Experimental and Numerical Methods

HOT-WIRE AND HOT-FILM ANEMOMETRY

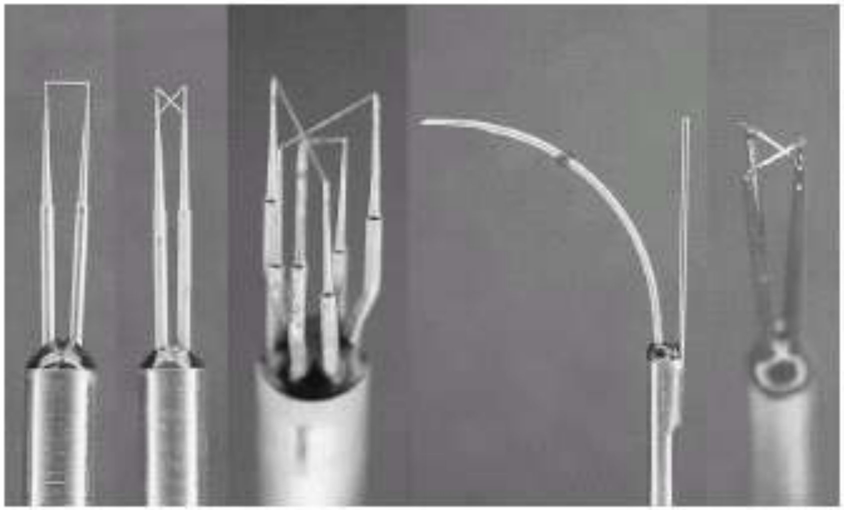


Fig. 3.1 Hot-wire and hot-film probes. (Photographs provided by and used with permission of TSI, Inc.)

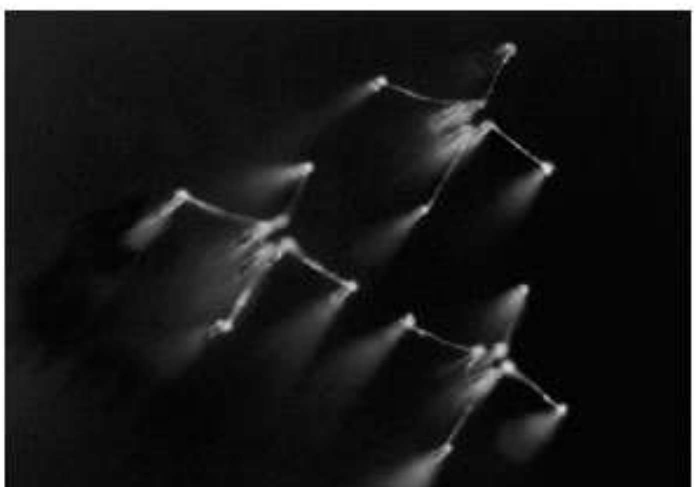


Fig. 3.4 Twelve-sensor hot-wire probe to measure velocity and velocity gradient components.

$$R_s = R_f[1 + \alpha(T_s - T_f)],$$

$$\frac{dQ}{dt} = P - F,$$

which describes the thermal energy balance. Here Q is the internal energy of the sensor, $P \equiv IE \equiv I^2 R_s$ is the electrical input power to the sensor, with I the current through the sensor, E the voltage drop across the sensor, and R_s the sensor resistance.

F represents the total rate of heat transferred from the sensor

$$F = q_c + q_p + q_s + q_r,$$

where q_c is the heat transfer rate due to convection from the sensor by the flow, q_p is the heat transfer rate due to conduction to the prongs supporting the ends of the sensors, q_s is the conductive heat transfer rate to the quartz substrate (for hot-films only), and q_r is the heat transfer rate due to radiation from the sensor.

$$cm\frac{dT_s}{dt} = P(I, T_s) - F(U, T_s),$$

$Q \equiv cmT_s$, where c is the specific heat and m is the mass of the sensor.

Free convection small when

$$R_{e_d} > 2G_r^{1/3},$$

where the Grashof number, $G_r \equiv gd^3\beta(T_w - T_f)/\nu^2$, criterion for neglecting free convection can be stated as

$$\frac{U}{2}\left[\frac{1}{g\nu\beta(T_w - T_f)}\right]^{1/3} > 1.$$

Forced Convection Hot-Wire Cooling

$$q_c = hS(T_w - T_f), \tag{3.7}$$

where h is the convective heat transfer coefficient and $S = \pi d\ell$ is the surface area of the sensor wire of length ℓ and diameter d. Unfortunately, h is *not* a constant;

Nusselt Number

$$N_u = \frac{hd}{k_f}, \tag{3.8}$$

where k_f is the thermal conductivity of the fluid. N_u indicates the difference in the heat transfer rate to the fluid when the fluid is moving compared to what it would be if the fluid were stationary relative to the sensor.

$$N_u = N_u\left(R_{e_d}, P_r, M_a, G_r, K_n, \frac{\ell}{d}, r_T, \gamma, \theta\right), \qquad (3.9)$$

where, in addition to the parameters already defined, P_r is the Prandtl number, M_a is the Mach number, K_n is the Knudsen number, $r_T = (T_w - T_f)/T_f$ is the temperature overheat ratio, γ is the ratio of specific heats of the fluid, and θ is the angle between the normal to the axis of the sensor and the velocity vector at the midpoint of the sensor. Fortunately, the effects of M_a, G_r, K_n, and ℓ/d are negligible for appropriate choices of operating conditions and sensor dimensions.

(3.9) reduces to

$$N_u = N_u(R_{e_d}, P_r, r_T)$$

For gas flows with nearly constant P_r

$$N_u = \left[M(r_T) + N(r_T)R_{e_d}^n\right]\left(1 + \frac{r_T}{2}\right)^m$$ *King's law of convective cooling*

for m = 0 and substituting expressions above for N_u and q_c

$$q_c = \left(M' + N'U^n\right)(T_w - T_f)$$

if the *temperature* of the wire is constrained to be constant and the fluid temperature changes are negligible, the heat transfer rate will depend on the local flow velocity alone. Moreover, since $dT_s/dt \approx 0$ in this case, (3.4) shows that

$$\frac{E^2}{R_w} = \left(M' + N'U^n\right)\frac{R_w - R_f}{\alpha R_f} \tag{3.13}$$

or

$$E^2 = A + BU^n, \tag{3.14}$$

where the new constants A and B are the products of M' and N' with $(R_w - R_f)/\alpha R_f$. This is the form in which King's law is usually given.

To account for the sensor wire cooling from the velocity component tangential to the sensor as well as to that from the normal component, an "effective" velocity U_e is usually assumed, so that (3.14) becomes

$$E^2 = A + BU_e^n. \tag{3.15}$$

Alternatives to King's Law

the effective velocity U_e is frequently expressed as a function of a qth-order polynomial of the voltage E, as in 2

$$U_{e_j} = \sum_{m=1}^{q} A_{mj} E_j^{m-1}, \tag{3.16}$$

where A_{mj} are the polynomial coefficients for the jth sensor in a multisensor probe.

For example

$$U_{e_j}^2 = A_{1j} + A_{2j}E_j + A_{3j}E_j^2 + A_{4j}E_j^3 + A_{5j}E_j^4$$

Another alternative is to construct a

table of ordered values of E, U_1, U_2, and U_3 to establish the relationship between the measured voltage and the velocity components, although this does not necessarily have to be single valued.

