Stalking: Aggressive Shadowing of a Noisy Trajectory

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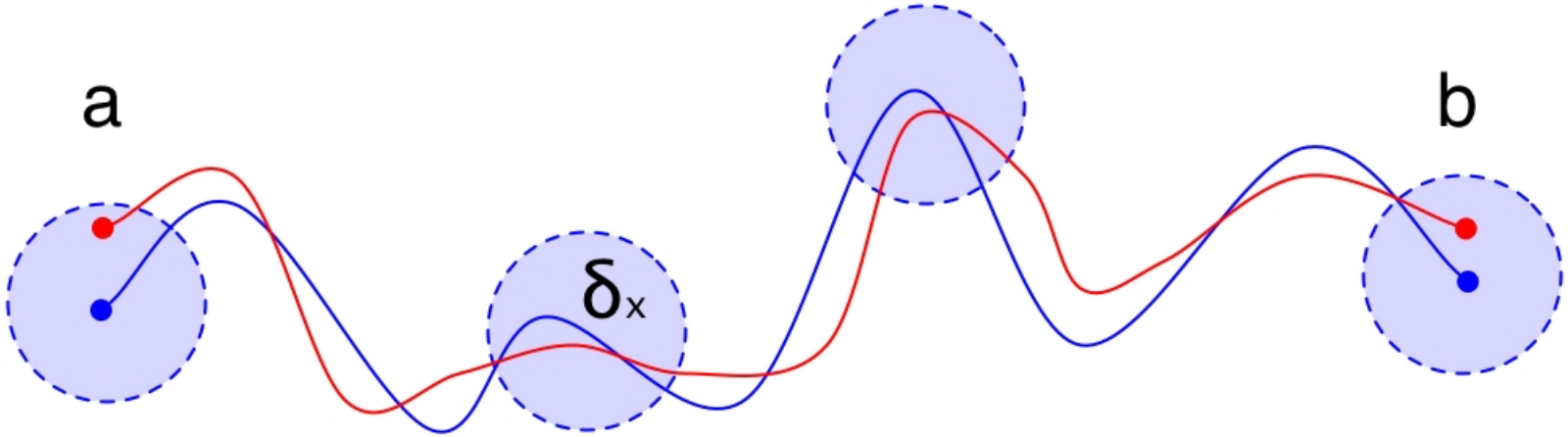
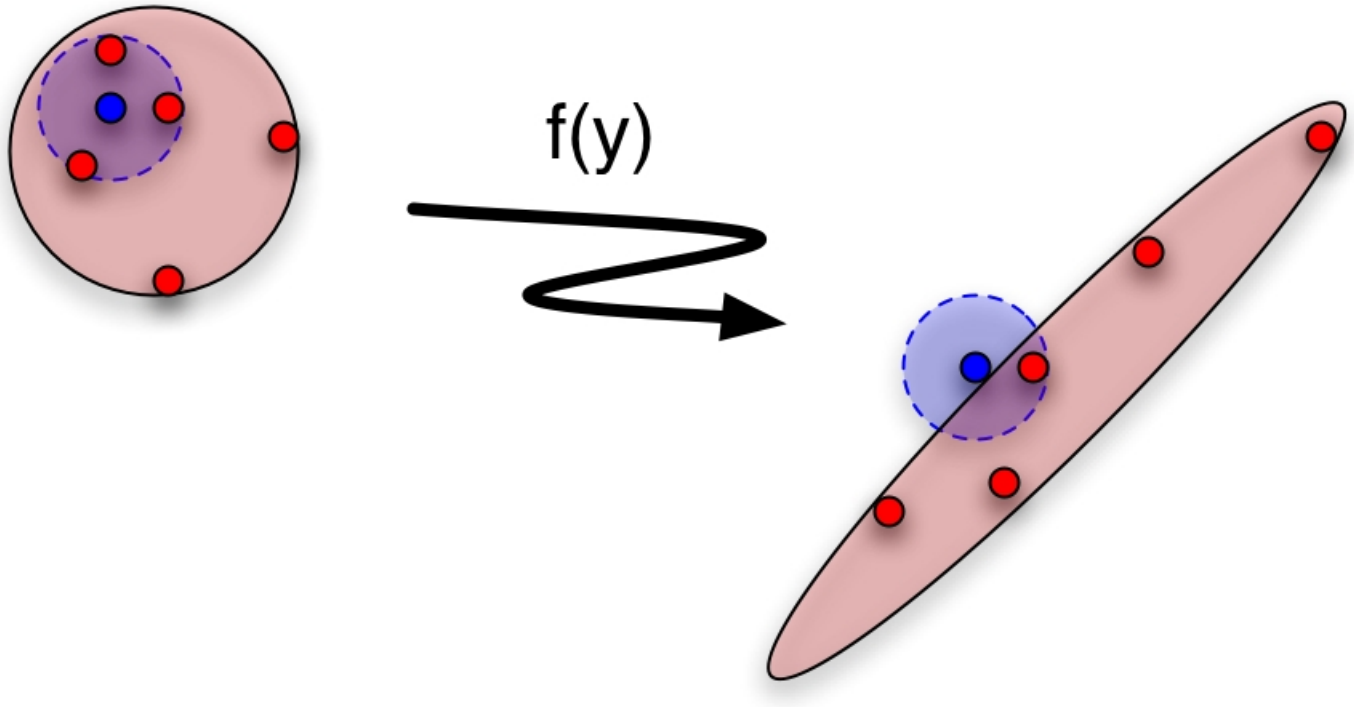
δ_x-Shadowing Definition

