2010 Tutorial School on Fluid Dynamics: Topics in Turbulence
University of Maryland, May 24 - 28, 2010

Multiphase Flow Simulation Based on Unstructured Grid

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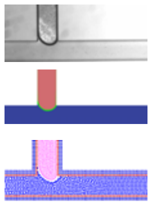
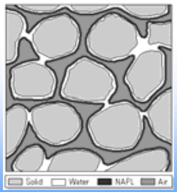
Outline

- Background
- Bubble Packing Method
- VOF method Based on the Unstructured Grid
- Remark

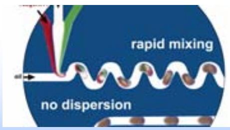
Background

- The difficulty to simulate multiphase flows is the presence of deforming interfaces
- Meshless method
  - SPH, MPS…
- Grid based method
  - Interface tracking method
  - Interface capturing method (VOF, Levelset)

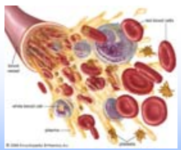




Organic liquids pollution in subsurface environment, Linda & Scott 1998



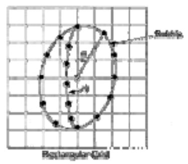
Helen Song et al.

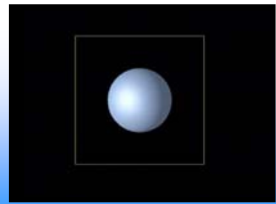


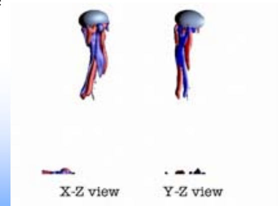
Microcirculation

Background

- Interface-tracking methods
  - Overdid and Tryggvason, 1992
  - explicitly treat the interface as a discontinuity. Usually, it is specified by an ordered set of marker points, connected by an interpolation curve. The markers are advanced in the Lagrangian manner and then redistributed to obtain the best resolution of the interface.
  - Difficult to deal with topology change



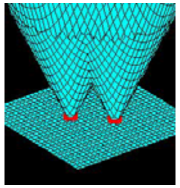




Background

- Interface capturing method – Level set
  - Osher & Sethian,1988
  - The idea is that instead of moving the red front, one might try and instead move the surface.

$$\frac{\partial \phi}{\partial t} + \vec{u} \cdot \nabla \phi = 0 \qquad \kappa = \nabla \cdot \frac{\nabla \phi}{|\nabla \phi|}$$



Sethian

- The curvature can be computed accurately and the smoothness of discontinuous physical quantities near interfaces is very good
- However, the LS method produces more numerical error, especially when the interfaces experience severe stretching or tearing. The conservation of mass is not guaranteed

Background

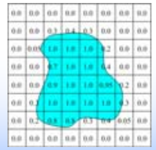
- In the VOF method, a volume fraction function f, whose value lies between 0 and 1, is defined to denote whether a space is occupied by the dispersed phase or continuous phase.
  - $f = 1$: dispersed phase
  - $f = 0$: continuous phase
  - $0 < f < 1$: contains both the dispersed and continuous phases

f distribution

- For a given flow field, the standard advection equation governs the evolution of f:

$$\frac{\partial f}{\partial t} + (\mathbf{u} \cdot \nabla) f = 0$$

- Accurate algorithms can be used to advect the volume fraction function so that the mass is conserved while still maintaining a sharp representation of the interfaces
- Because the volume fraction function f is a step function, it is difficult to obtain the accurate curvature and smooth the discontinuous physical quantities near the interfaces



Fluid distribution denoted by F

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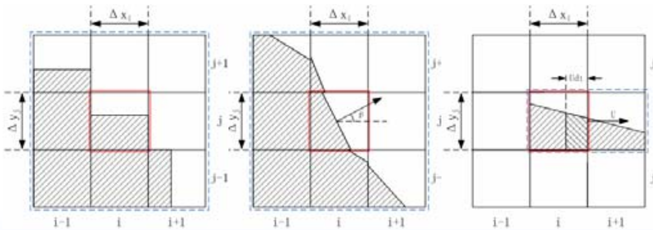
Background

- Interface reconstruction



SLIC Method (Noh & Woodward,1976)

PLIC Method (Youngs, 1982)

FLAIR Method (Ashgriz & Poo, 1991)

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Objective

- To develop high quality unstructured grid generation method
- To develop VOF method based on unstructured grid
- To analyze the influence of cell regularity on results