Effective Cooling Velocity

velocity vector cooling a hot-wire or hot-film sensor

$$\mathbf{V} = U_n\mathbf{n} + U_b\mathbf{b} + U_t\mathbf{t}, \tag{3.18}$$

where **n**, **b**, and **t** are unit vectors in the normal, binormal, and tangential directions

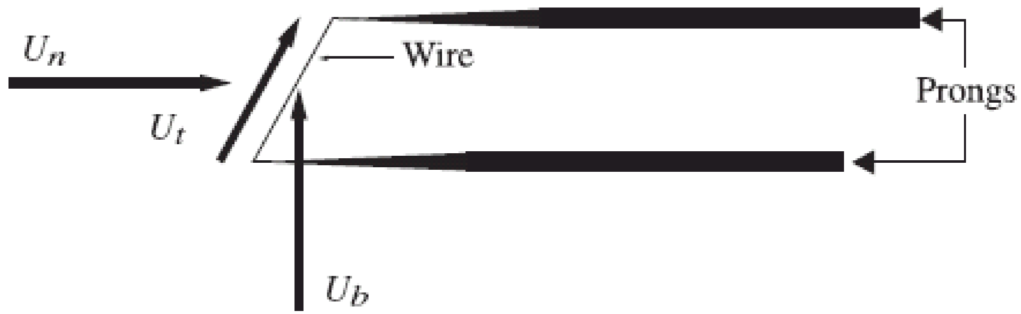


Fig. 3.2 *Orientations of normal (U_n), tangential (U_t), and binormal (U_b) velocity components with respect to a hot-wire or hot-film sensor.*

Jorgensen's cooling law

$$U_e^2 = U_n^2 + h^2U_b^2 + k^2U_t^2$$

The value of h depends on, among other influences, the aerodynamic blockage of the flow by the prongs and is usually close to unity. The value of k depends on the aspect ratio of the sensor, ℓ/d; for $\ell/d \approx 200$, $k \approx 0.2$.

In the laboratory coordinate system

$$\mathbf{V} = U_1\mathbf{i} + U_2\mathbf{j} + U_3\mathbf{k},$$

Relating the two decompositions of V

$$U_n = n_1U_1 + n_2U_2 + n_3U_3,$$

$$U_b = b_1U_1 + b_2U_2 + b_3U_3,$$

$$U_t = t_1U_1 + t_2U_2 + t_3U_3,$$

where the coefficients n_i, b_i, and t_i ($i = 1, 2, 3$) can be written in terms of sines and cosines of the angles of inclination of the sensors to the laboratory coordinate system

Substituting U_n, U_b, and U_t

$$U_{e_j}^2 = a_{1_j}U_{1_j}^2 + a_{2_j}U_{2_j}^2 + a_{3_j}U_{3_j}^2 + a_{4_j}U_{1_j}U_{2_j} + a_{5_j}U_{1_j}U_{3_j} + a_{6_j}U_{2_j}U_{3_j}, \qquad (3.24)$$

where the coefficients a_{n_j} ($n = 1, \ldots, 6$) are products of the geometry coefficients, n_i, b_i, and t_i, in (3.21) through (3.23) with the weighting factors, h and k, in (3.19).

Multisensor Probes

Velocity Component Measurements

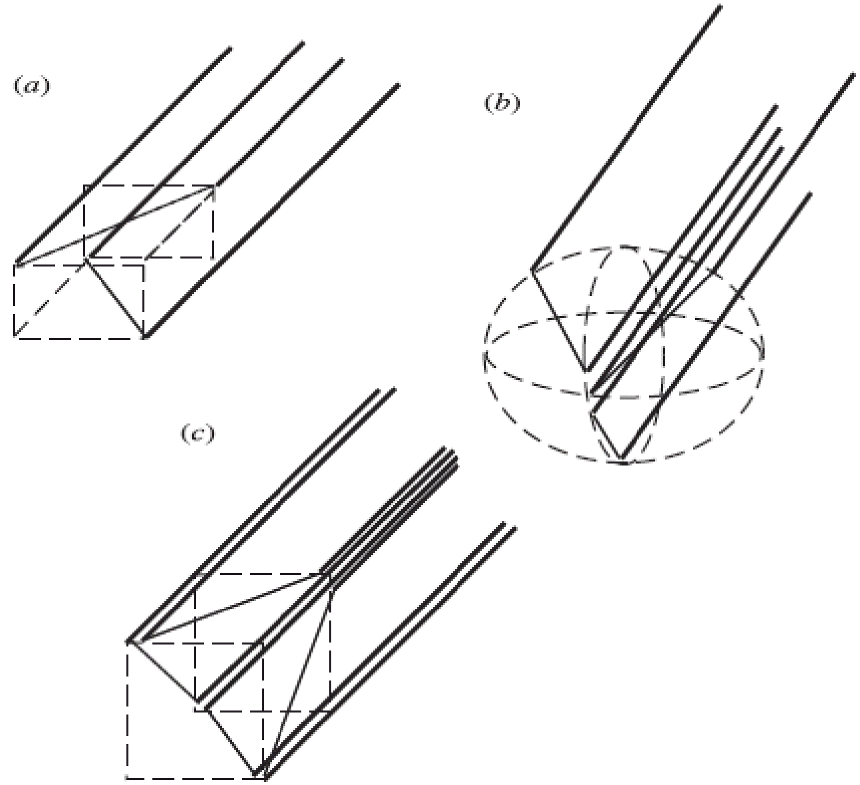


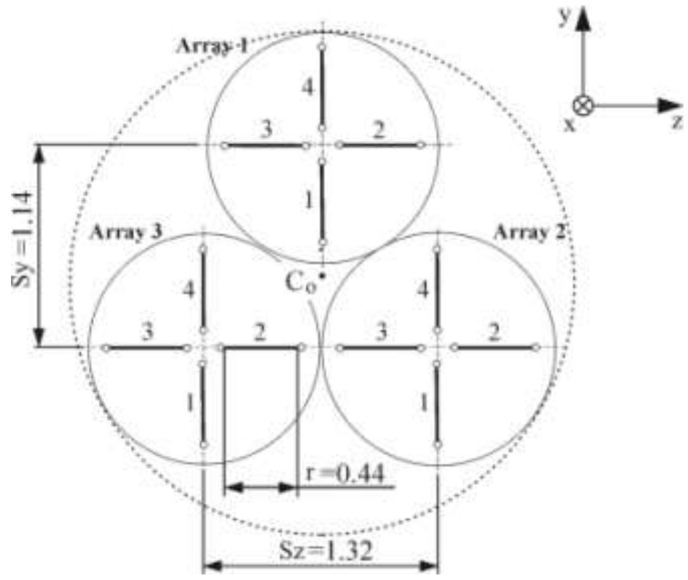
Fig. 3.3 *Sketches of (a) two-, (c) three-, and (c) four-sensor hot-wire probes.*

Two component X- or V-array probes

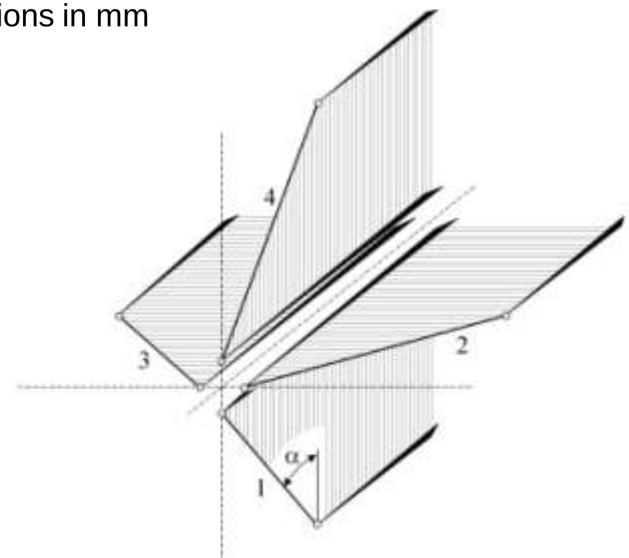
$$a_{1_j} U_{1_j}^2 + a_{2_j} U_{2_j}^2 + a_{4_j} U_{1_j} U_{2_j} = A_{1_j} + A_{2_j} E_j + A_{3_j} E_j^2 + A_{4_j} E_j^3 + A_{5_j} E_j^4$$

$(j = 1, 2)$ in the two unknown velocity components U_1 and U_2.