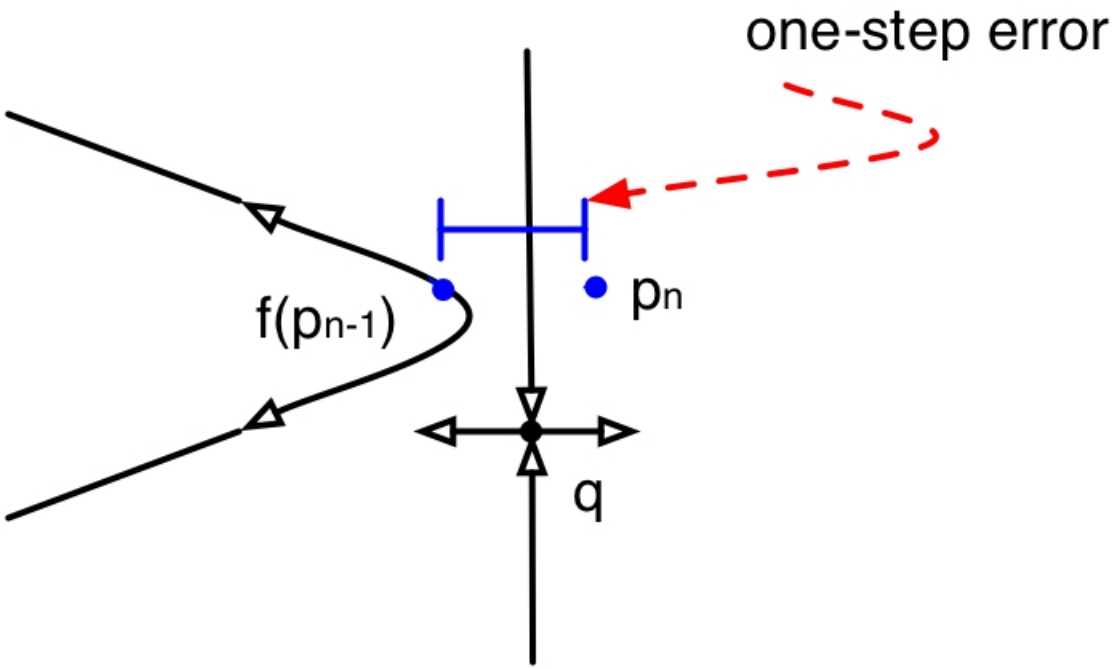
Given a system $y_{n+1} = f(y_n)$, a tolerance δ_x, and a sequence $\{p_n\}_{n=a}^{b}$, a true orbit $\{x_n\}_{n=a}^{b}$, $x_{n+1} = f(x_n)$, δ_x-*shadows* $\{p_n\}_{n=a}^{b}$ on $a \leq n \leq b$ if $|x_n - p_n| < \delta_x$



Noise may push the target solution outside of the ensemble ellipse. This is fine, so long as *some* ensemble member is still consistent with the target (red and blue overlap).



However, if a contracting direction begins to expand, or there is a near tangency of the stable and unstable manifolds, we may have a shadowing failure or *glitch*.



