler equations

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Abstract: A class of solutions of the incompressible 3D Euler equations is considered of the form $U(x, y, z, t) = \{u_1(x, y, t), u_2(x, y, t), u_3(x, y, z, t)\}$ where $u_3(x, y, z, t) = z\gamma(x, y, t) + w(x, y, t)$. This structure is reminiscent of the Burgers vortex. The resulting two-dimensional PDEs for γ and $u(x, y, t) = \{u_1, u_2\}$ give numerical indications of singular (infinite energy) behaviour (Ohkitani & Gibbon 2000). This has been confirmed analytically by Constantin (2000). Physically the corresponding vortices open in a flower-like manner but vanish when this class of solutions fails to be sustained at the singular time. Additionally, two exact, cylindrically symmetric solutions of the 3D Euler equations that become singular in finite time have also been found recently by Gibbon, D R Moore & J T Stuart (2002). These solutions blow-up everywhere so they also have infinite energy.