12-Sensor Hot-Wire Probe



Dimensions in mm



Vukoslavčević & Wallace. (1996)

Meas. Sci & Tech. 10

12-sensor Probe Data Processing for Simultaneous velocity vector and velocity gradient tensor measurements

Taylor's series expansion of velocity components about probe cross-stream plane centroid to center of the jth sensor over the measured distances, C_j and D_j.

$$U_{1_j} = U_{1_o} + C_j \frac{\partial U_1}{\partial y} + D_j \frac{\partial U_1}{\partial z}$$

$$U_{2_j} = U_{2_o} + C_j \frac{\partial U_2}{\partial y} + D_j \frac{\partial U_2}{\partial z}$$

$$U_{3_j} = U_{3_o} + C_j \frac{\partial U_3}{\partial y} + D_j \frac{\partial U_3}{\partial z}$$

12 Cooling equations for each of the j sensors in terms of the three velocity components at the probe centroid and the six velocity gradients in the cross-stream plane.

$$P_j = A_{1_j} + A_{2_j} E_j + A_{3_j} E_j^2 + A_{4_j} E_j^3 + A_{5_j} E_j^4$$

is a polynomial of the measured voltages, Ej.

120 calibration coefficients, A_{ij} and k_{ij} to be determined .

$$\begin{aligned} f_j \equiv & - P_j + U_{1_o}^2 + 2C_j U_{1_o} \frac{\partial U_1}{\partial y} + 2D_j U_{1_o} \frac{\partial U_1}{\partial z} \\ & - k_{2j} \left[U_{2_o}^2 + 2C_j U_{2_o} \frac{\partial U_2}{\partial y} + 2D_j U_{2_o} \frac{\partial U_2}{\partial z} \right] \\ & - k_{3j} \left[U_{3_o}^2 + 2C_j U_{3_o} \frac{\partial U_3}{\partial y} + 2D_j U_{3_o} \frac{\partial U_3}{\partial z} \right] \\ & - k_{4j} \left[U_{1_o} U_{2_o} + C_j \left(U_{1_o} \frac{\partial U_2}{\partial y} + U_{2_o} \frac{\partial U_1}{\partial y} \right) + D_j \left(U_{1_o} \frac{\partial U_2}{\partial z} + U_{2_o} \frac{\partial U_1}{\partial z} \right) \right] \\ & - k_{5j} \left[U_{1_o} U_{3_o} + C_j \left(U_{1_o} \frac{\partial U_3}{\partial y} + U_{3_o} \frac{\partial U_1}{\partial y} \right) + D_j \left(U_{1_o} \frac{\partial U_3}{\partial z} + U_{3_o} \frac{\partial U_1}{\partial z} \right) \right] \\ & - k_{6j} \left[U_{2_o} U_{3_o} + C_j \left(U_{2_o} \frac{\partial U_3}{\partial y} + U_{3_o} \frac{\partial U_2}{\partial y} \right) + D_j \left(U_{2_o} \frac{\partial U_3}{\partial z} + U_{3_o} \frac{\partial U_2}{\partial z} \right) \right] = 0 \end{aligned}$$

System of equations solved by minimizing the error function $\sum f_j = 0$ iteratively at each time step.

System of equations for 12-sensor probe in terms of error function f_j

$$
\begin{aligned}
f_j \equiv & - P_j + U_{1_o}^2 + 2C_j U_{1_o} \frac{\partial U_1}{\partial y} + 2D_j U_{1_o} \frac{\partial U_1}{\partial z} \\
& - k_{2j} \left[U_{2_o}^2 + 2C_j U_{2_o} \frac{\partial U_2}{\partial y} + 2D_j U_{2_o} \frac{\partial U_2}{\partial z} \right] \\
& - k_{3j} \left[U_{3_o}^2 + 2C_j U_{3_o} \frac{\partial U_3}{\partial y} + 2D_j U_{3_o} \frac{\partial U_3}{\partial z} \right] \\
& - k_{4j} \left[U_{1_o} U_{2_o} + C_j \left(U_{1_o} \frac{\partial U_2}{\partial y} + U_{2_o} \frac{\partial U_1}{\partial y} \right) + D_j \left(U_{1_o} \frac{\partial U_2}{\partial z} + U_{2_o} \frac{\partial U_1}{\partial z} \right) \right] \\
& - k_{5j} \left[U_{1_o} U_{3_o} + C_j \left(U_{1_o} \frac{\partial U_3}{\partial y} + U_{3_o} \frac{\partial U_1}{\partial y} \right) + D_j \left(U_{1_o} \frac{\partial U_3}{\partial z} + U_{3_o} \frac{\partial U_1}{\partial z} \right) \right] \\
& - k_{6j} \left[U_{2_o} U_{3_o} + C_j \left(U_{2_o} \frac{\partial U_3}{\partial y} + U_{3_o} \frac{\partial U_2}{\partial y} \right) + D_j \left(U_{2_o} \frac{\partial U_3}{\partial z} + U_{3_o} \frac{\partial U_2}{\partial z} \right) \right] \\
& = 0,
\end{aligned}
\qquad (3.29)
$$

where $k_{nj} = a_{nj}/a_{1j}, n = 2, \ldots, 6, j = 1, \ldots, 12$, and P_j is the right-hand side of (3.25). The system of equations (3.29) can be solved at each time step by minimizing the error function given by $\sum f_j^2$ using Newton's method.

Taylor's Frozen Turbulence Hypothesis
to determine streamwise gradients

$$\frac{dU_i}{dx} = -\frac{1}{U_c}\frac{dU_i}{dt}$$

setting the acceleration equal to zero in the N-S equations

$$\frac{\partial U_i}{\partial t} + U_1\frac{\partial U_i}{\partial x} + U_2\frac{\partial U_i}{\partial y} + U_3\frac{\partial U_i}{\partial z} = 0.$$

Rearranging (3.31) yields

$$\frac{\partial U_i}{\partial x} = -\frac{1}{U_1}\left(\frac{\partial U_i}{\partial t} + U_2\frac{\partial U_i}{\partial y} + U_3\frac{\partial U_i}{\partial z}\right).$$