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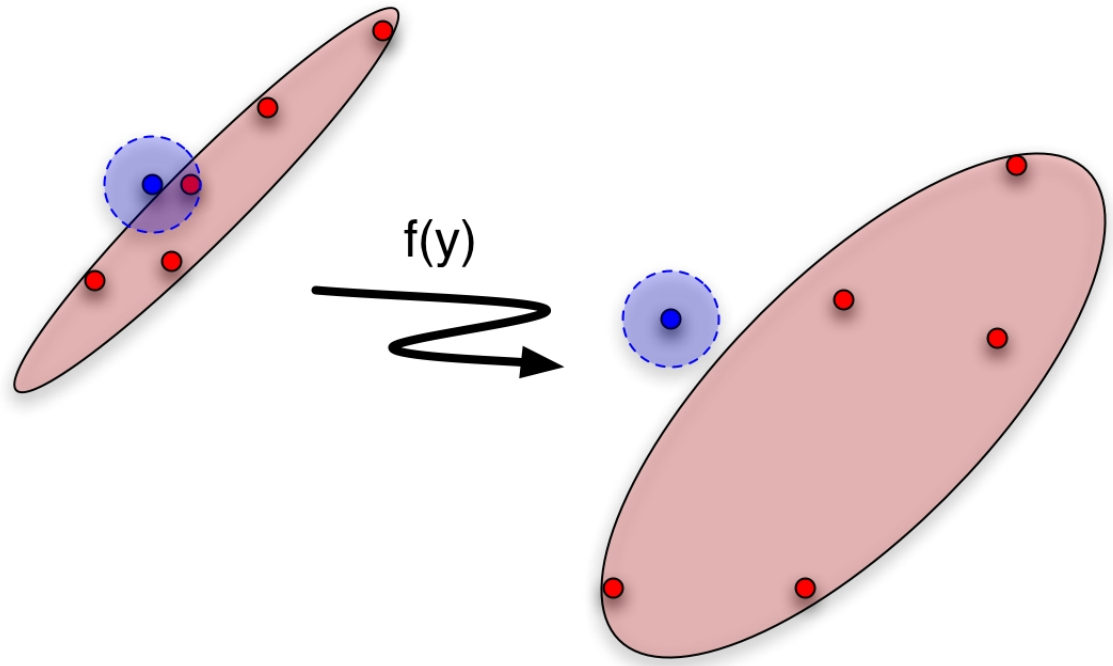