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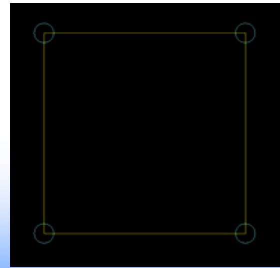
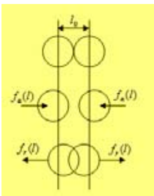
Bubble Packing Method

- K.Shimada, 1995
- Add bubbles with virtual mass into domain
- Define a force for each bubble to adjust the bubble's position, i.e., optimal node's distribution
- Bubble population control
  - To delete the overlapping bubbles
  - To add bubbles to fill the gaps
- Connect these nodes by Delaunay triangulation to generate the mesh



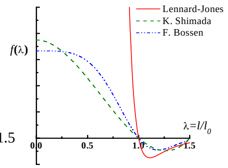
Bubble Packing Method



$l > l_0$: attractive force

$l < l_0$: repulsive force

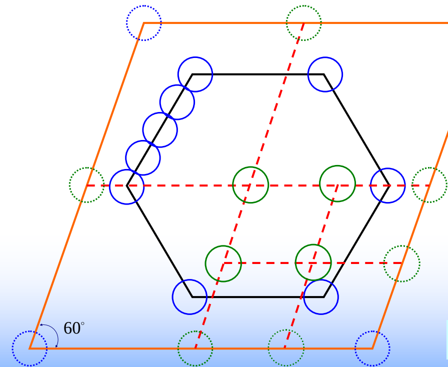
$$f(\lambda)=\begin{cases} l_0 \cdot k_0\left(\frac{5}{4}\lambda^3-\frac{19}{8}\lambda^2+\frac{9}{8}\right), & 0\le\lambda\le 1.5 \\ 0, & \lambda<0, \lambda>1.5 \end{cases}$$



$$m_i \frac{d^2\mathbf{x}_i(t)}{dt^2}+c_i\frac{d\mathbf{x}_i(t)}{dt}=\mathbf{f}_i(t) \qquad (i=1,2,\ldots,n)$$

Forth-order Runge-Kutta method is utilized to solve the bubble motion equations so as to obtain the optimal positions of bubbles

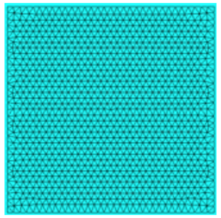
Bubble Packing Method



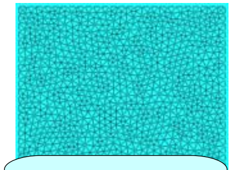
The conditions for further subdivision:

$$d-(r_1+r_2)\ge(r_1+r_2)/2$$

Bubble Packing Method



Bubble Packing Method

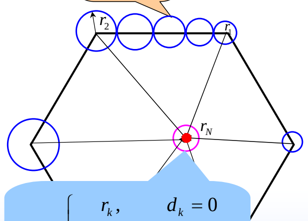


$$\eta_g = \frac{1}{m}\sum_{i=1}^{m}(0.5 - \frac{r_i}{R_i})$$

r/R	0~ 0.1	0.1~ 0.2	0.2~ 0.3	0.3~ 0.4	0.4~ 0.42	0.42~ 0.44	0.44~ 0.46	0.46~ 0.48	0.48~ 0.5	0.5	Total number	Geometric irregularity
D_S	0	0	0	29	11	63	221	646	1308	0	2278	0.02126210
BPM	0	0	0	16	12	24	64	146	704	1310	2276	0.00630635

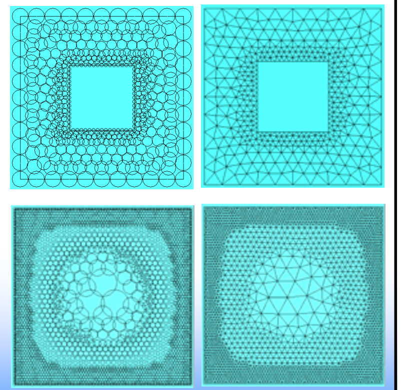
Bubble Packing Method

$$r_N = \frac{r_1 + r_2}{2}$$

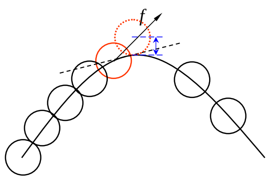


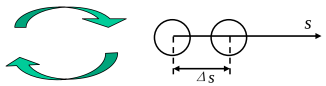
$$r_N = \begin{cases} r_k, & d_k = 0 \\ \sum_{k=1}^{m} \omega_k \cdot r_k & d_k \neq 0 \end{cases}$$