Streamwise wavenumber approximated from frequency

$$k_x \approx \frac{2\pi f}{\overline{U_1}}$$

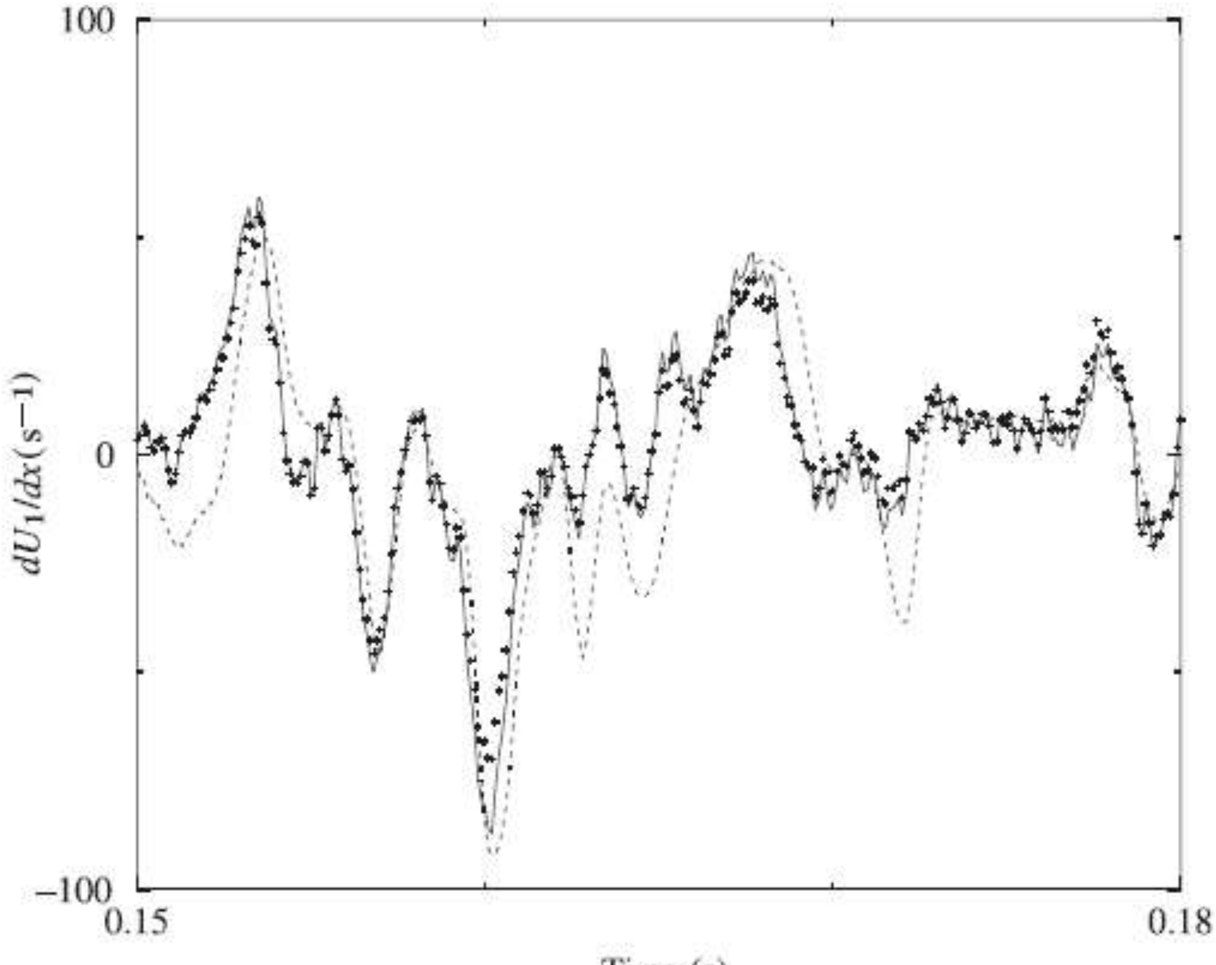


Fig. 3.5 *Comparison of time-series signals determined from Taylor's hypothesis [Eqs. (3.30) —— and (3.32) +++] and from the continuity equation (· · ·) using mixing-layer data from a 12-sensor probe. (From [31].)*

Constant-Current Operation

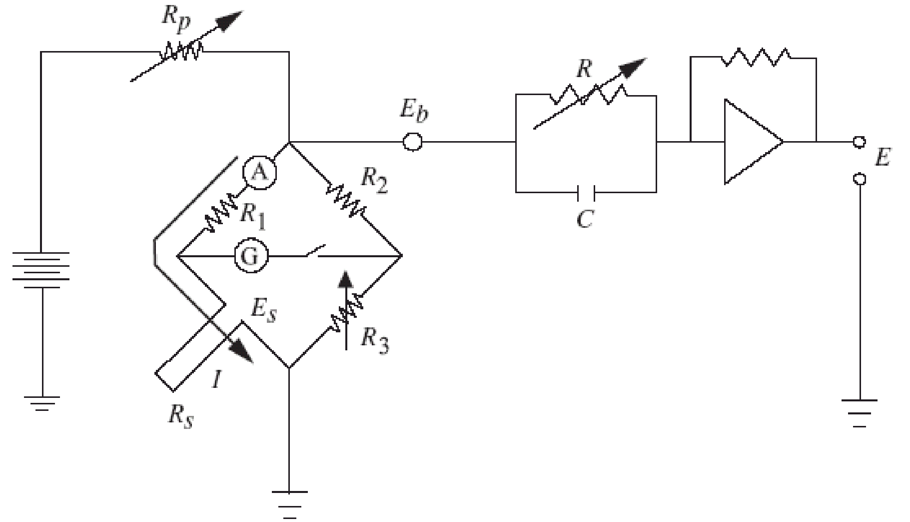


Fig. 3.6 *Constant-current hot-wire anemometer circuit. (Adapted from [38].)*

Constant-Temperature Operation

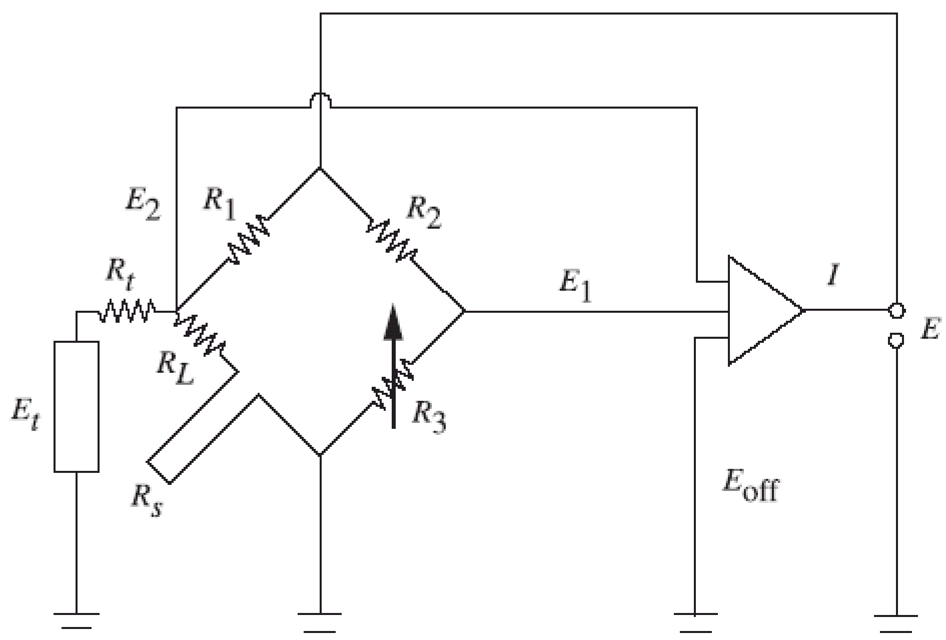


Fig. 3.7 *Constant-temperature hot-wire anemometer circuit. (Adapted from [8*

Study of Spatial Resolution Effects using DNS

P. V. Vukoslavčević · N. Beratlis · E. Balaras · J. M. Wallace · O. Sun

Exp Fluids
DOI 10.1007/s00348-008-0544-y

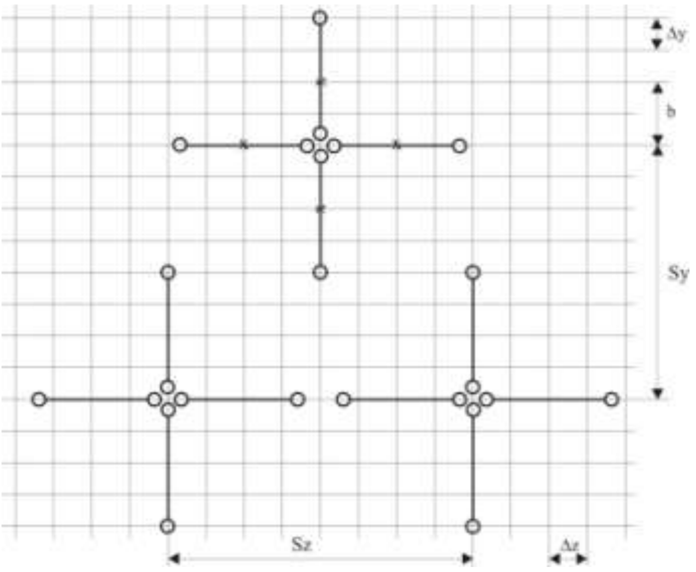
Virtual experiment

Database: DNS of a minimal channel flow at $Re_\tau = 200$

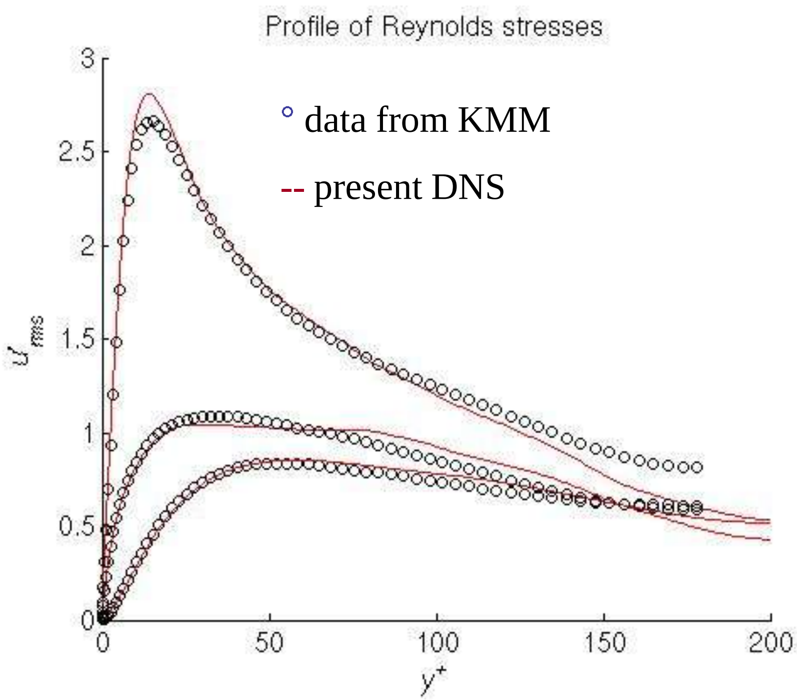
Grid resolution: $\Delta x^+ = \Delta y^+ = \Delta z^+ = 1$ $(400 \times 192 \times 400)$