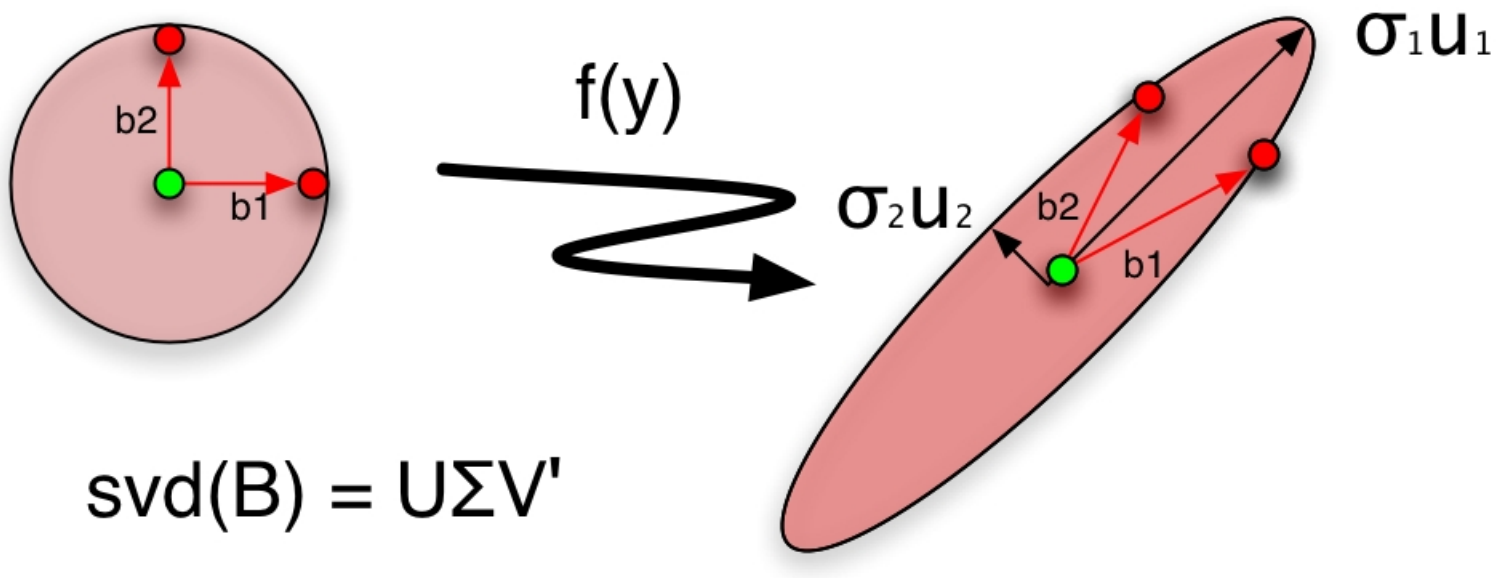
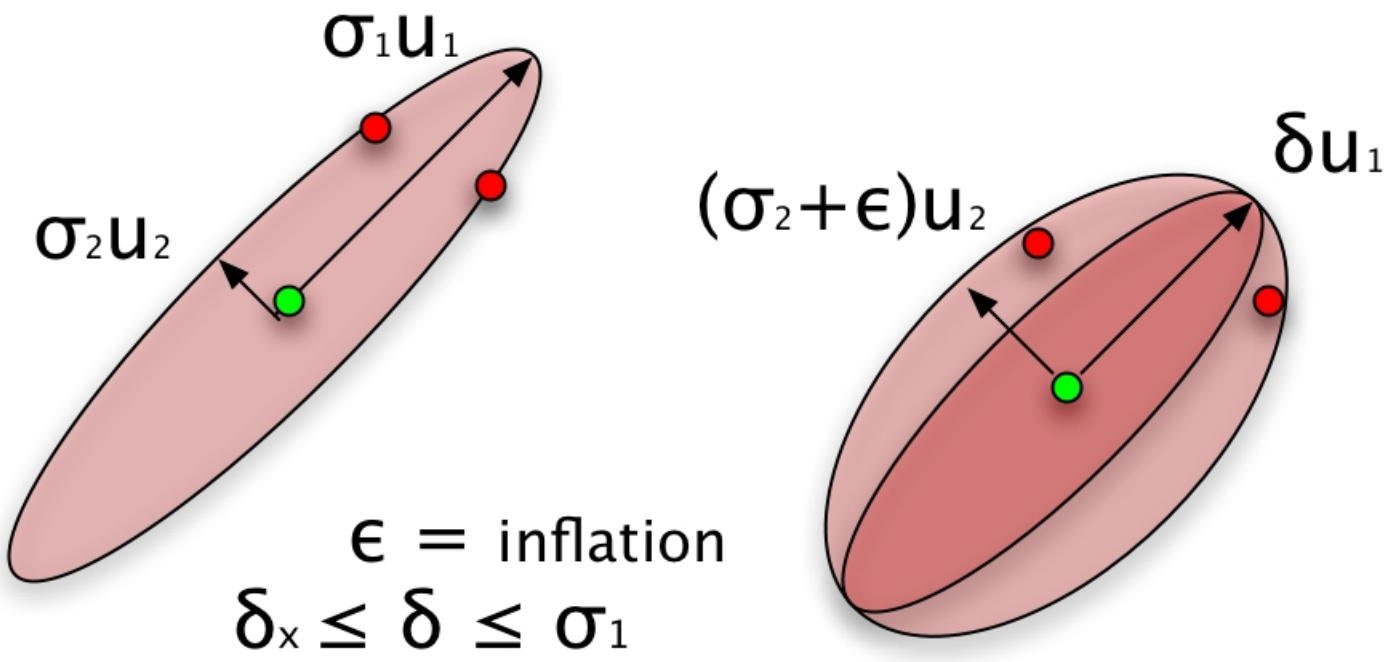