$$\omega_k = \frac{1}{d_k^2} \Big/ \sum_{k=1}^{m} \frac{1}{d_k^2}$$



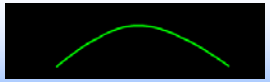
Bubble Packing Method



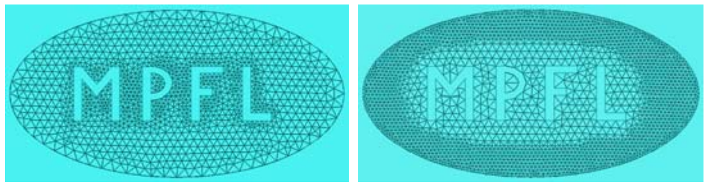


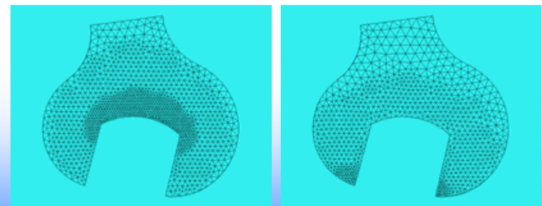
$$\begin{cases} x = \varphi(t) \\ y = \phi(t) \end{cases} \quad (a \le t \le b)$$

$$s(t) = \int_a^t \sqrt{\varphi'^2(t) + \phi'^2(t)}\,dt$$

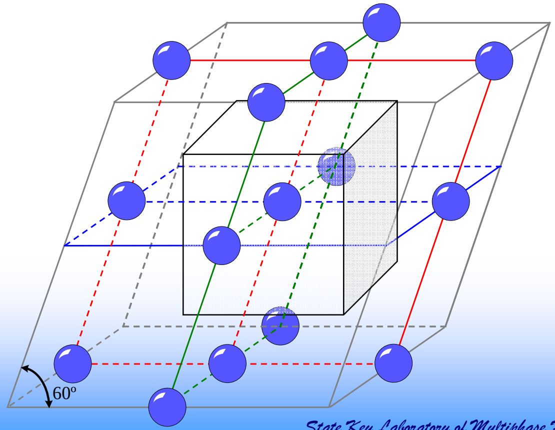


Bubble Packing Method

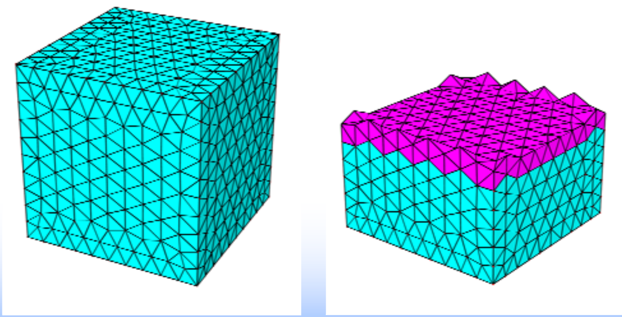




Bubble Packing Method



Bubble Packing Method



Outline

- Background
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- VOF method Based on the Unstructured Grid
- Remark

VOF

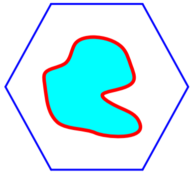
$$\frac{\partial f}{\partial t}+(\mathbf{u}\cdot\nabla)f=0$$

$$\frac{\partial}{\partial t}\iint_{\Delta S_i} f\,\mathrm{d}S+\sum_{k=1}^{3}\int_{l_k}(\mathbf{u}\cdot\vec{n}_k f)\,\mathrm{d}l=0,\quad k=1,2,3$$

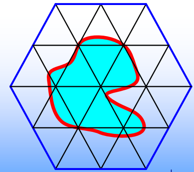
$$\left(\iint_{\Delta S_i} f\,\mathrm{d}S^{n+1}-\iint_{\Delta S_i} f\,\mathrm{d}S^{n}\right)+\sum_{k=1}^{3}\int_{l_k}(f\cdot u_k\,\mathrm{d}t)\,\mathrm{d}l=0$$

$$F_i^{n+1}-F_i^{n}=\frac{1}{\Delta S_i}\sum_{k=1}^{3}\int_{l_k}(f\cdot u_k\,\mathrm{d}t)\,\mathrm{d}l=F_i^{n}-\sum_{k=1}^{3}\Delta F_{i,k}$$

Problem :
orientation and location of the interface?