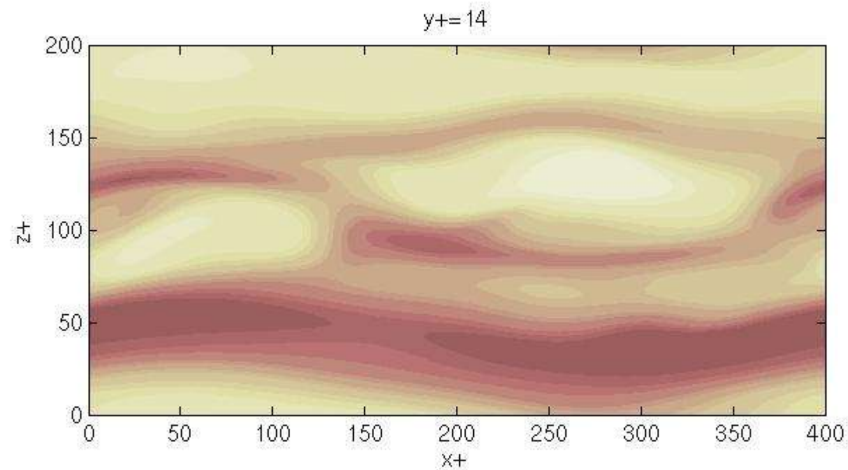
Virtual probe with S_y = 8 Δy over the numerical grid where Δy is 1 viscous length

$$S_y^+ = 2,\ 4,\ 8,\ 12$$

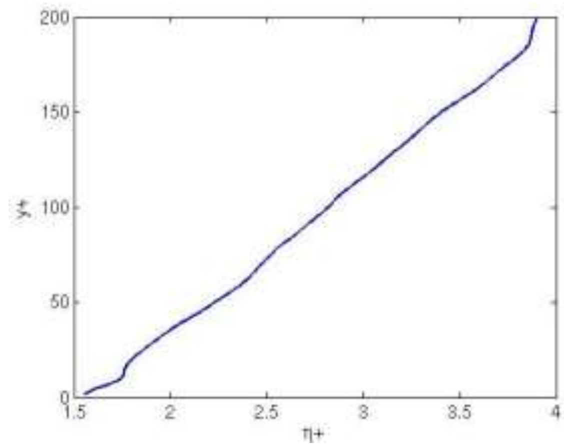


DNS database



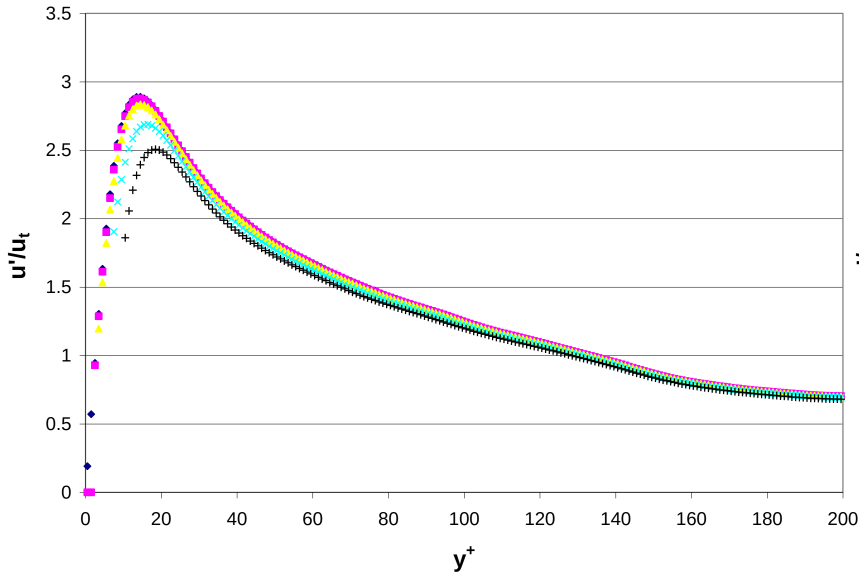


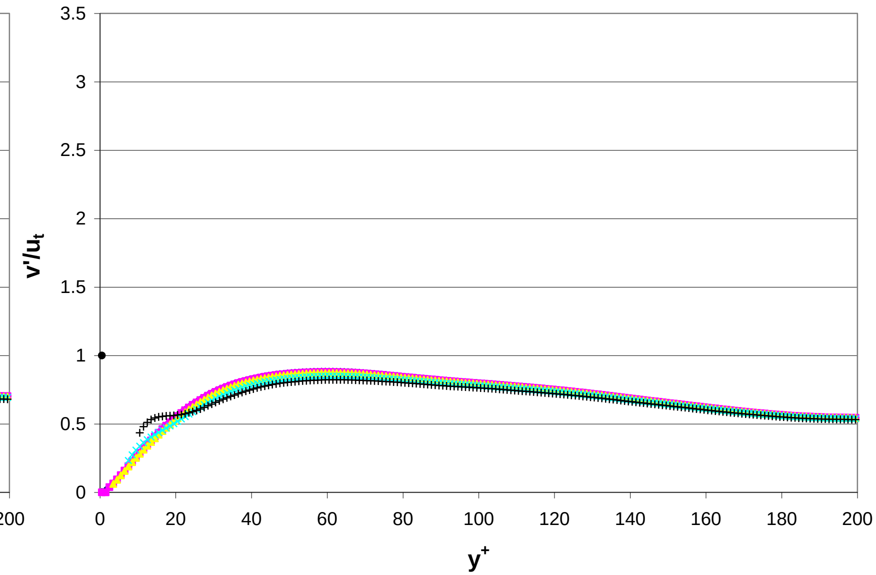
High and low speed streaks at an instant in time, in a plane parallel to the wall at y+=14