A few words about Breeding

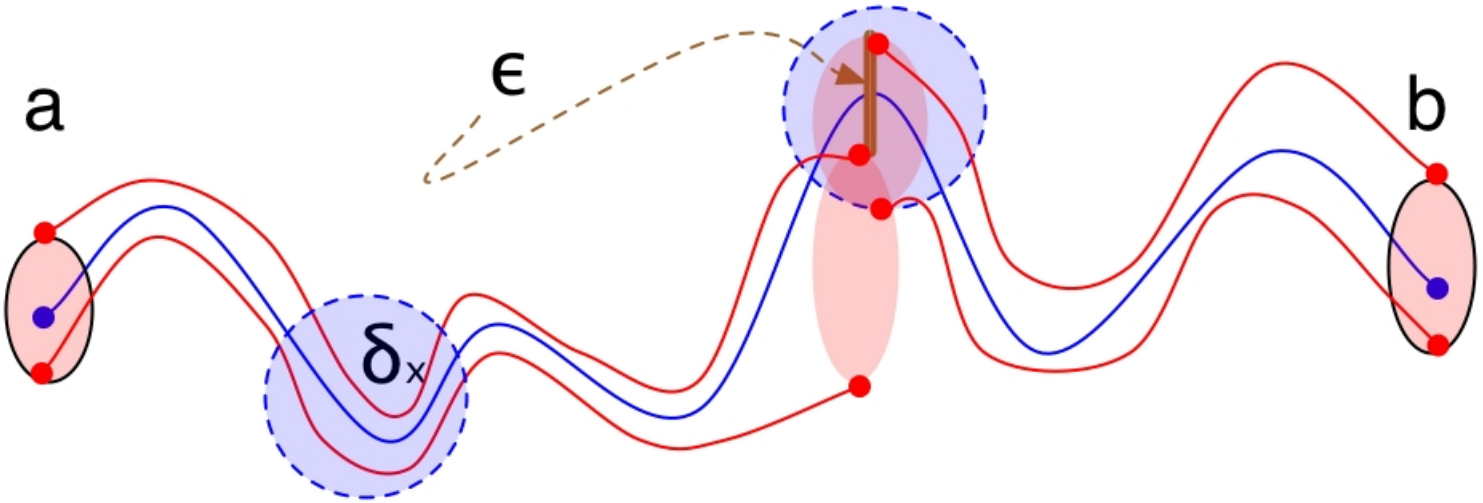


Ensemble variance inflation



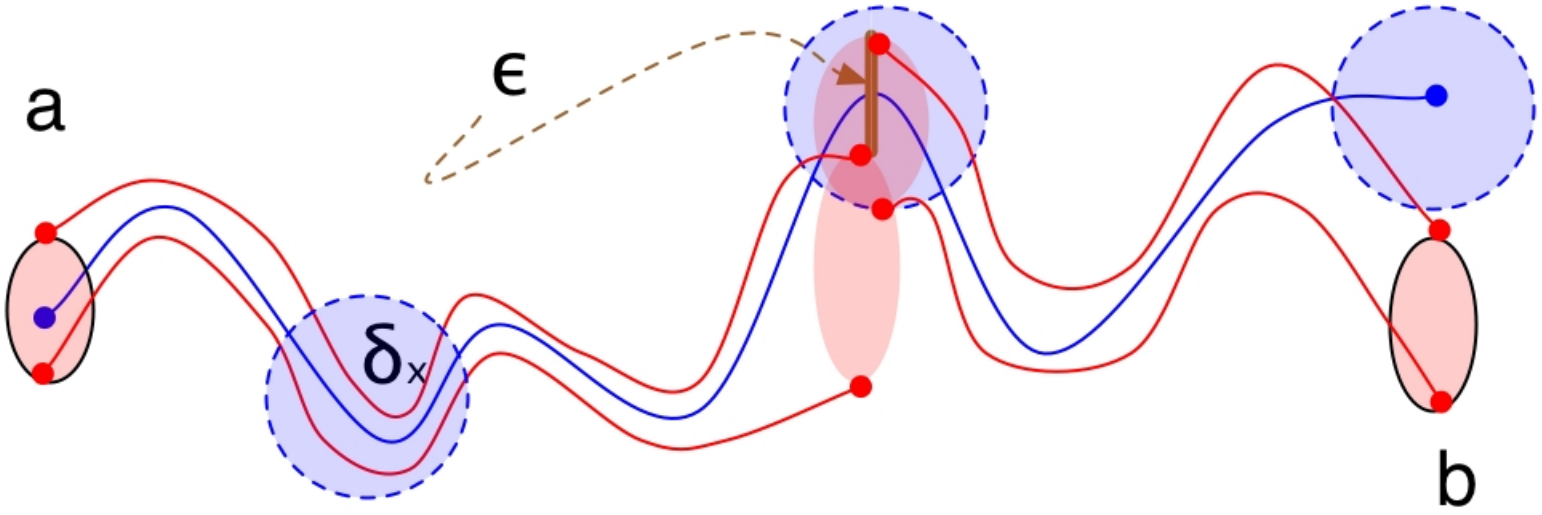
δ_x-Stalking Definition

Given a system $y_{n+1} = f(y_n)$, a tolerance δ_x, and a sequence $\{p_n\}_{n=a}^{b}$, an ε-pseudo orbit $\{x_n\}_{n=a}^{b}$, $|x_{n+1} - f(x_n)| < \varepsilon$, δ_x-*stalks* $\{p_n\}_{n=a}^{b}$ on $a \le n \le b$ if $|x_n - p_n| < \delta_x$