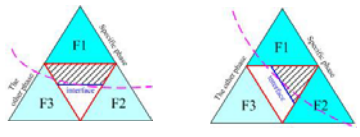


$$F_i=\frac{1}{\Delta S_i}\iint_{\Delta S_i} f(x,y,t)\,\mathrm{d}s$$

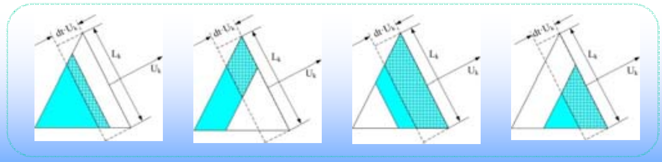


VOF

- SLIC – VOF :
  A linear segment is set through each cell as the interface parallel with one side of the cell
- To judge the interface normal:
- K=(F1−F2)−(F2−F3)
  (F1>F2>F3)

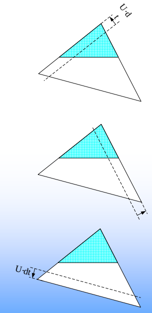


K>0, (F1 >> F2, F3)   K<0, (F1, F2 >> F3)

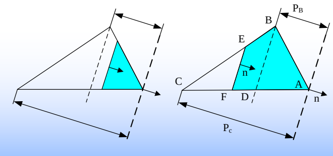


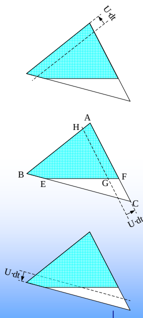
VOF

- PLIC – VOF : A piecewise linear segment defined by a slope and intercept is set through each cell as the interface
- The slope of the line is given by the interface normal
- the intercept follows from invoking volume conservation



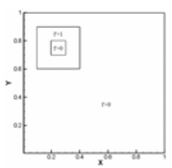
$$\vec{n} = \frac{\nabla f}{|\nabla f|} \quad \begin{cases} n_x = \frac{2}{3} \cdot \sum_{j=1}^{3} \frac{(x_j - x_0)\cdot(F_j - F_0)}{(x_j - x_0)^2 + (y_j - y_0)^2} \\ n_y = \frac{2}{3} \cdot \sum_{j=1}^{3} \frac{(y_j - y_0)\cdot(F_j - F_0)}{(x_j - x_0)^2 + (y_j - y_0)^2} \end{cases}$$





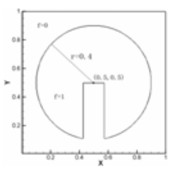
$B \notin$ specific phase $B \in$ specific phase

VOF



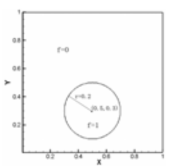
$u = 0.4$
$v = -0.4$

1. Advection model
(A hollowed square in advection)



$u = -\pi(y - 0.5)$
$v = \pi(x - 0.5)$

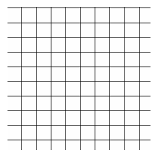
2. Zalesak model
(Rotation of a slotted circle)



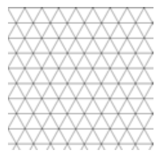
$u = -\pi \cdot \cos\pi(x - 0.5)\sin\pi(y - 0.5)$
$v = \pi \cdot \sin\pi(x - 0.5)\cos\pi(y - 0.5)$

3. Shearing Flow
(A circle in shearing flow)

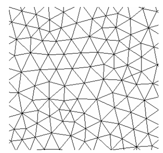
VOF



Structured grid
(STR, 6400cells)



Bubble Packing
(BPM, 7600)



Delaunay Triangulation
(DTM, 7600)