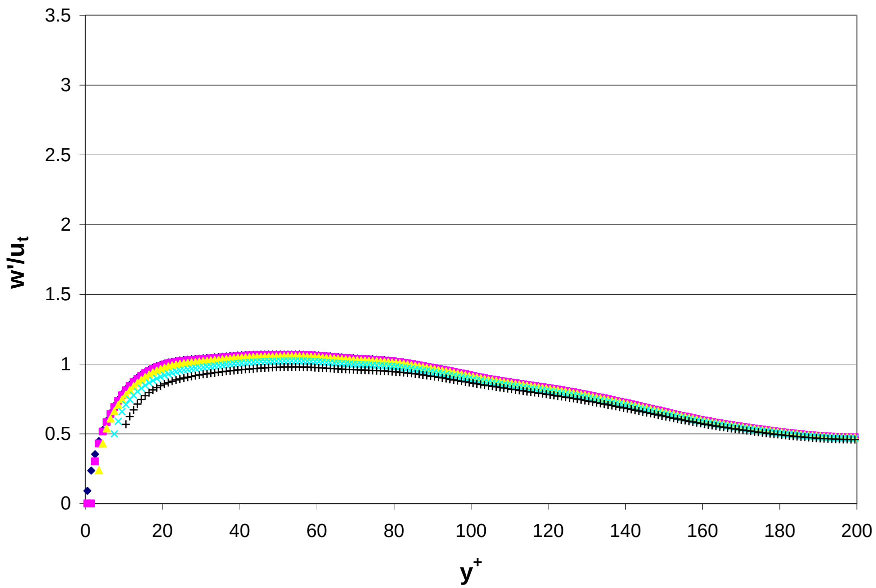


Ratio of Kolmogorov to viscous length scale

Velocity Statistics - RMS

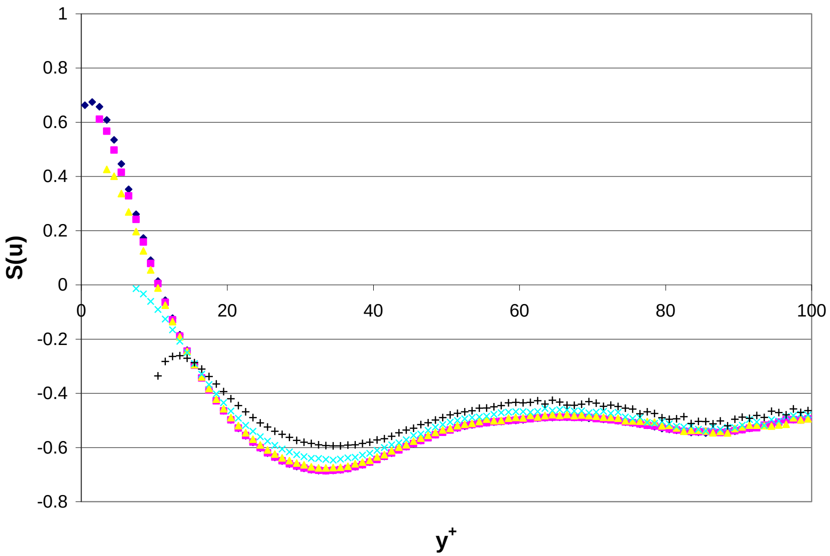


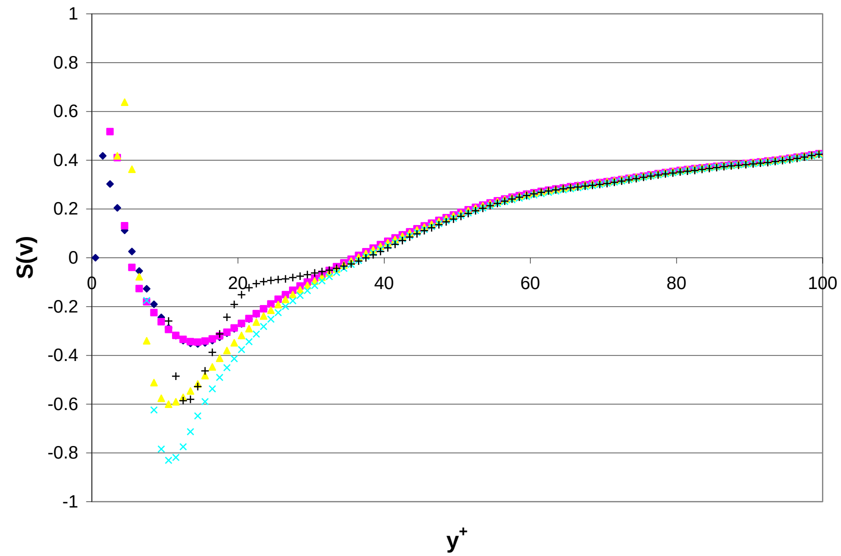


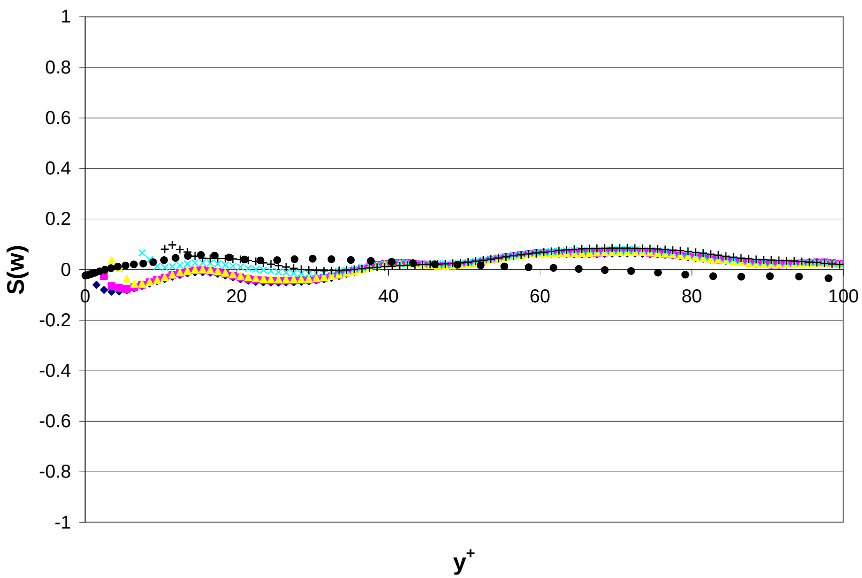


♦ DNS					
		y+ = 15		= 150	
■ S+=2	→	1.2 η	→	0.6 η	
▲ S+=4	→	2.4 η	→	1.2 η	
x S+=8	→	4.8 η	→	2.4 η	
+ S+=12	→	7.2 η	→	3.6 η	