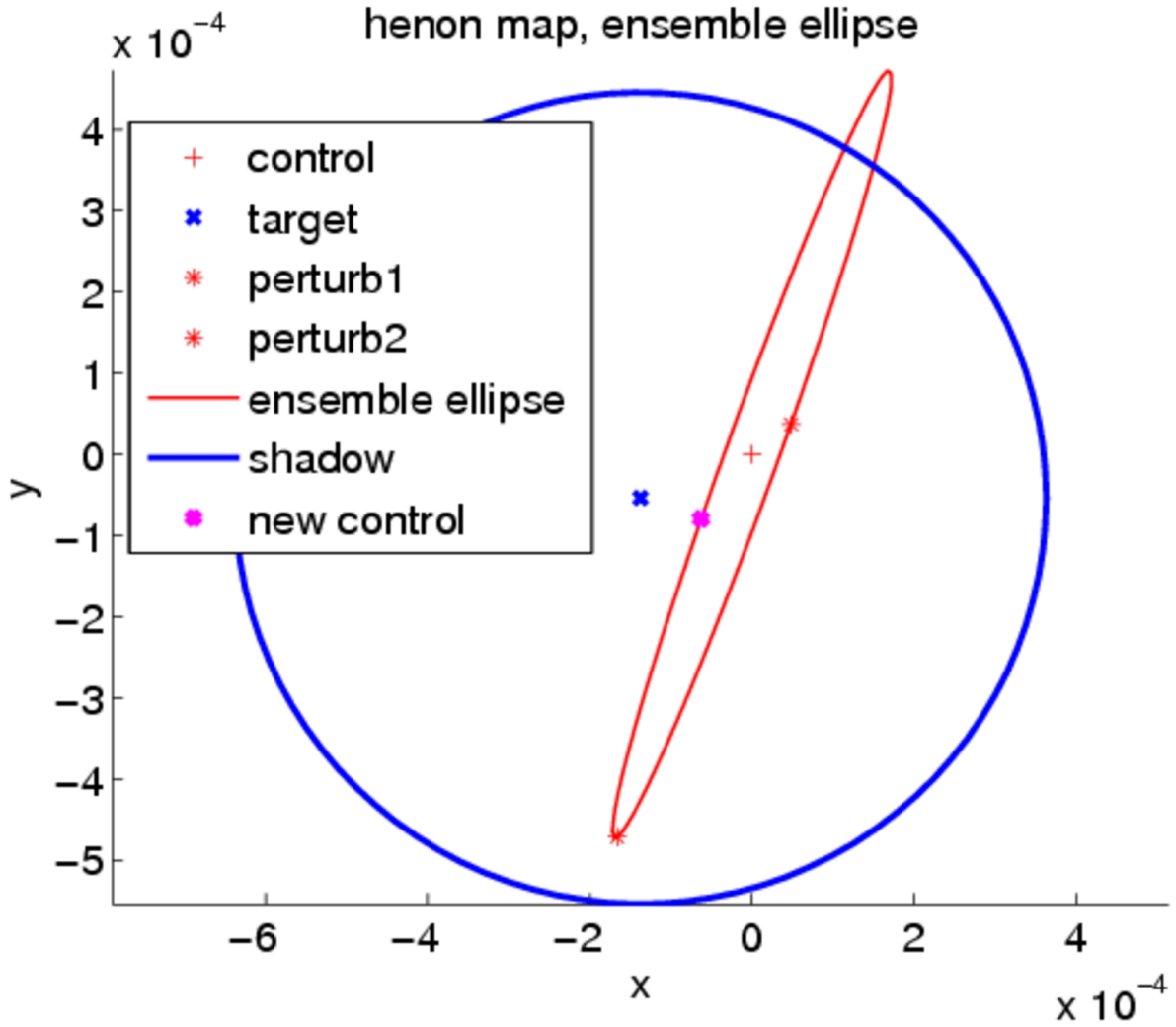


δ_x-Stalking Failure

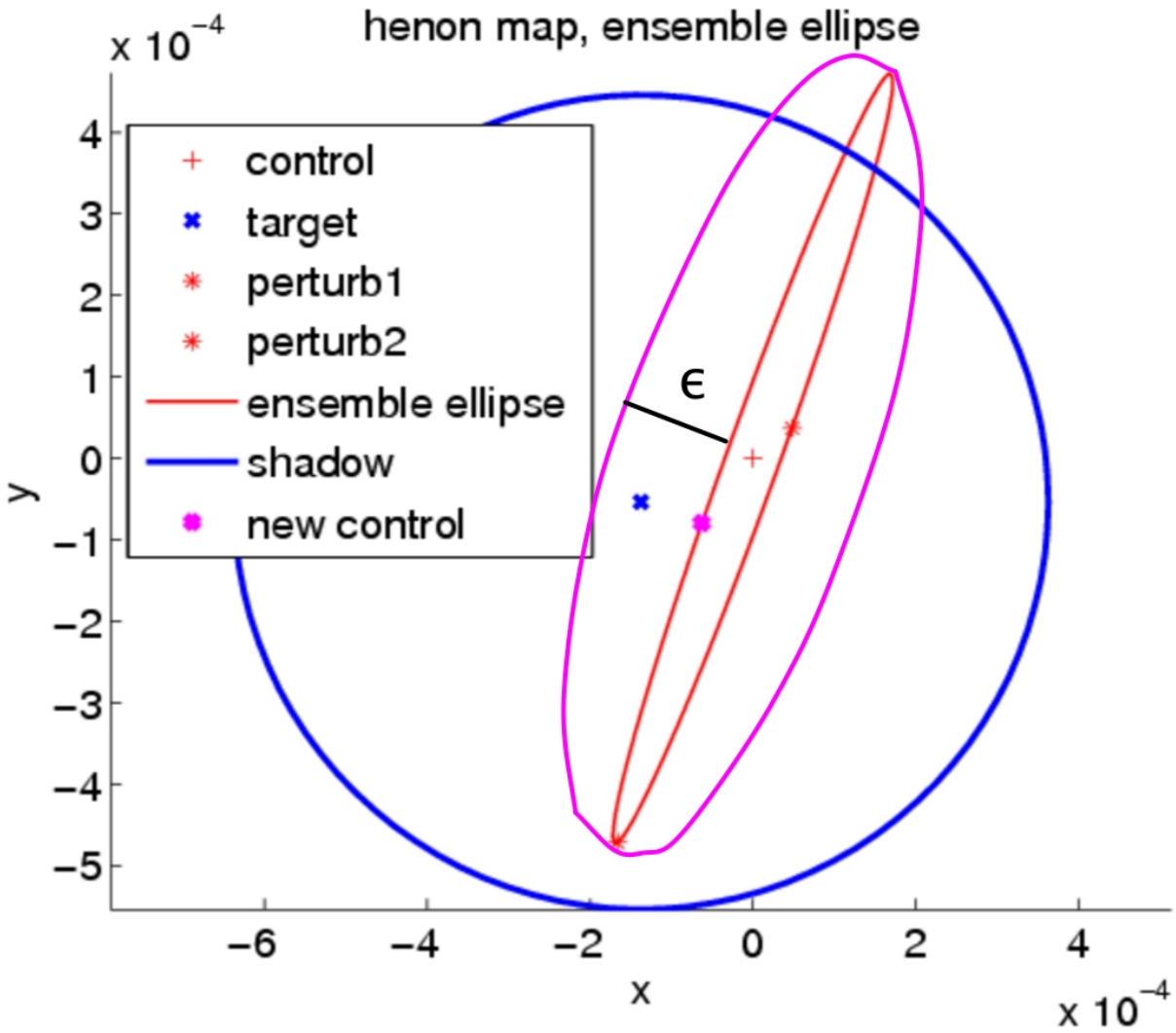
Given a system $y_{n+1} = f(y_n)$, a tolerance δ_x, and a sequence $\{p_n\}_{n=a}^{b}$, an ε-pseudo orbit $\{x_n\}_{n=a}^{b}$, $|x_{n+1} - f(x_n)| < \varepsilon$, δ_x-*stalks* $\{p_n\}_{n=a}^{b}$ on $a \le n \le b$ if $|x_n - p_n| < \delta_x$