Relative distortion

$$E_d = \frac{\sum_i \left| f_i^e(T) - f_i^n \right|}{\sum_i f_i^0}$$

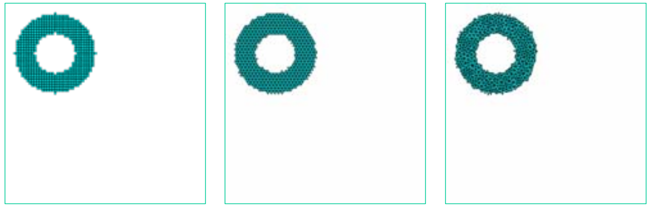
Geometrical error

$$E_g = \sum_i \left(\Delta S_i \left| f_i^e(T) - f_i^n \right| \right)$$

Relative mass conservation error

$$E_m = \frac{\left| \sum_i f_i^e(T) - \sum_i f_i^n \right|}{\sum_i f_i^0}$$

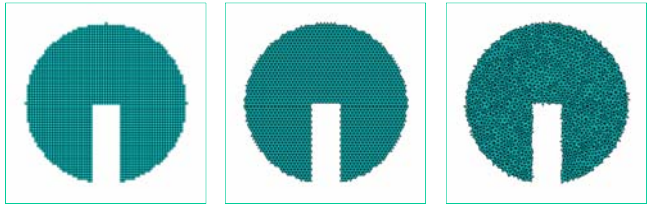
VOF



STR, 6400 cells | BPM, 7600 cells | DTM, 7600 cells

Grid Type	E_d	E_g	E_m
STR	2.35e-1	2.21e-2	-3.24e-3
BPM	9.21e-2	8.68e-3	-5.00e-3
DTM	9.63e-2	9.05e-3	-3.07e-2

VOF



STR, 6400 cells | BPM, 7600 cells | DTM, 7600 cells

Grid Type	E_d	E_g	E_m
STR	5.17e-2	2.31e-2	-1.99e-2
BPM	5.48e-2	2.44e-2	-2.55e-2
DTM	1.07e-1	4.77e-2	-1.00e-1

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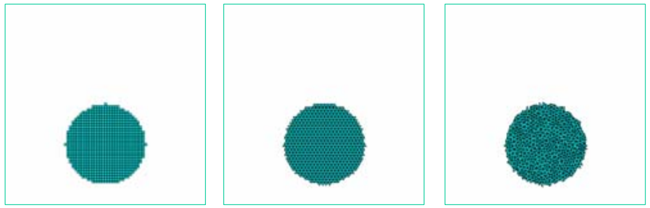
VOF



STR, 6400 cells | BPM, 7600 cells | DTM, 7600 cells

Grid Type	E_d	E_g	E_m
STR	6.52e-2	8.16e-3	-2.02e-2
BPM	1.64e-1	2.04e-2	-1.20e-1
DTM	2.67e-1	3.35e-2	-2.65e-1

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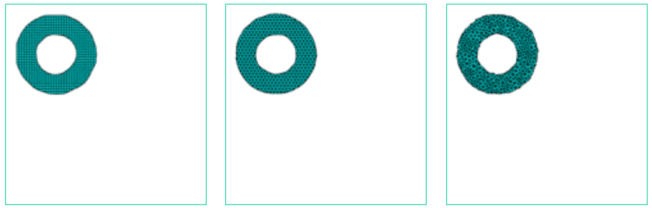
VOF



STR, 6400 cells | BPM, 7600 cells | DTM, 7600 cells

Grid Type	E_d	E_g	E_m
STR	5.89E-02	5.53E-03	1.64E-07
BPM	5.42E-02	5.11E-03	-4.01E-03
DTM	6.26E-02	5.88E-03	-3.74E-03

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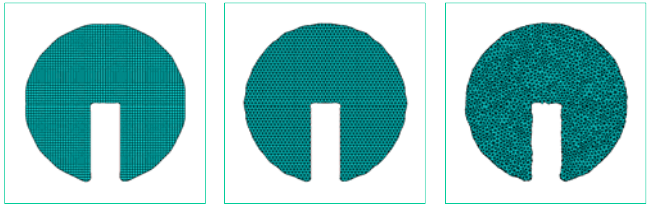
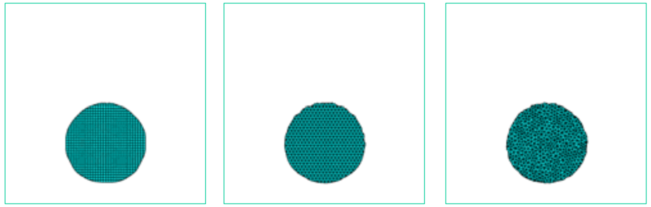
VOF



STR, 6400 cells   BPM, 7600 cells   DTM, 7600 cells

Grid Type	E_d	E_g	E_m
STR	1.86E-02	8.21E-03	1.65E-06
BPM	2.51E-02	1.12E-02	-1.04E-02
DTM	3.16E-02	1.40E-02	-2.34E-02

VOF



STR, 6400 cells   BPM, 7600 cells   DTM, 7600 cells

Grid Type	E_d	E_g	E_m
STR	2.91E-02	3.67E-03	3.43E-05
BPM	6.61E-02	8.26E-03	-5.59E-02
DTM	1.16E-01	1.45E-02	-1.13E-01

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Thank you for your attention