Velocity Skewness

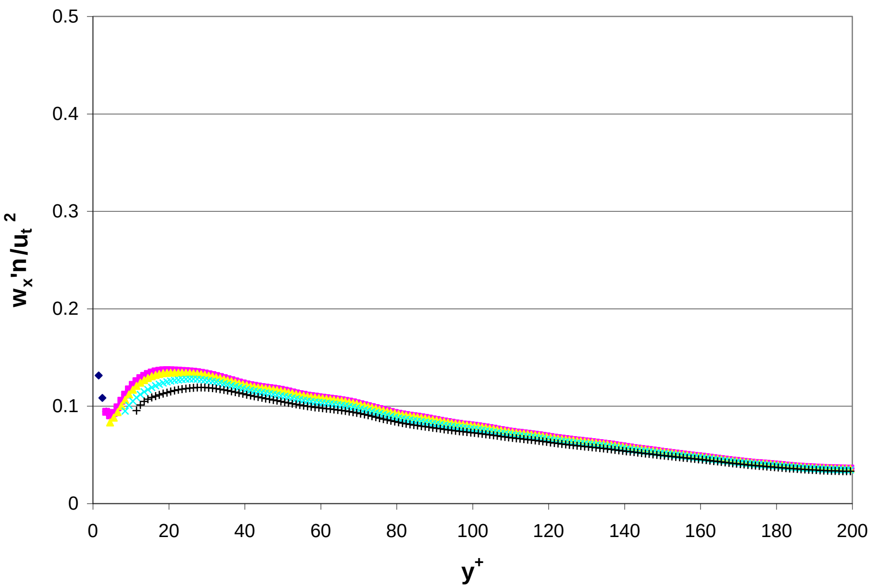


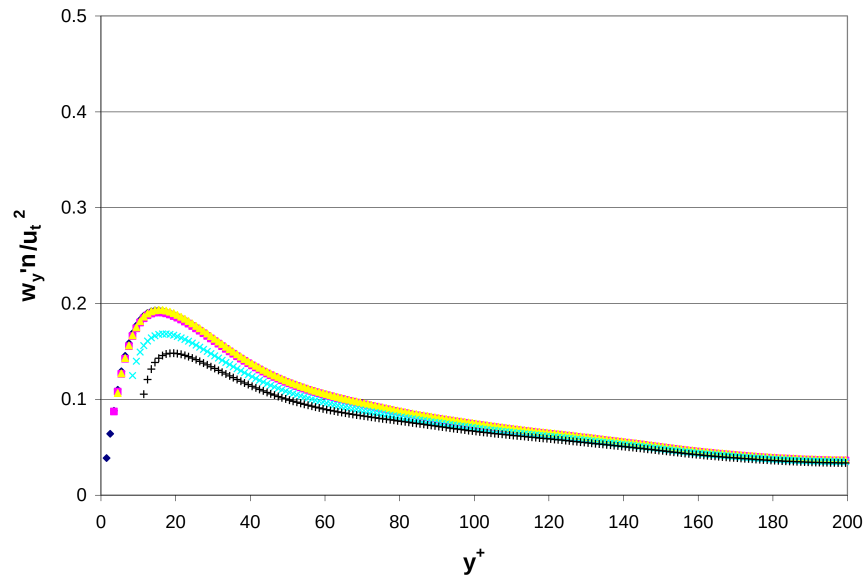


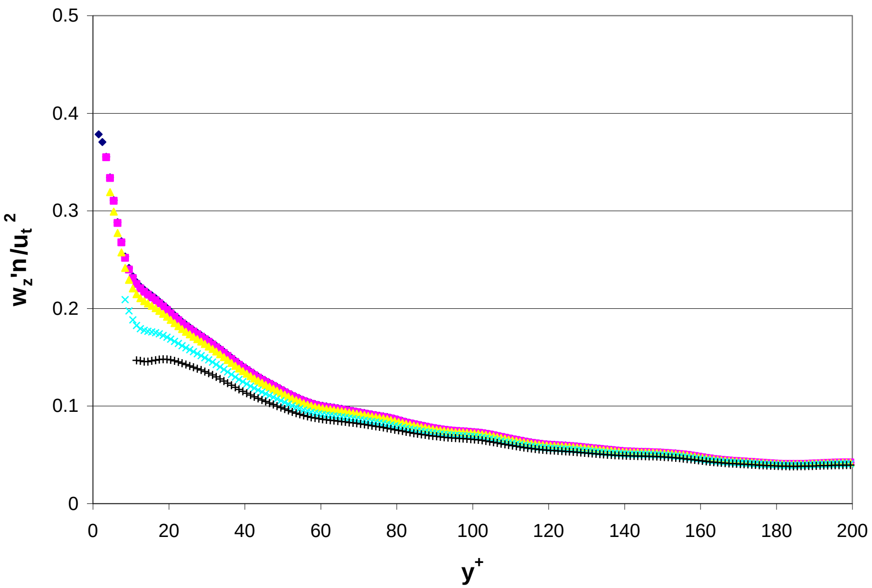


♦ DNS			
	y+ = 15		= 150
■ S+=2 →	1.2 η	→	0.6 η
▲ S+=4 →	2.4 η	→	1.2 η
x S+=8 →	4.8 η	→	2.4 η
+ S+=12 →	7.2 η	→	3.6 η

Vorticity Statistics - RMS



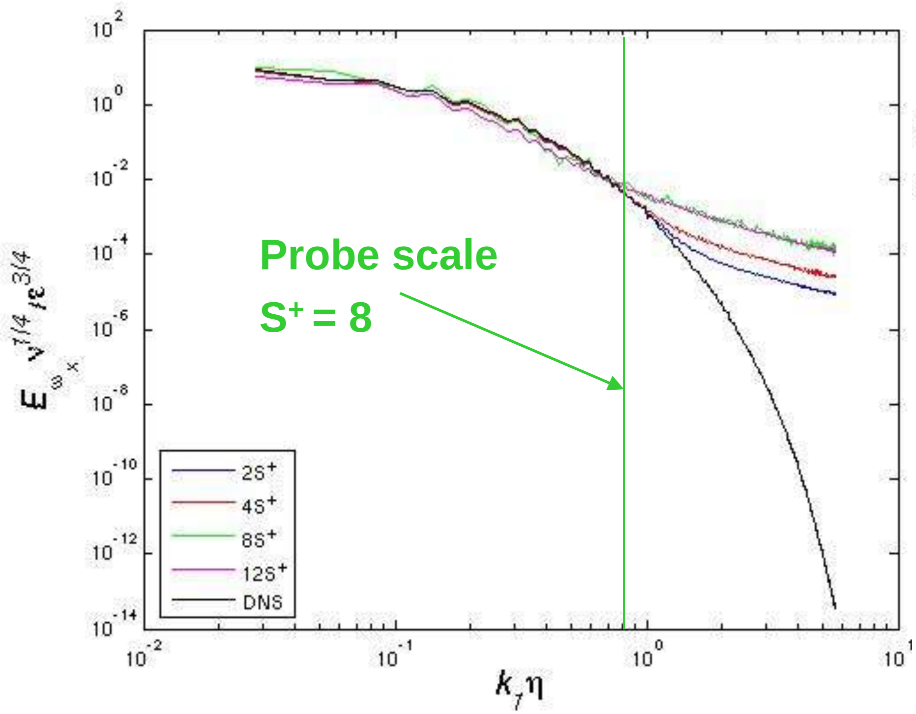


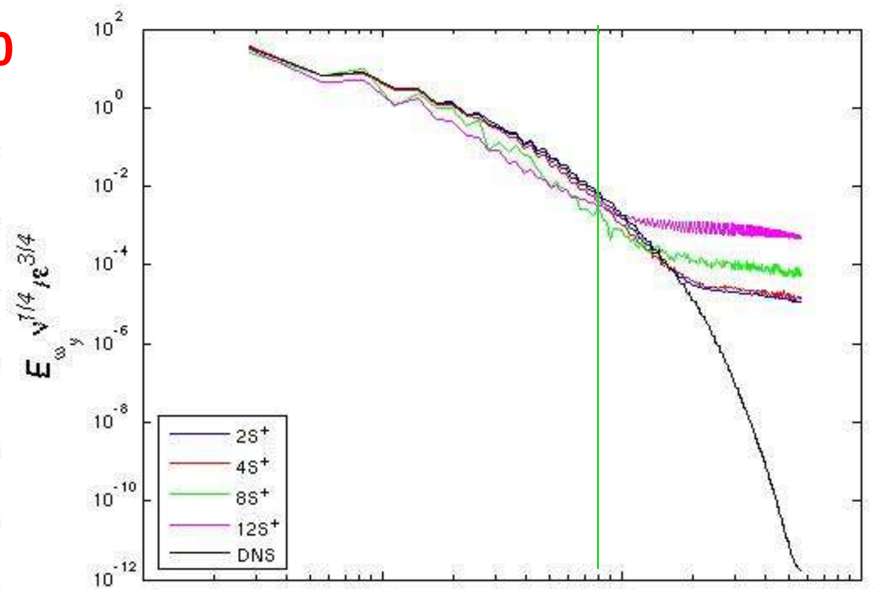


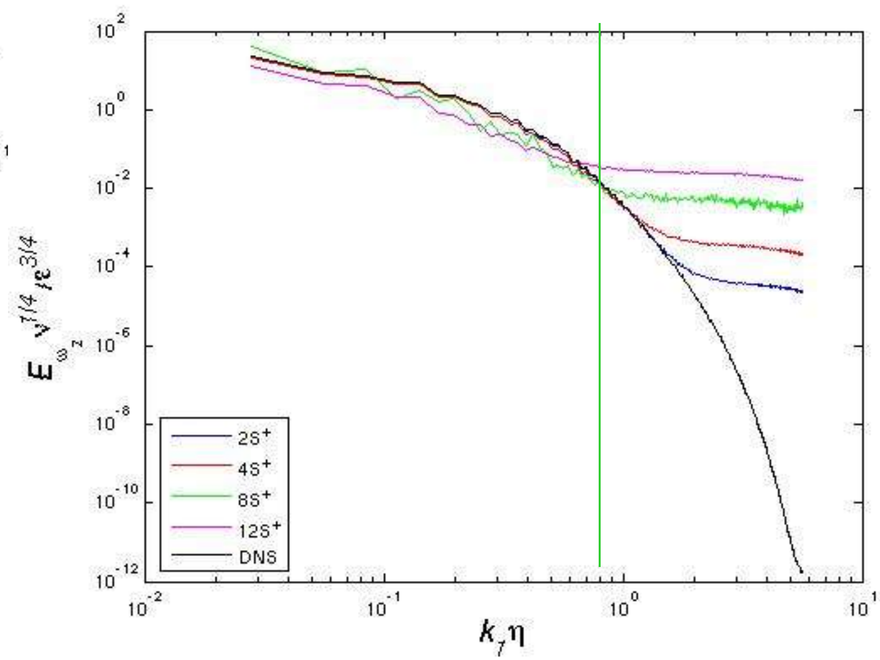
♦ DNS				
		y+ = 15		= 150
■ S+=2	→	1.2 η	→	0.6 η
▲ S+=4	→	2.4 η	→	1.2 η
x S+=8	→	4.8 η	→	2.4 η
+ S+=12	→	7.2 η	→	3.6 η