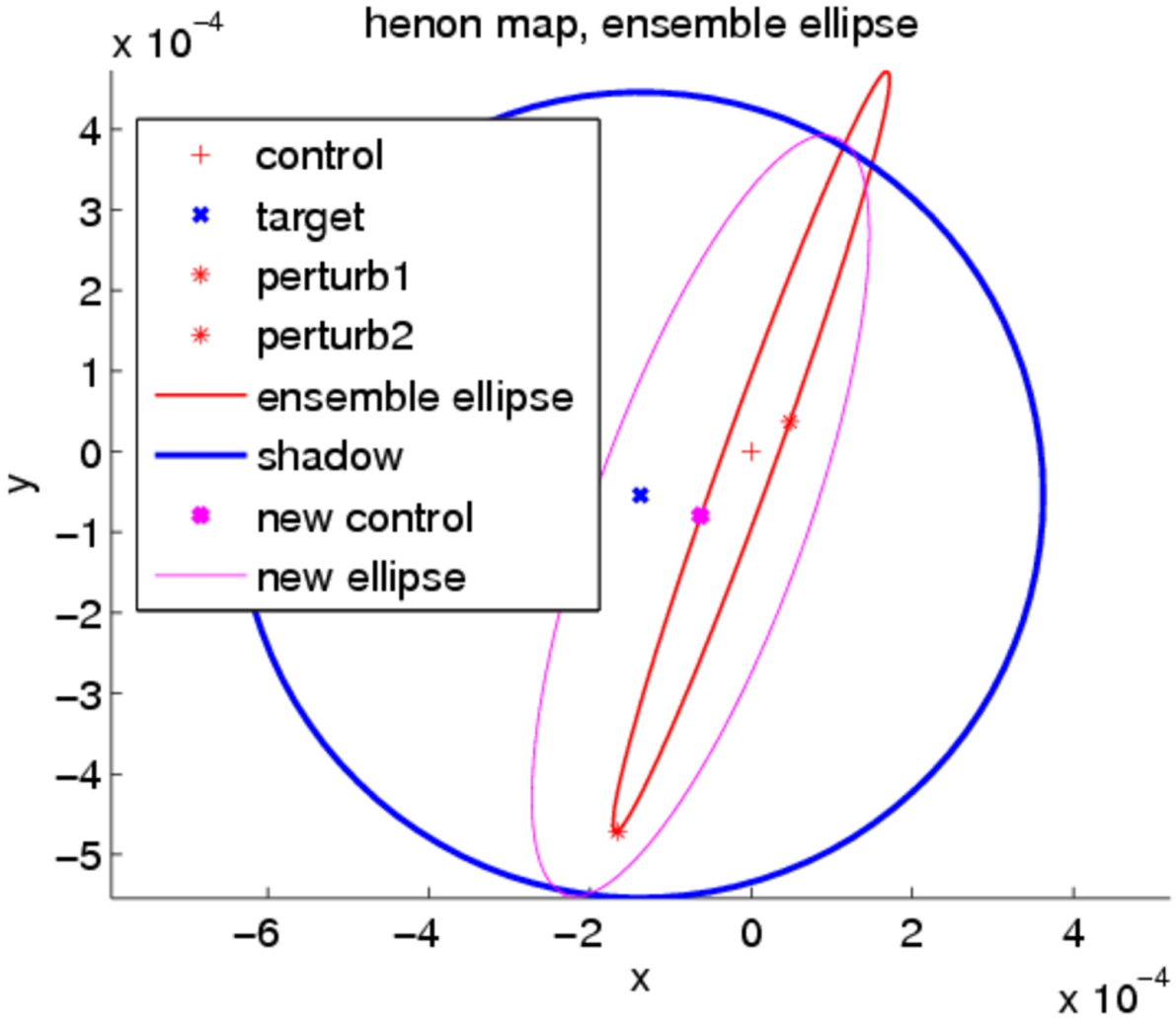
Iteration of the Henon map leaves the truth outside ellipse



Step 1: inflate contracting directions by ε



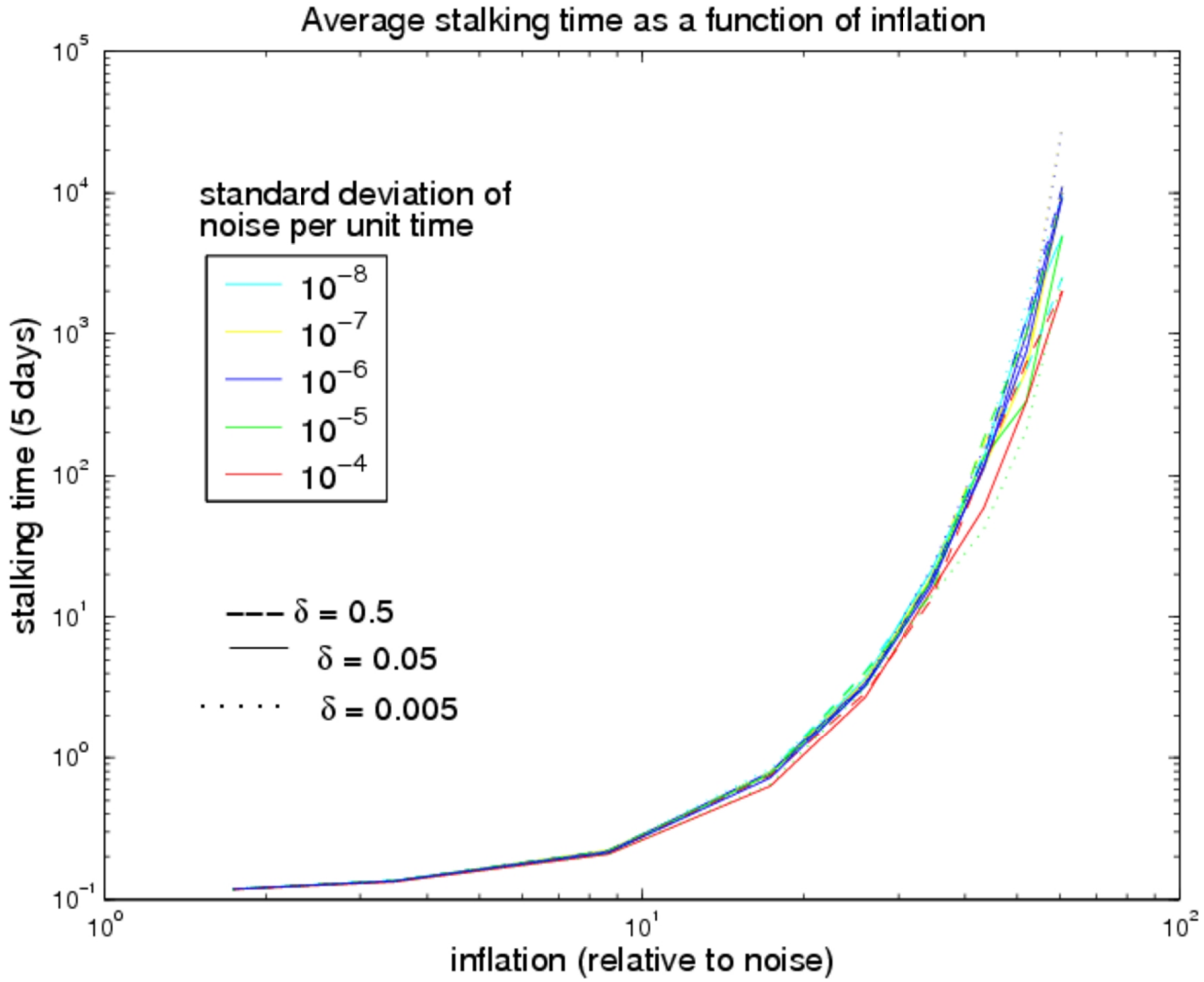
Step 2: rescale expanding directions



Goal

- Fixing the following parameters:
  - variance inflation (ε)
  - rescale of expanding axes ($\delta_x \leq \delta \leq \sigma_i$)
  - stalking distance (δ_x)
  - noise (δ_f), where $|p_{n+1} - f(p_n)| < \delta_f$
- How long will x_n stalk p_n?

Stalking time in Lorenz 40-D system for parameters ε (inflation), δ_f (noise), δ (rescale). Failure occurs when target leaves ellipse.



Conclusion

- Agreement across $O(\delta_f)$ (noise) and $O(\varepsilon)$ (ensemble variance inflation) indicates a substantial relationship between stalking time and parameters.
- Stalking solutions to the Lorenz 40-D model require inflation $>>$ noise, what if we increase the stalking distance?

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