Vorticity Spectra

@ $y^+ = 20$



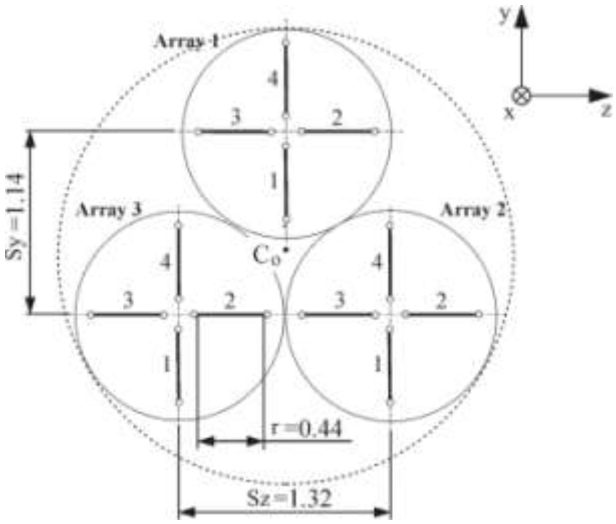




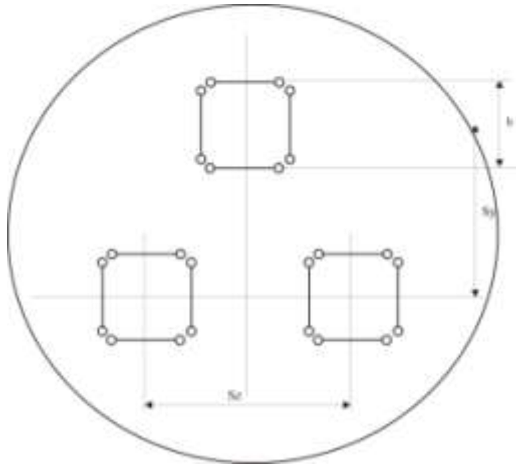
— **DNS**

	y+ = 15		= 150
— S+ = 2 →	1.2 η	→	0.6 η
— S+ = 4 →	2.4 η	→	1.2 η
— S+ = 8 →	4.8 η	→	2.4 η
— S+ = 12 →	7.2 η	→	3.6 η

12-sensor hot-wire probes

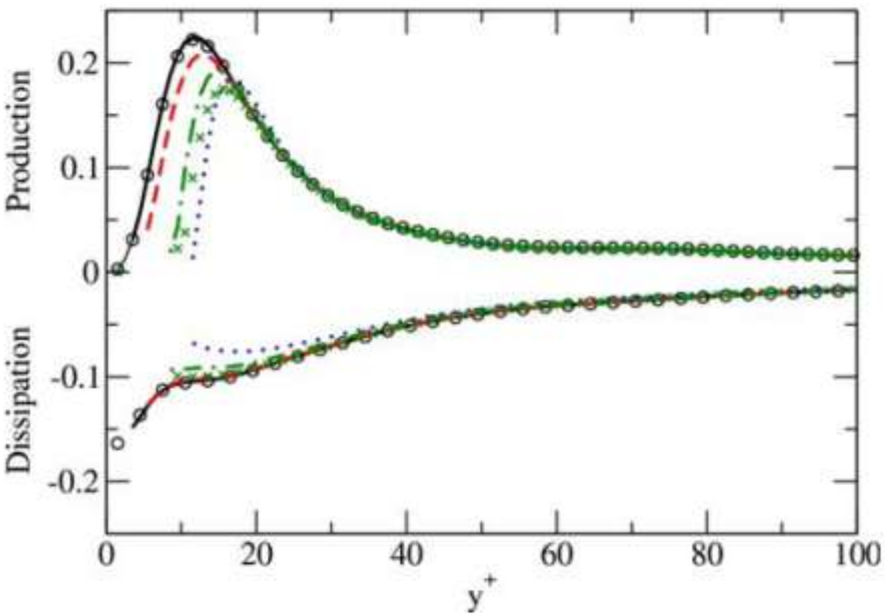


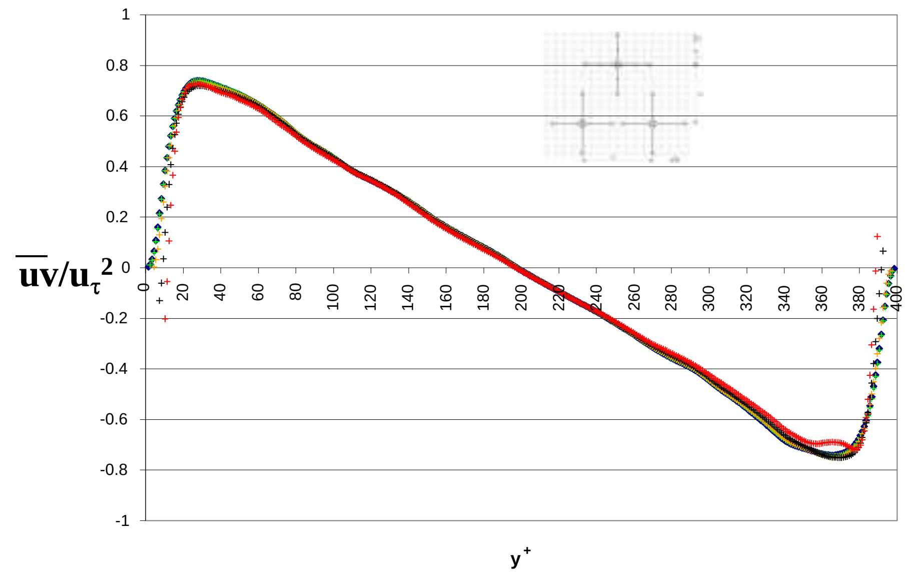
Dimensions in mm

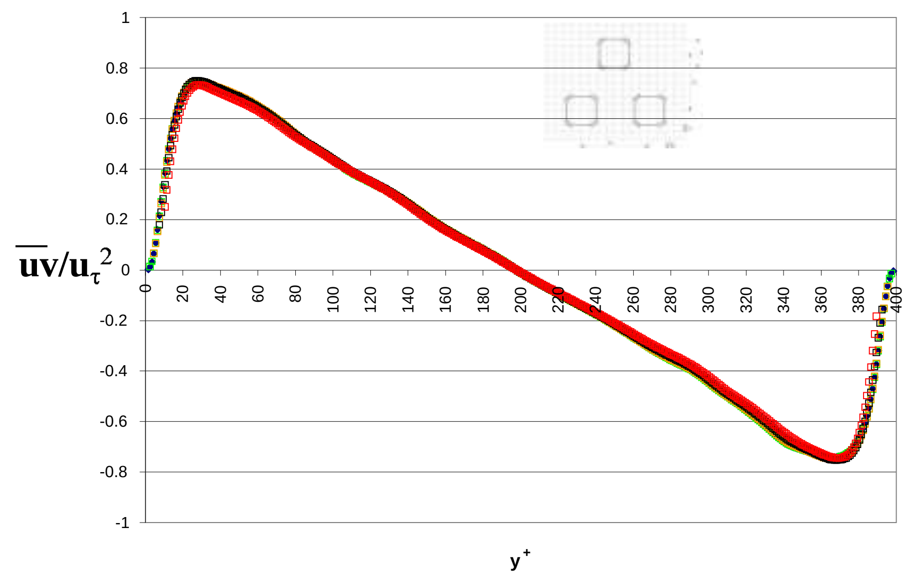


Plus and square configuration of 12- sensor probes to measure velocity and velocity gradient properties of turbulent flows

Turbulent Kinetic Energy Production & Dissipation and Reynolds shear stress







♦ **DNS**

$y^+ = 15$ → $= 150$

+ □ $S^+=2$ → 1.2 η → 0.6 η

+ □ $S^+=4$ → 2.4 η → 1.2 η

+ □ $S^+=8$ → 4.8 η → 2.4 η

+ □ $S^+=12$ → 7.2 η → 3.6 η